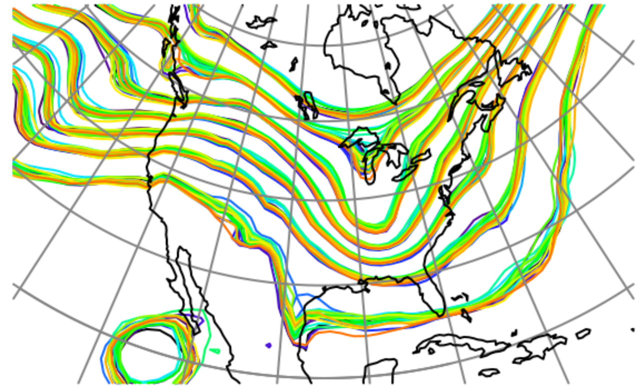


Exploiting Nonlinear Relations between Observations and State Variables in Ensemble Filters

Jeff Anderson, NCAR Data Assimilation Research Section

Schematic of a Sequential Ensemble Filter

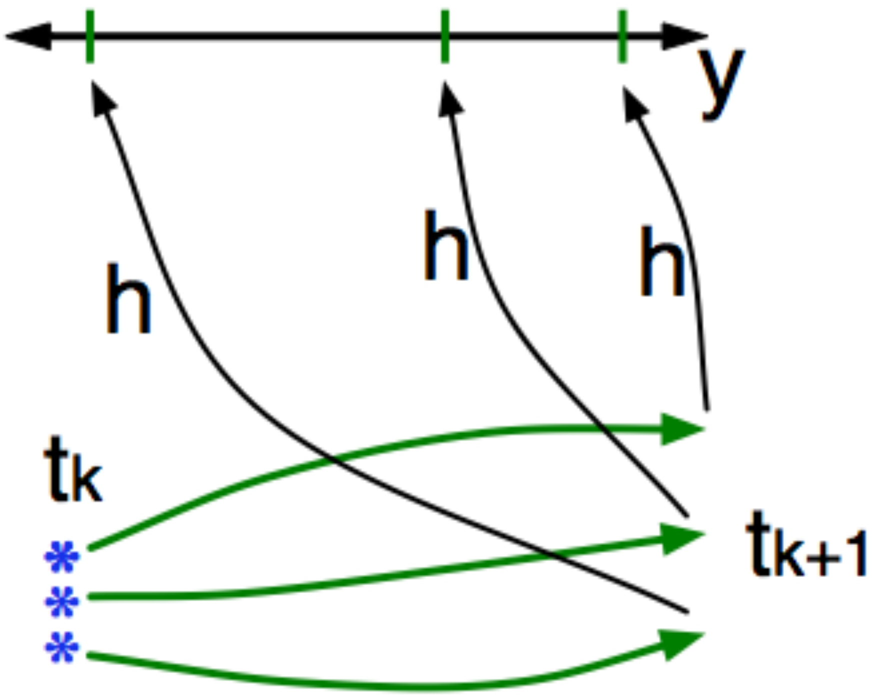
1. Use model to advance ensemble (3 members here) to time at which next observation becomes available.

Ensemble state estimate after using previous observation (analysis)

Ensemble state at time of next observation (prior)

Schematic of a Sequential Ensemble Filter

2. Get prior ensemble sample of observation, $y = h(x)$, by applying forward operator **h** to each ensemble member.

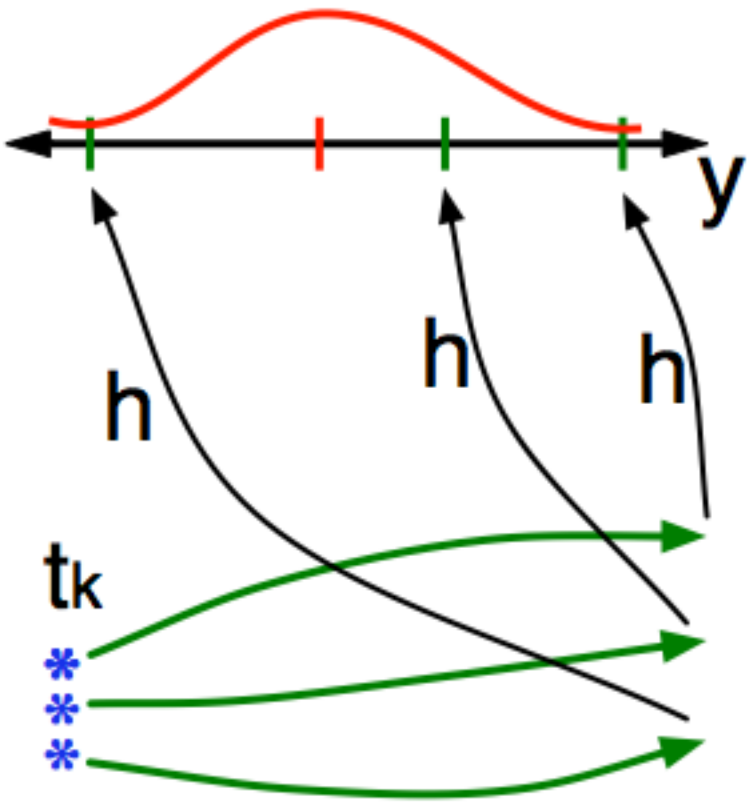


Theory: observations from instruments with uncorrelated errors can be done sequentially.

Can think about single observation without (too much) loss of generality.

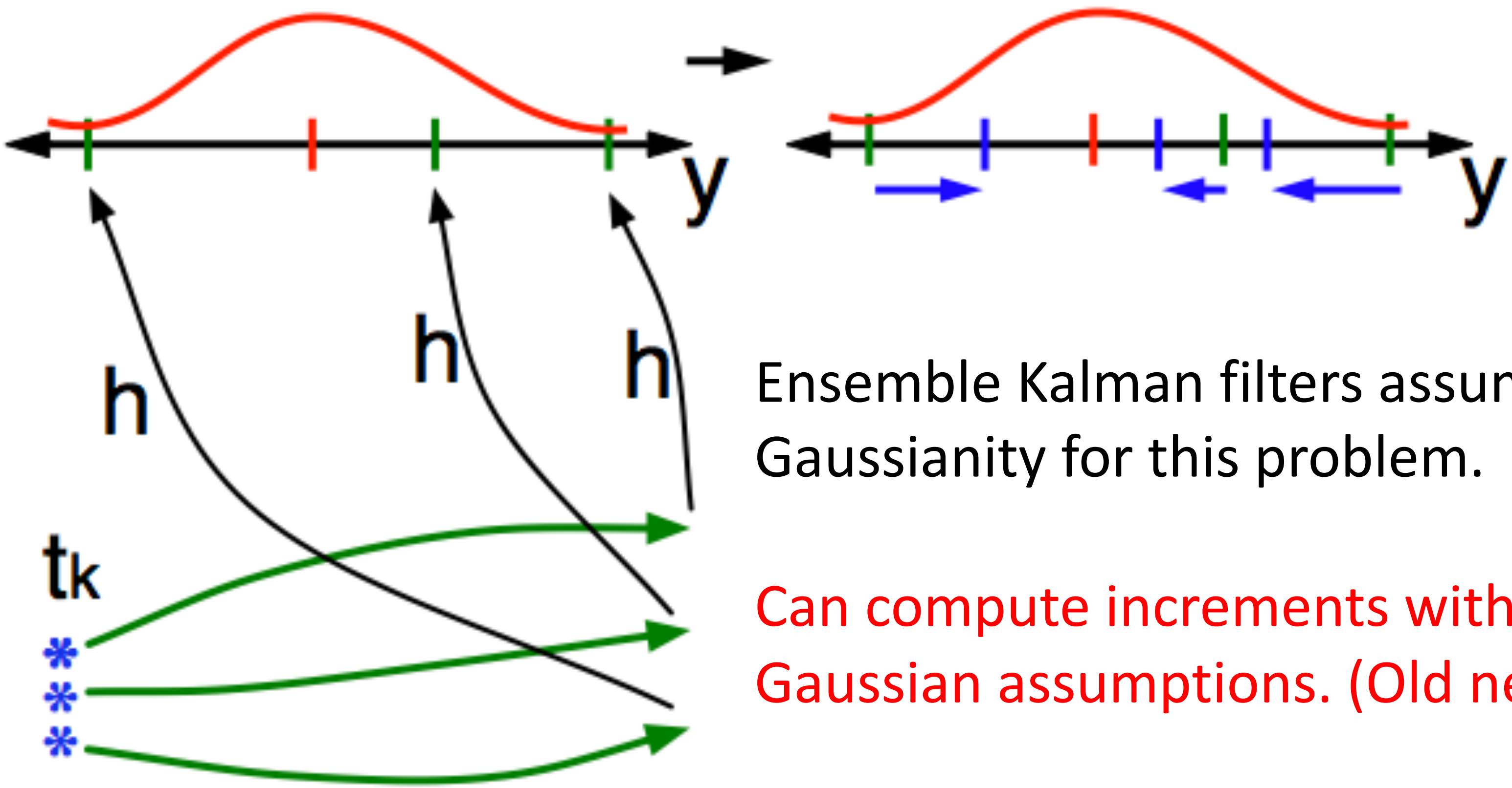
Schematic of a Sequential Ensemble Filter

3. Get observed value and observational error distribution from observing system.



Schematic of a Sequential Ensemble Filter

4. Find the increments for the prior observation ensemble (this is a scalar problem for uncorrelated observation errors).

