

# Uncertainties of estimates of inertio-gravity energy in the atmosphere



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## Motivation

- Divergent tropical circulations crucial for understanding the climate but unreliable from present (re)analysis
- Unclear how large part of the global atmospheric energetics pertains to the divergent motion i.e. inertio-gravity waves.
- Large-scale equatorial waves in recent years diagnosed from different mass-field observations and models, but exact quantification of their variance and dynamical relevance not completely understood.

## Objectives

- Apply normal mode functions (NMFs) to the analysis and forecast fields to quantify percentage of energy contained in balanced (Rossby) and inertio-gravity (IG) motions.
- Estimate energy spectra and propagation properties for the large-scale IG motion, with emphasis on the equatorial waves. In particular, how much of the large-scale tropical variability is associated with the Kelvin wave (KW), mixed Rossby-gravity (MRG) wave, other IG waves?

## Datasets

Four analysis datasets for July 2007, global fields every 6 hours

- DART/CAM: ensemble mean of an 80-member ensemble produced by the DART system. For details see <http://www.image.ucar.edu/DARes/DART/>. The CAM version is 3.1, horizontal resolution T85, 26 vertical levels up to 3.5 hPa.
- ECMWF operational analyses: 12-hour 4D-Var system, Cycle 32r2, T799 interpolated to N64 grid, 91 vertical level up to 0.01 hPa.
- NCEP operational analyses: grid-point 3D-Var system, T382 interpolated to N64 grid, 64 vertical levels up to 0.32 hPa.
- NCEP/NCAR reanalyses: 3D-Var system, T62 interpolated to T47 grid, 28 vertical levels up to 2.7 hPa. An old system compared to the operational NCEP data.

## Normal mode expansion

Applied set of orthogonal modes was derived by Kasahara and Puri (MWR, 1981). Basic idea in the present application is to select the subset of modes which provides the optimal fit (best correlation and variance fit to the input grid-point fields) ⇔ tuning of the truncation parameters  $N_v, N_m, N_k$

- $N_m$  – number of vertical modes, index  $m$
- $N_v$  – number of meridional modes per wave type, index  $v$
- $N_k$  – number of zonal waves, index  $k$

$$\mathbf{X}(\lambda, \phi, z, t) = \sum_{m=1}^{N_m} \sum_{v=0}^{N_v-1} \sum_{k=-N_k}^{N_k} \chi_{kmv}(t) \mathbf{S}_m \Pi_{kmv}(\lambda, \phi, z)$$

input data vector      common expansion coefficient      normalization matrix

$$\mathbf{X} = (u, v, P/g)^T \cdot P = gz + RT_v \ln(p_p)$$

$$\Pi_{kmv}(\lambda, \phi, z) = \Phi_m(z) \cdot H_{kmv}$$

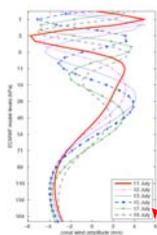
vertical normal modes      Hough functions

$$\langle \Pi_{kmv}, \Pi_{k'v'm'} \rangle = \delta_{kk'} \delta_{mm'} \delta_{vv'}$$

orthogonal 3D expansion basis

$$\mathbf{S}_m = \begin{pmatrix} (gH_m)^{3/2} & 0 & 0 \\ 0 & (gH_m)^{3/2} & 0 \\ 0 & 0 & H_m \end{pmatrix}$$

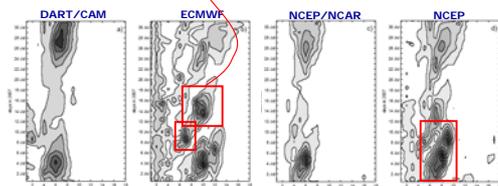
## Modal-space diagnosis of the vertical energy propagation by the Kelvin waves



The best agreement between the datasets is for the Kelvin wave. Below is shown the zonal wavenumber  $k=1$  Kelvin wave evolution in July 2007.

On the left is the zonal wind component in the Kelvin wave at subsequent days showing the downward phase propagation (the ECMWF system).

The difference in the depth of the atmosphere in DART/CAM and NCEP/NCAR on one hand and ECMWF and NCEP on the other appears to be one reason for different propagation properties as well as energy levels in datasets.

