Statistical Properties of Turbulent Flows

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Turbulent flows are everywhere: the atmosphere, the oceans, the solar wind and solar convection zone, the interstellar medium and beyond. Physical conditions are vastly different but they share a property of having large Reynolds number $Re = U_0 L_0/v$ where U_0 and L_0 are characteristic velocity and length scale and v is the viscosity; the ratio of nonlinear inertial to viscous accelerations, Re is the governing parameter of the equations of motion in the simplest case (one may also have to consider the Rossby number measuring rotation, the Nusselt number measuring convective flux, the Froude number for stratified flows, the Prandtl number measuring relative dissipative processes of say the temperature or the magnetic field vs. the velocity, etc.). When Re >> 1, a very large number of modes are a priori relevant and a major problem of turbulence is hence to solve for the statistical properties of such flows [1]. Progress can be made using a variety of techniques and contrasting them: theory, models, experiments, observations, and numerical simulations, either direct -- i.e., dealing with the primitive equations of motion, DNS -- or Large Eddy Simulations (LES) using some form of modeling for the unresolved small scales of the flow under study.

 In this context, we shall describe results stemming from both high-resolution DNS and models of LES of three-dimensional Navier-Stokes and magnetohydrodynamic (MHD, including coupling to a magnetic field) and pose several questions that may be relevant from a statistical point of view ([2] and references therein). Different initial conditions and/or forcing functions will be considered (Taylor-Green, Beltrami (ABC), Orszag-Tang, and ABC plus random fluctuations in the small scales), and resolutions of up to 1536^3 regularly spaced grid points will be employed. At each grid point, the three components of the velocity field, and in the MHD case, of the magnetic induction, are computed over several characteristic times, leading to data sets of substantial size. As examples, three problems are:

- **The problem of universality of statistical properties of turbulence, at either small or large scales or both:** Take a flow at a given **Re** at a given resolution of 512^3 points, and examine three snapshots; are they different in their statistical properties, specifically in their distribution function or its parameters? The three sets can come from three different times of a same statistically-steady run, or from three DNS runs differing in regard to initial condition, forcing, omission of certain dynamics, or otherwise. What about statistics of the energy dissipation?

- **Parameter optimization:** Take a 1024^3 Navier-Stokes DNS; find the optimal value, in a norm to be defined, for the parameter a_v of the Lagrangian Averaged model at a given (lower) resolution (see [3] for a rapid derivation of the model, and references therein). Does the optimal choice of a_v change with **Re**? Then do the same thing for MHD, with now two parameters $a_{v,m}$; in that latter case, what is the optimal ratio of a_v / a_m as a function of the magnetic Prandtl number $P_M = \nu\eta$, where η is the magnetic diffusivity?

- **Model-type optimization:** Given a high-resolution DNS together with several (related) LES (Leray, Clark, *a* models [4]; Smagorinsky?) at smaller resolutions but all at the same **Re**, can you differentiate between them statistically? Which model appears "best," in some measure to be determined? Are they criteria to determine the "best" measure? What about scaling with Reynolds number?

References

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