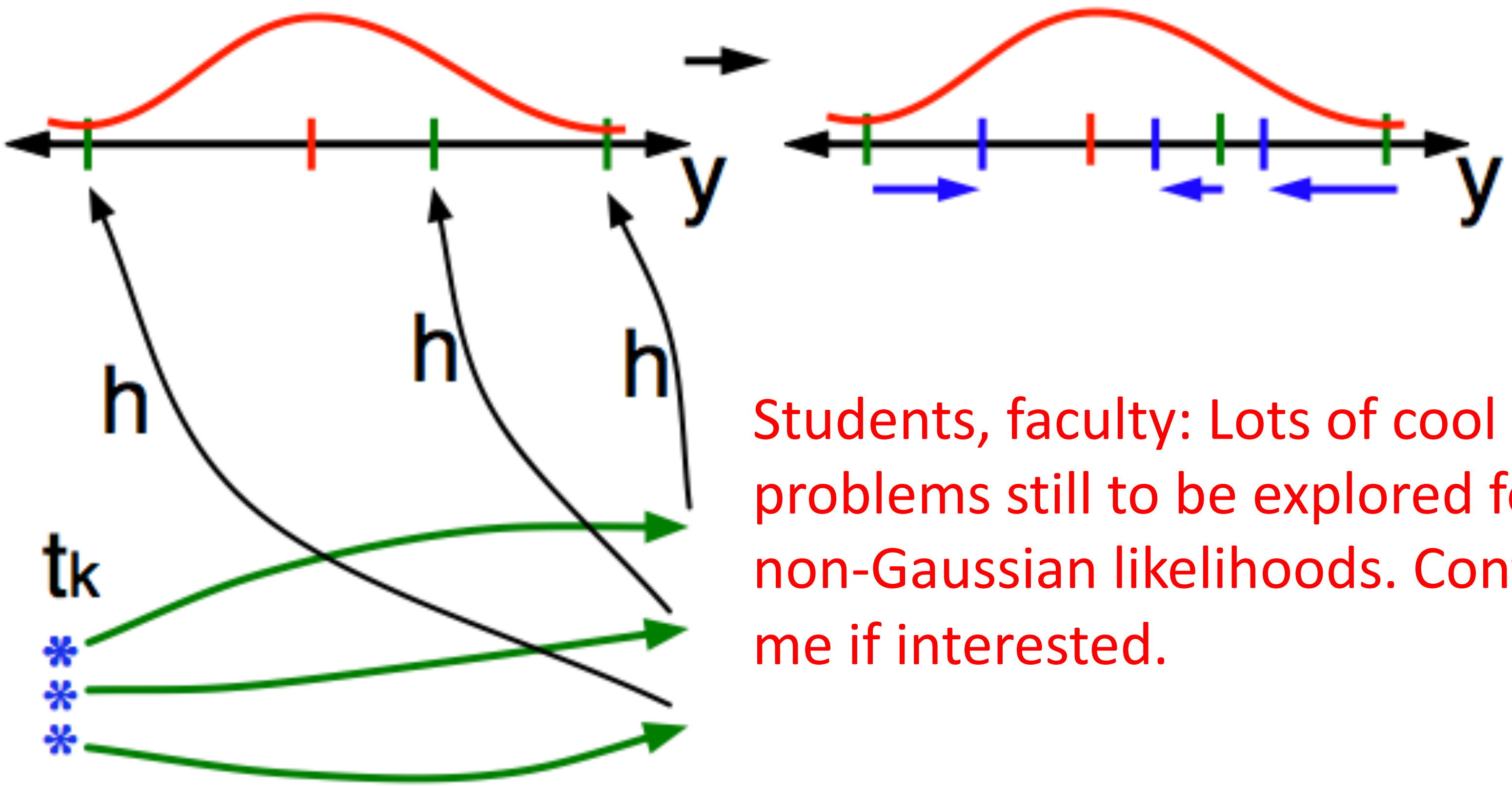


Ensemble Kalman filters assume Gaussianity for this problem.

Can compute increments without Gaussian assumptions. (Old news).

Schematic of a Sequential Ensemble Filter

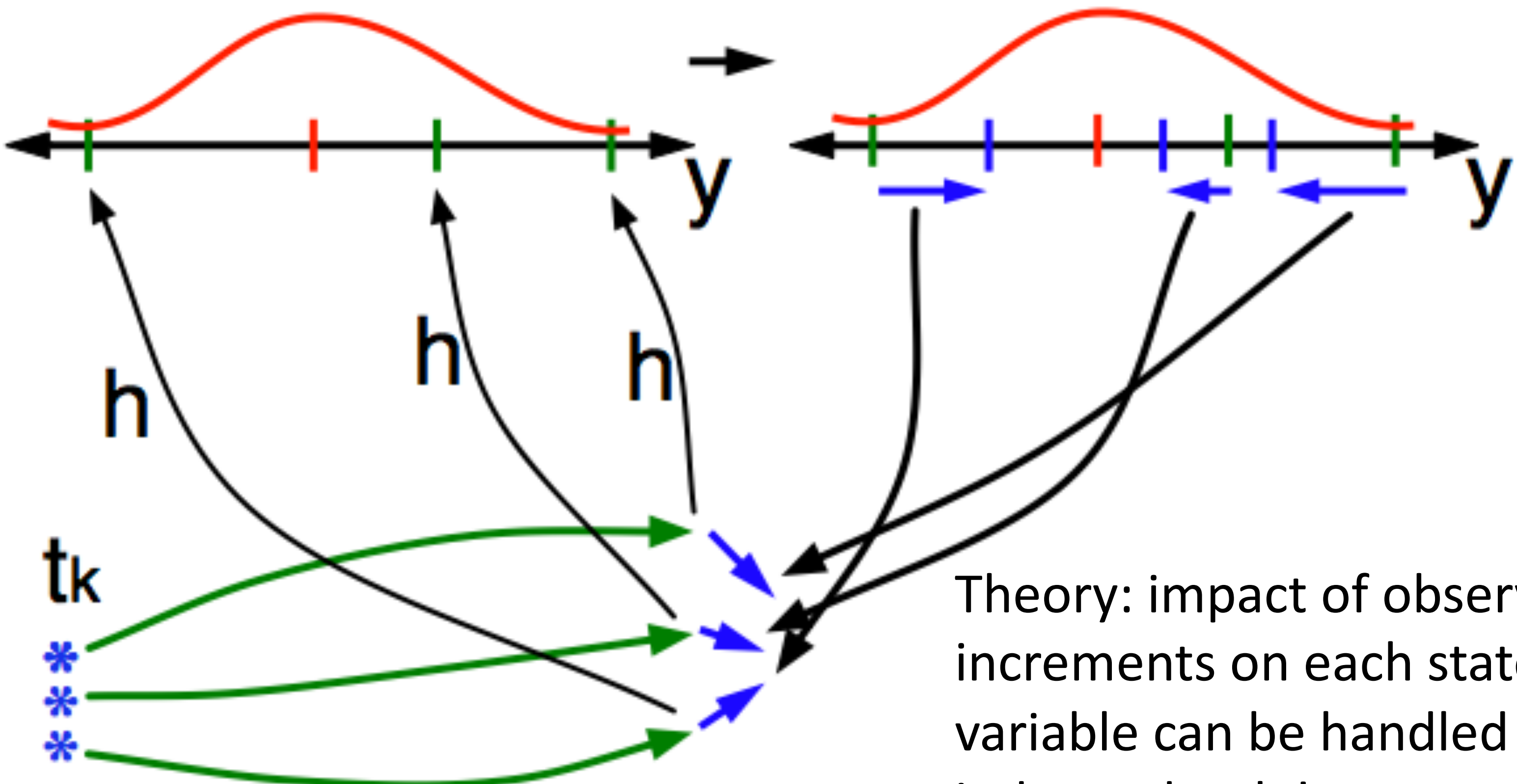
4. Find the increments for the prior observation ensemble (this is a scalar problem for uncorrelated observation errors).