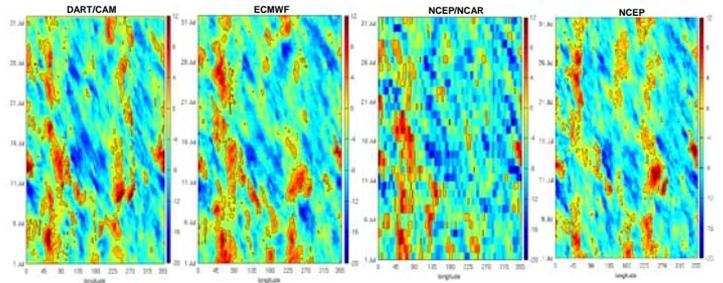


## Conclusions

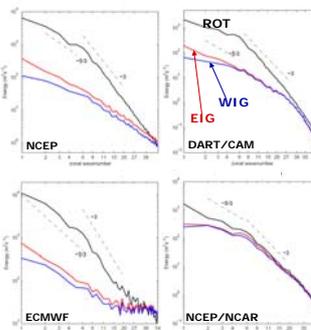


- The normal mode expansion allows us to quantify energy in various motions and to modify the traditional view of inertio-gravity motions as junk.
- The results show that the percentage of IG motion in the present NCEP, ECMWF and DART/CAM analysis systems is between 1% and 2% of the total energy field. In the wave part of the flow (zonal wavenumber  $k > 0$ ), the IG energy contribution is between 9% and 15%. On the contrary, the NCEP/NCAR reanalyses contain more IG motion, especially in the Southern Hemisphere extratropics.
- Each analysis contains more energy in the eastward IG motion than in its westward counterpart. The difference is about 2% to 3% of the total wave energy and it is associated with the motions projected onto the Kelvin wave in the tropics. The Kelvin wave contribution to the IG wave energy varies between 7% and 25%. The contribution of the MRG modes varies between 4% and 15%.
- The inverse projection shows that the bulk of the IG motion is confined to the tropics. The average tropical IG circulation in July 2007 is characterized by reverse flows in the upper and lower troposphere consistent with the ideas behind simple tropical models. For the successful reproduction of this circulation by the normal modes it is important that the expansion includes many vertical modes based on realistic temperature and stability profiles.

## Tropical winds in 4 analysis datasets in July 2007 zonal winds at 370 hPa, along 5°N



## Average energy spectra for balanced (ROT), eastward- (EIG) and westward (WIG) inertio-gravity motions



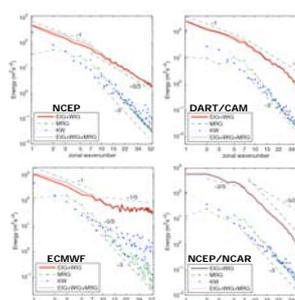
Averaging over one month

$$\sum_k \sum_n \sum_m g H_{eq} |\chi_{km}|^2 \quad [\text{m}^2 \text{s}^{-2}]$$

## Energy percentages in various motions

Dataset	Motion type			Motion type		
	ROT	EIG	WIG	ROT	EIG	WIG
DART/CAM	87.9	7.4	4.7	98.6	0.7	0.6
ECMWF	85.3	8.6	6.1	98.2	1.0	0.8
NCEP/NCAR	54.2	24.5	21.3	95.9	2.2	1.9
NCEP	91.3	5.8	2.9	98.7	0.8	0.5

## Average energy spectra for the Kelvin wave, mixed Rossby-gravity (MRG) and inertio-gravity motions (EIG+WIG)

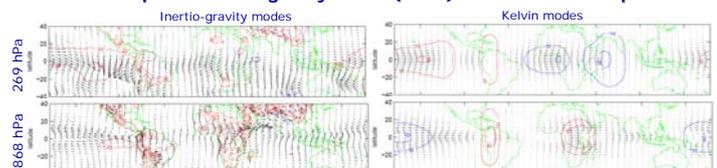


Energy ratio (%)	Dataset			
	DART/CAM	ECMWF	NCEP/NCAR	NCAR
MRG: (IG+MRG)	8	12	4	15
KW: (IG+MRG)	20	17	7	25
MRG: (WIG+MRG)	18	25	7	34
KW: EIG	36	19	13	45

At long zonal scales (zonal wavenumber  $k < 7$ ) derived energy spectra of the total IG wave ( $k > 0$ ) motion have slopes close to -1.

The KW spectra are at the longest scales fitted by a -5/3 law while the MRG energy spectrum appears flat. At shorter scales spectra for both modes follow a -3 law.

## Tropics as envisaged by A. Gill (1980): DART/CAM example



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