Students, faculty: Lots of cool problems still to be explored for non-Gaussian likelihoods. Contact me if interested.

Schematic of a Sequential Ensemble Filter

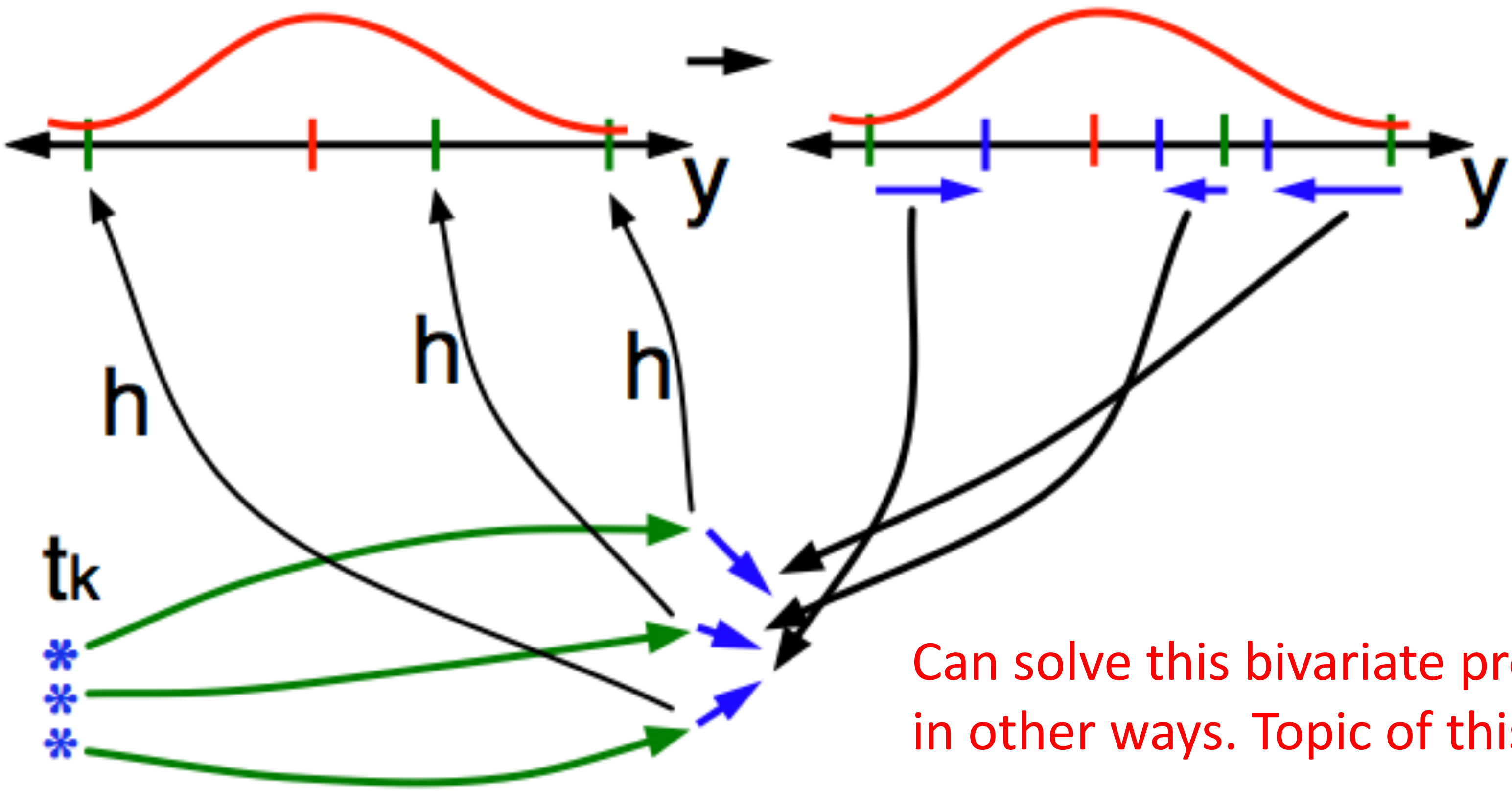
5. Use ensemble samples of y and each state variable to linearly regress observation increments onto state variable increments.



Theory: impact of observation increments on each state variable can be handled independently!

Schematic of a Sequential Ensemble Filter

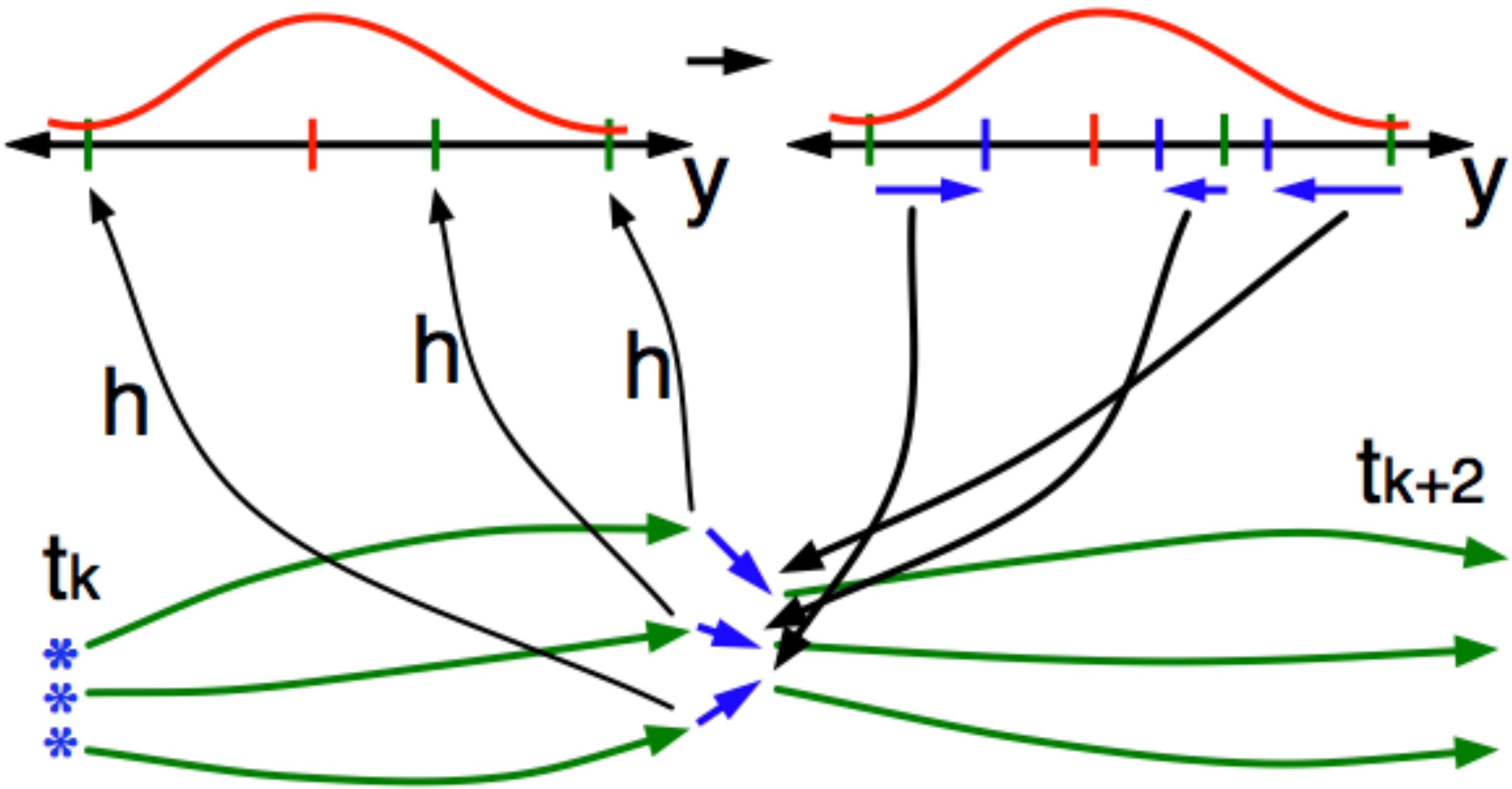
5. Use ensemble samples of y and each state variable to linearly regress observation increments onto state variable increments.



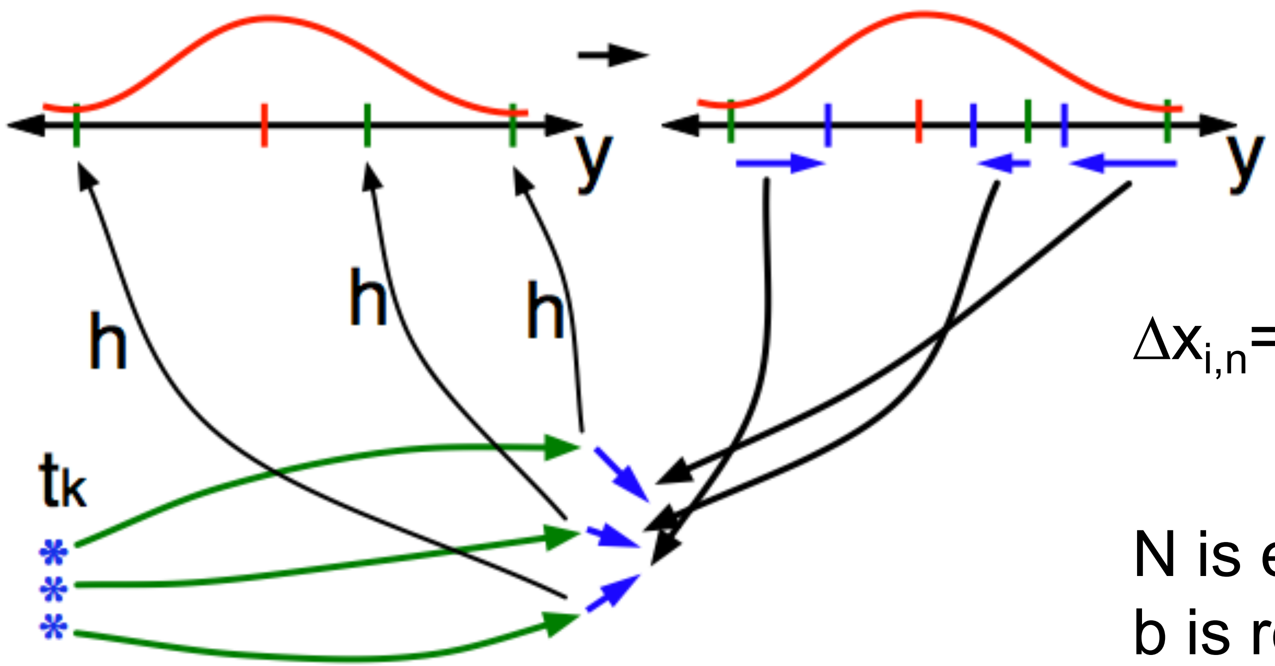
Can solve this bivariate problem in other ways. Topic of this talk.

Schematic of a Sequential Ensemble Filter

6. When all ensemble members for each state variable are updated, there is a new analysis. Integrate to time of next observation …



Standard ensemble filters just use bivariate sample linear regression to compute state increments.

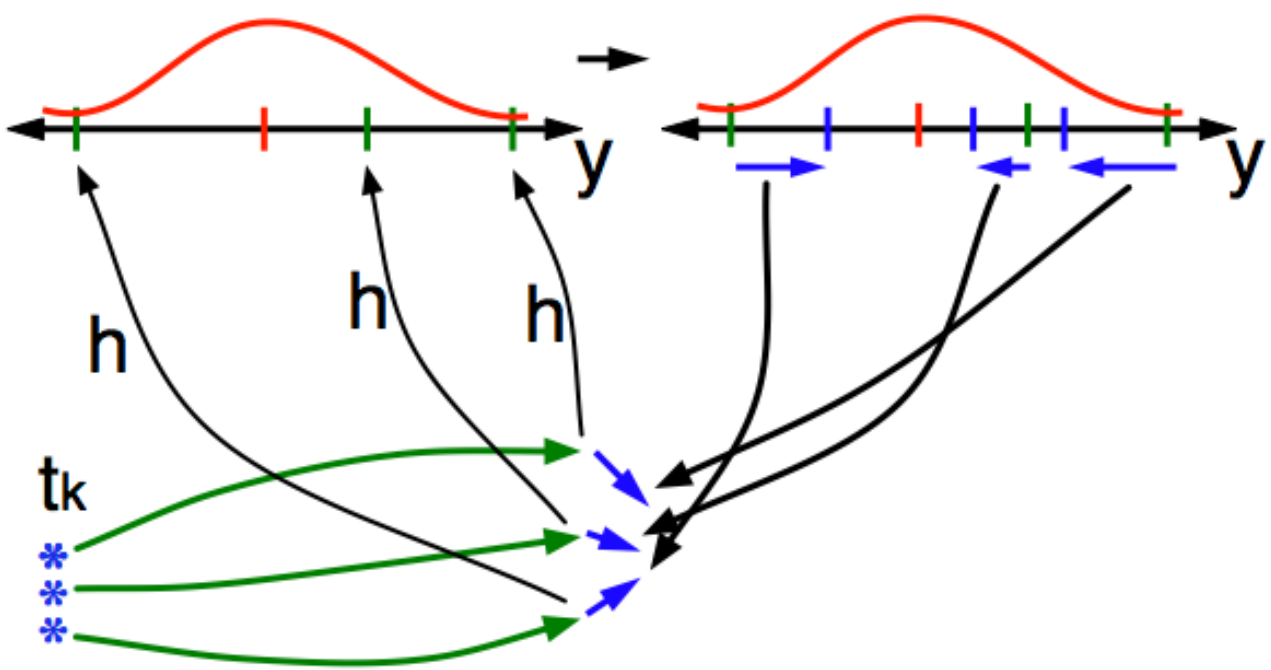


$\Delta x_{i,n} = b\Delta y_n,$
$n=1,\ldots N.$

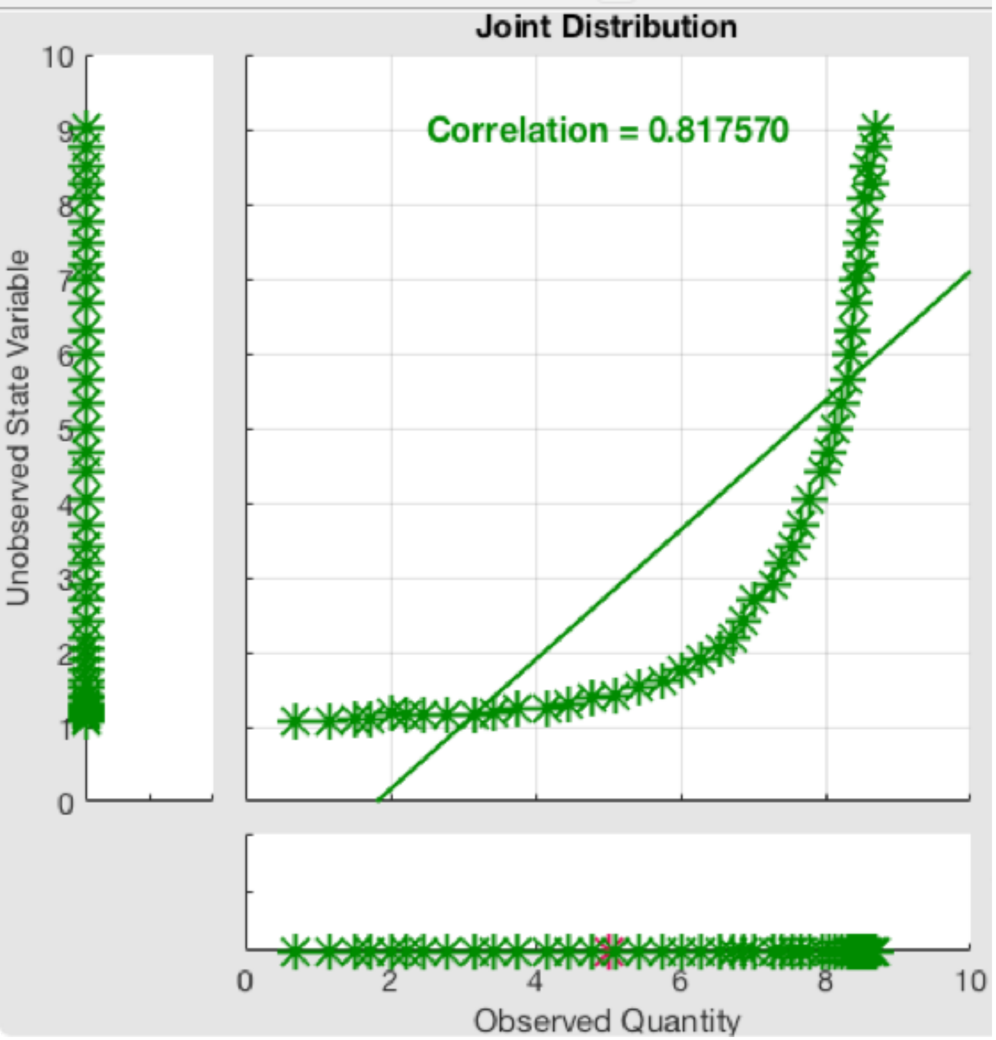
N is ensemble size.
b is regression coefficient.

Focus on the Regression Step

Will examine two additional ways to increment state given observation increments. Both still bivariate.



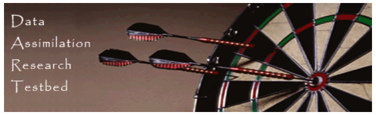
Nonlinear Regression Example



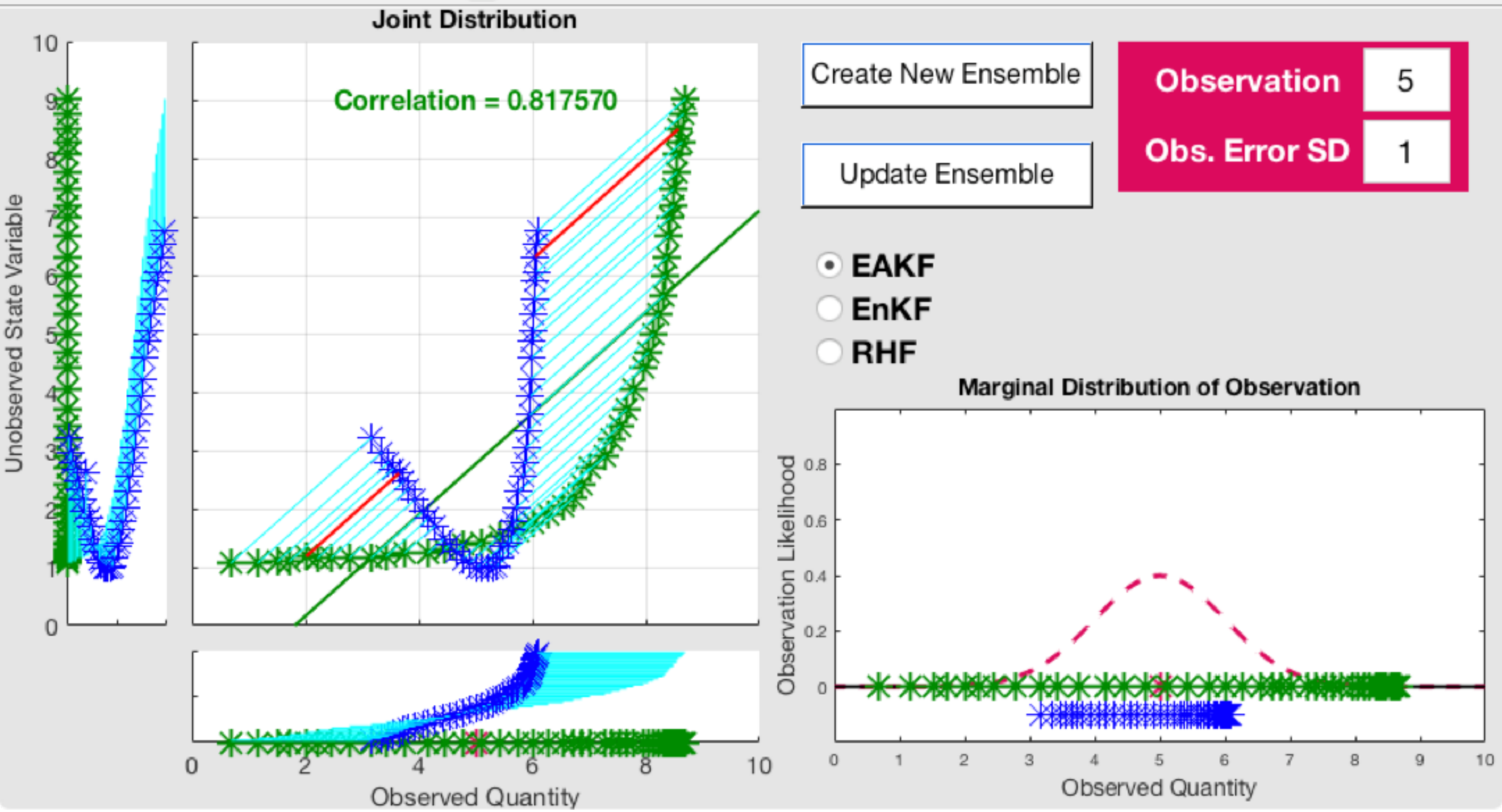
Try to exploit nonlinear prior relation between a state variable and an observation.

Example: Observation y~log(x).

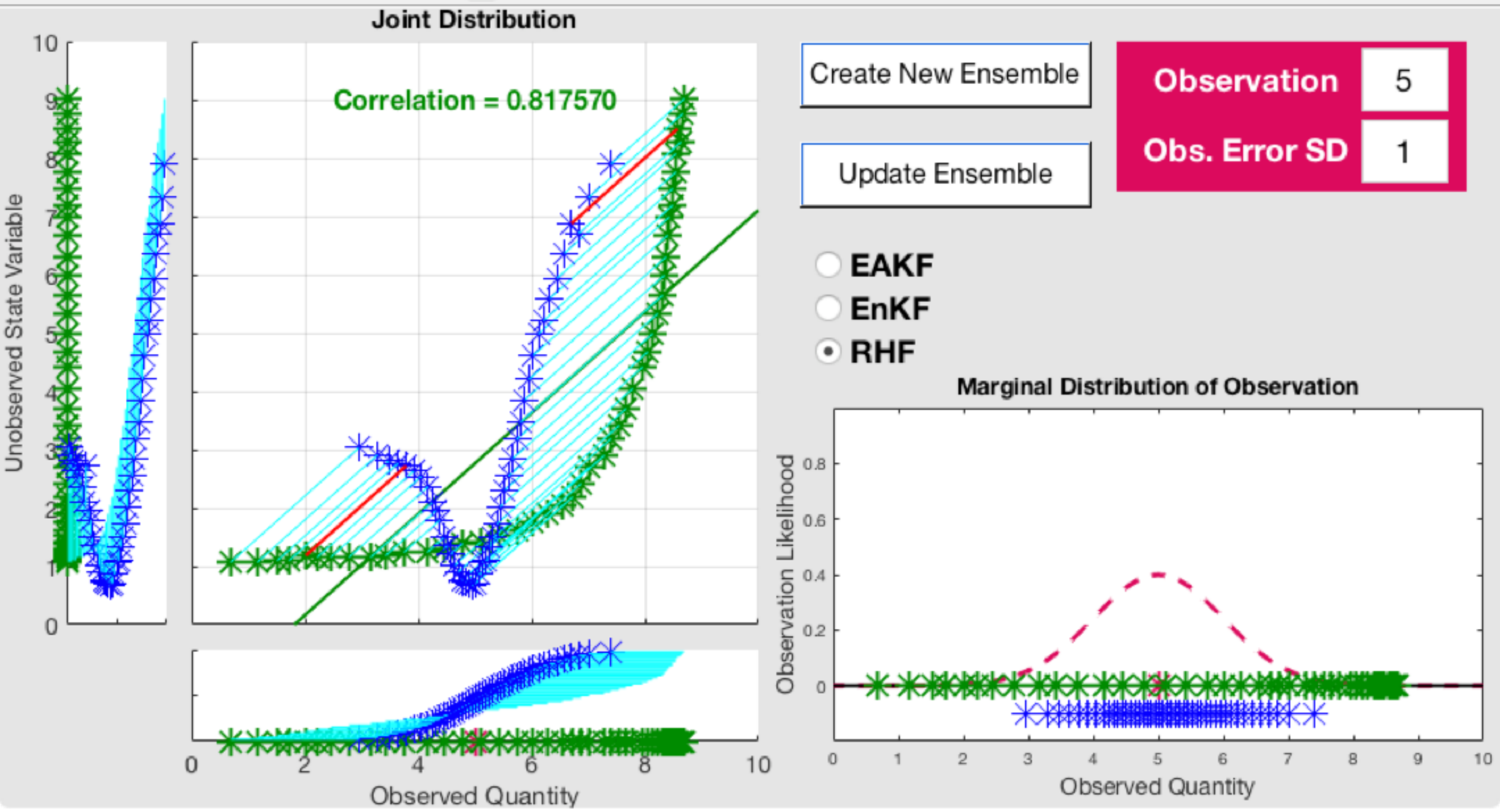
Also relevant for variables that are log transformed for boundedness (like concentrations).



Standard Ensemble Adjustment Filter (EAKF)

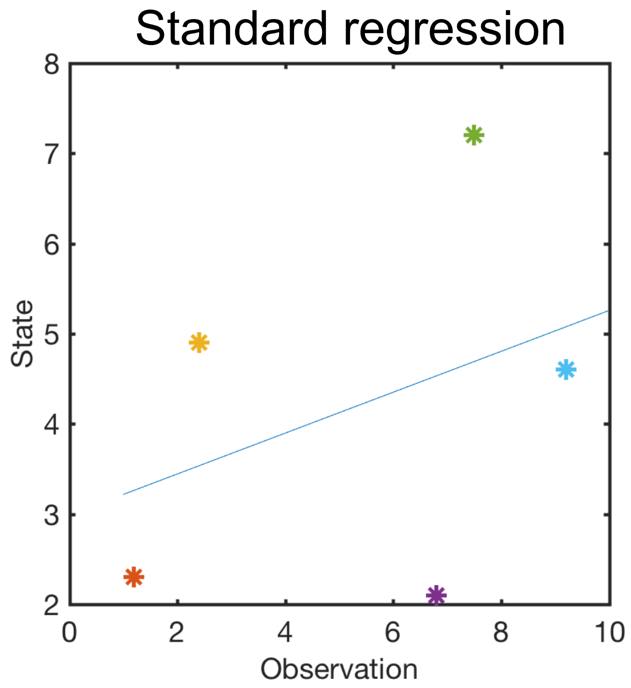


Standard Rank Histogram Filter (RHF)

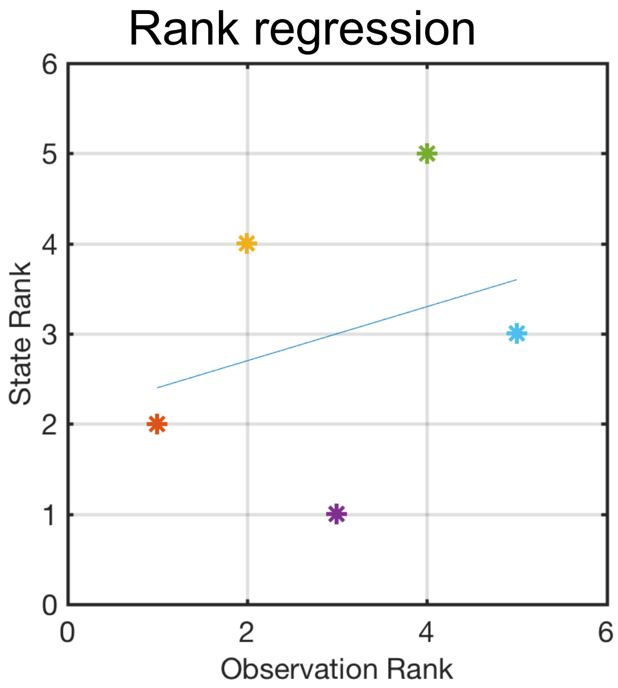


Rank Regression

1. Convert bivariate ensemble to bivariate rank ensemble.
2. Do least squares on bivariate rank ensemble.



Noisy
Relation.



Rank Regression

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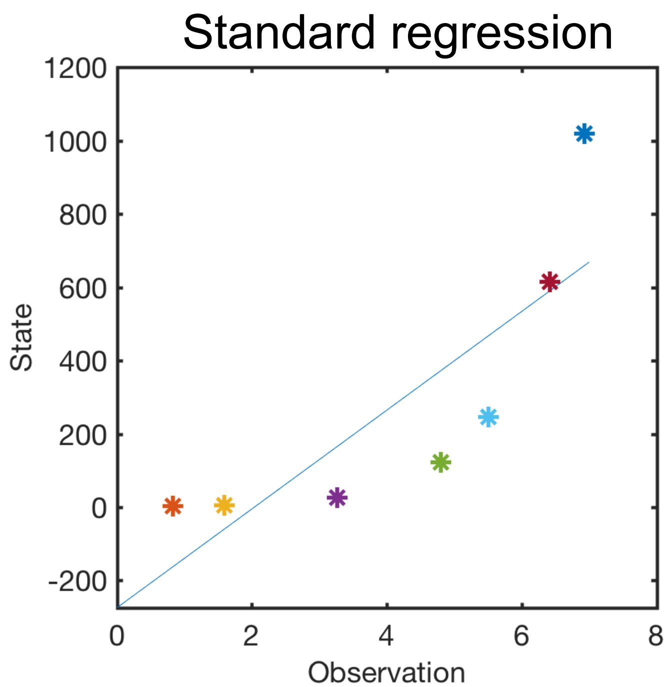
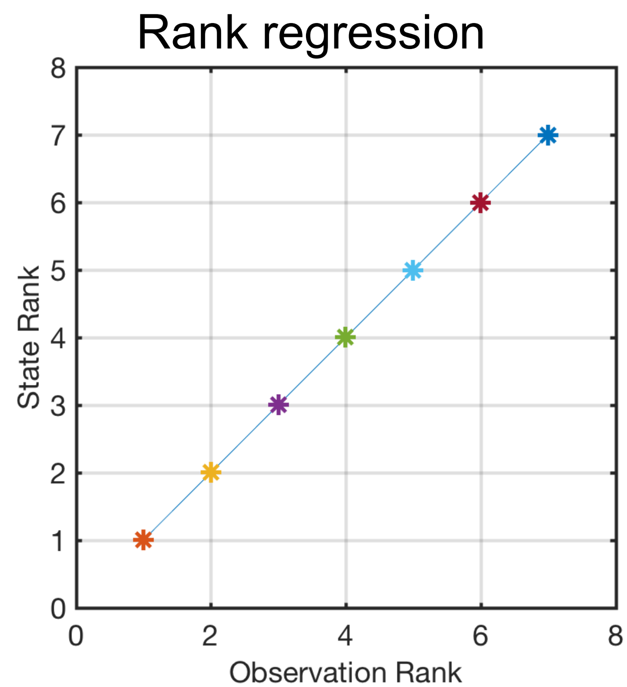


Monotonic relation.



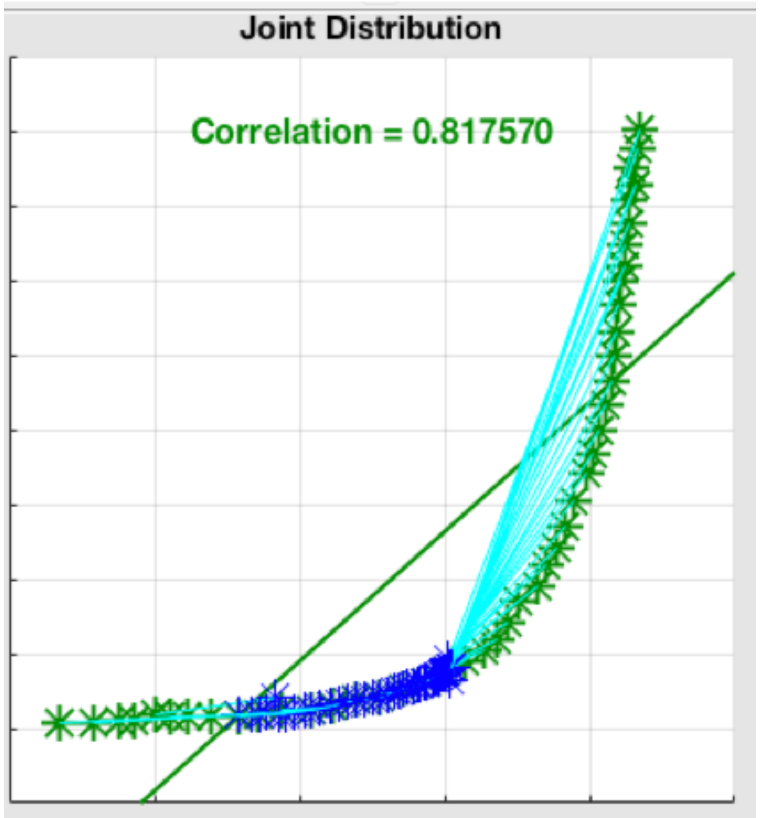
Rank Regression

1. Convert bivariate ensemble to bivariate rank ensemble.
2. Do least squares on bivariate rank ensemble.
3. Convert observation posteriors to rank.
   a. Extrapolate by assuming Gaussian tails on prior.
   b. Same as Rank Histogram filter.
4. Regress rank increments onto state ranks.

Rank Regression

1. Convert bivariate ensemble to bivariate rank ensemble.
2. Do least squares on bivariate rank ensemble.
3. Convert observation posteriors to rank.
   a. Extrapolate by assuming Gaussian tails on prior.
   b. Same as Rank Histogram filter.
4. Regress rank increments onto state ranks.
5. Convert posterior state ranks to state values.
6. If posterior rank is outside ‘legal’ values, use weighted average of extrapolation and standard regression.

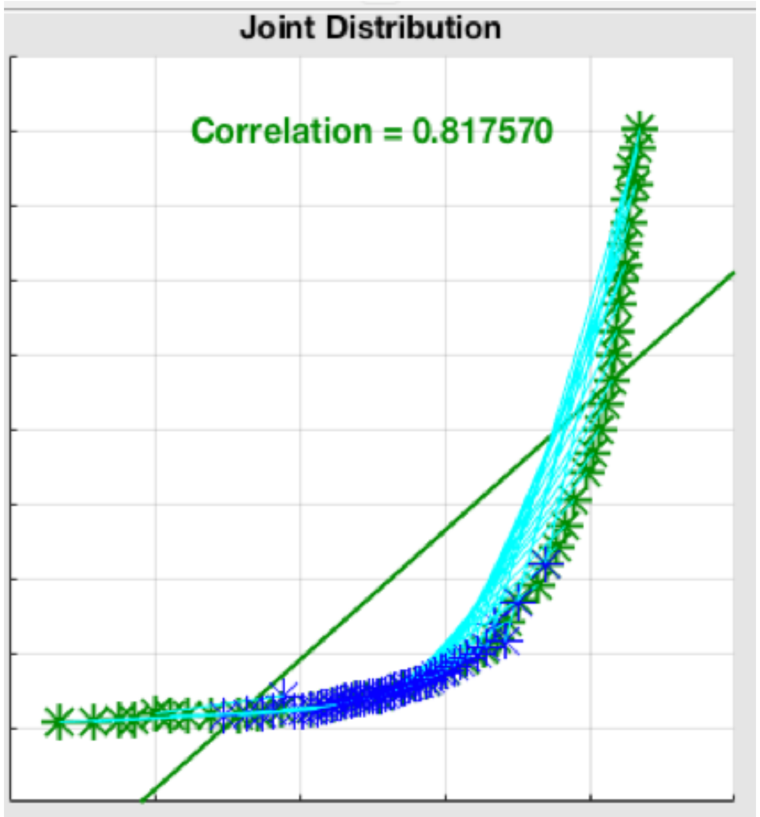
Nonlinear Regression Example



Rank regression with EAKF for observation marginal.

Follows monotonic ensemble prior ‘exactly’.

Nonlinear Regression Example

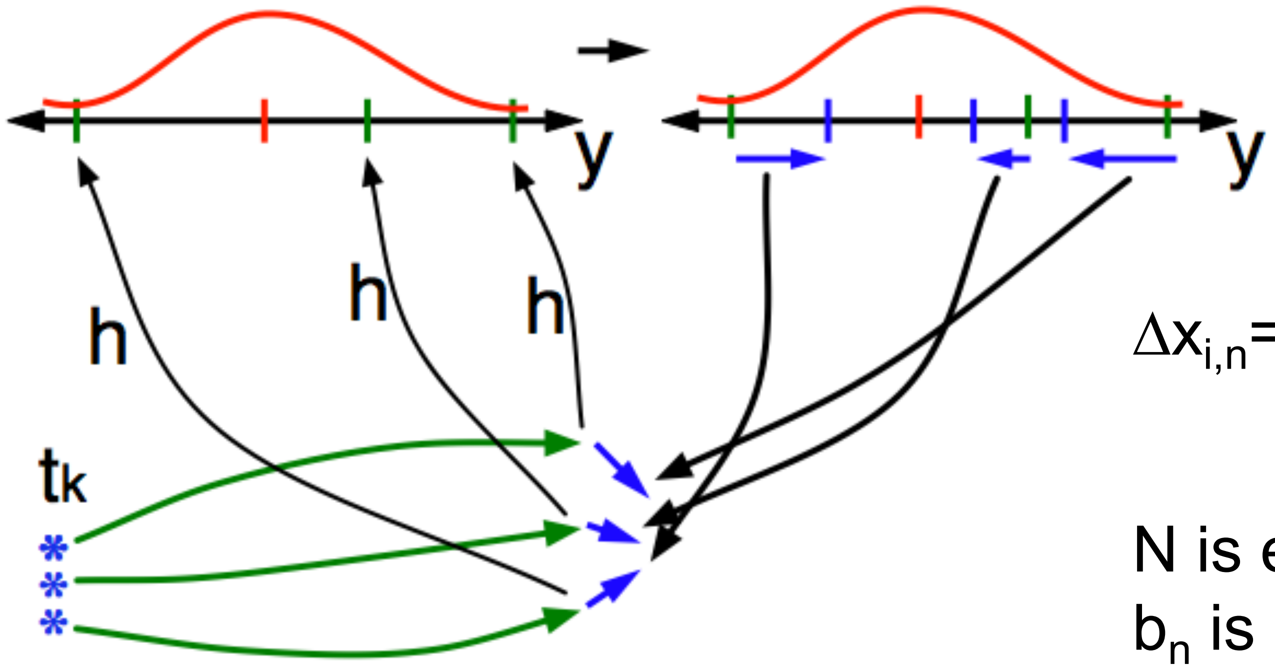


Rank regression with RHF for observation marginal.

Follows monotonic ensemble prior 'exactly'.

Focus on the Regression Step

Second approach, use different 'regression' for each ensemble member to compute increments for x_i



$\Delta x_{i,n} = b_n \Delta y_n$, $n=1,\ldots N$.

N is ensemble size.
b_n is 'local' regression coefficient.

Local Linear Regression

Relation between observation and state is nonlinear.

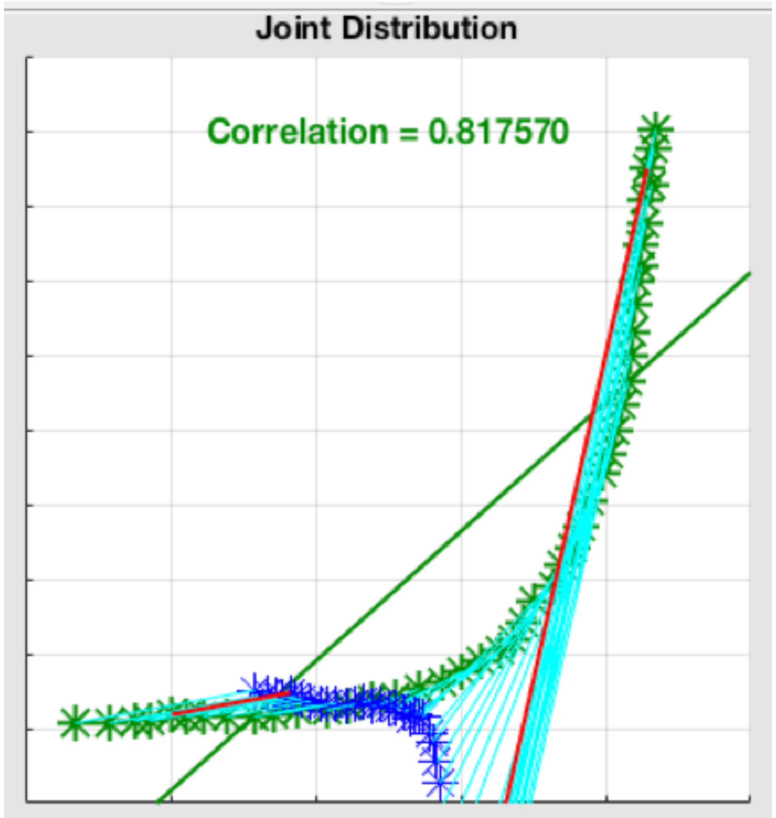
Try using 'local' subset of ensemble to compute regression.

What kind of subset?

Cluster that contains ensemble member being updated.

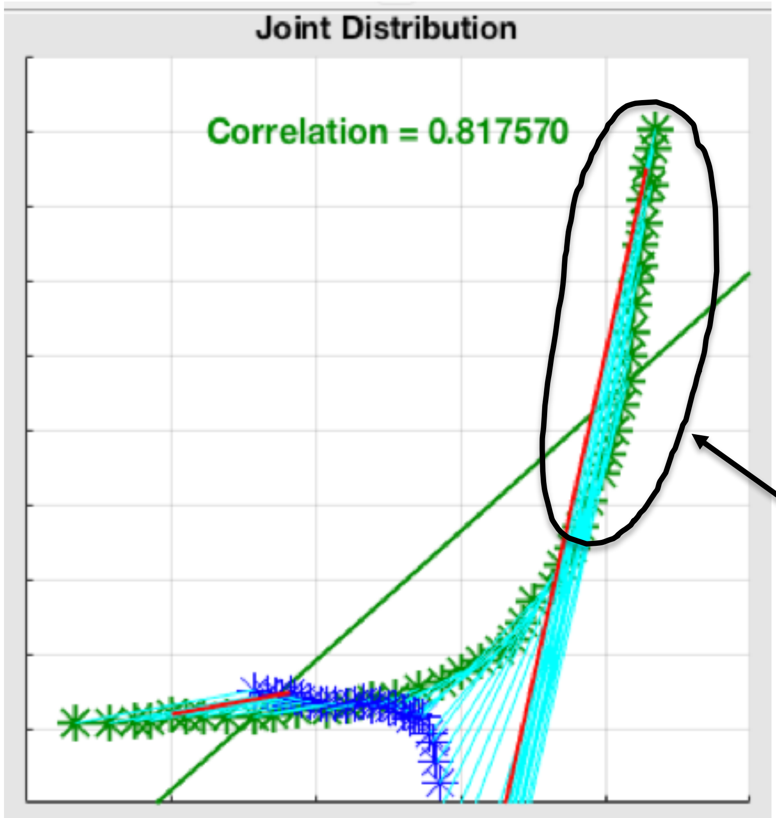
- Lots of ways to define clusters.
- Here, use naïve closest neighbors in (x,y) space.
- Vary number of nearest neighbors in subset.

Local Linear Regression



Local ensemble subset is nearest ½ . Regression approximates local slope of the relation.

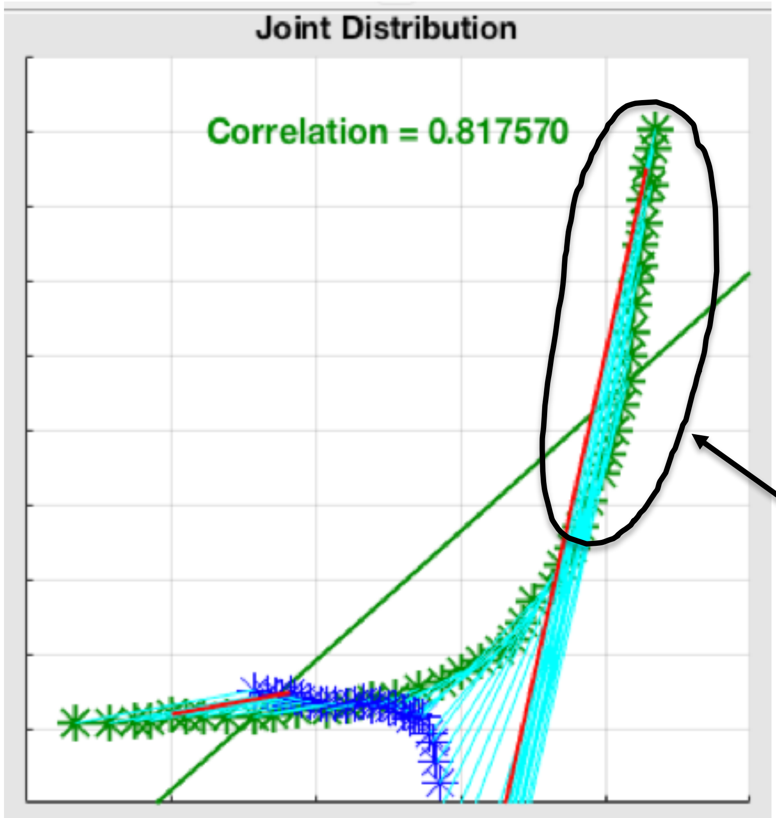
Local Linear Regression



Local ensemble subset is nearest ½ . Regression approximates local slope of the relation.

Highlighted red increment uses least squares fit to ensemble members in region.

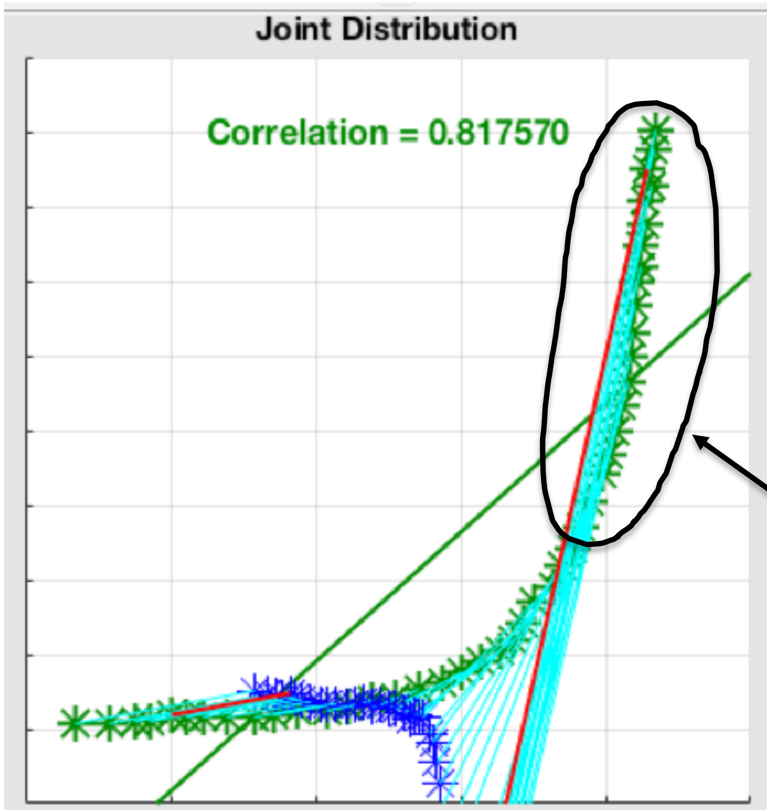
Local Linear Regression



Slope more accurate locally, but a disaster globally.

Highlighted red increment uses least squares fit to ensemble members in region.

Local Linear Regression



Note similarity to Houtekamer's method, except local ensemble members are used, rather than non-local.

Highlighted red increment uses least squares fit to ensemble members in region.

Local Linear Regression with Incremental Update

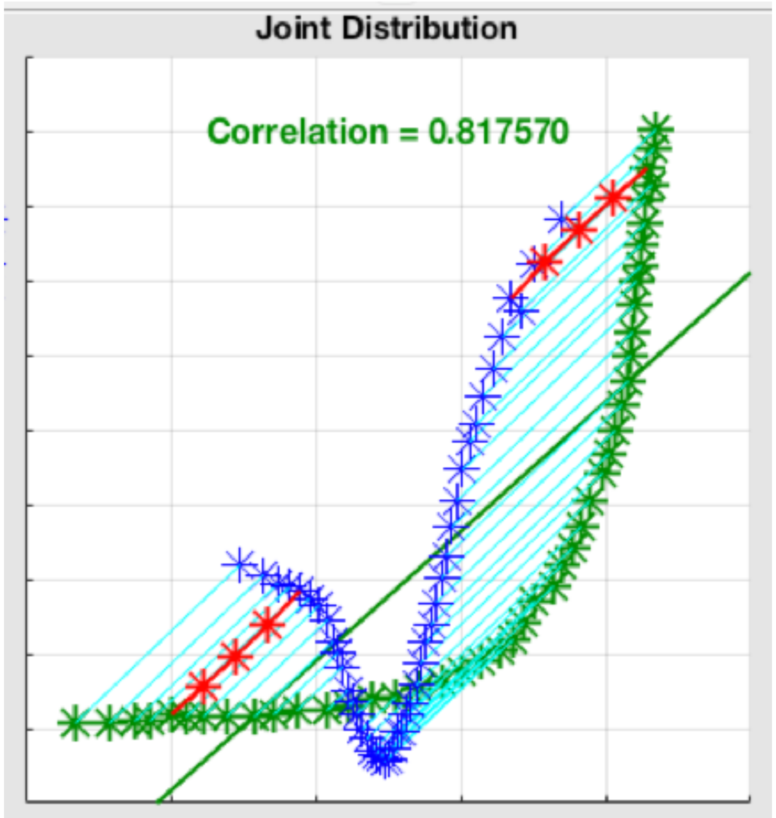
Local slope is just that, local.

Following it for a long way is a bad idea.

Will use a Bayesian consistent incremental update.

- Observation with error variance s.
- Assimilate k observations with this value.
- Each of these has error variance s/k.

Local Linear Regression with Incremental Update

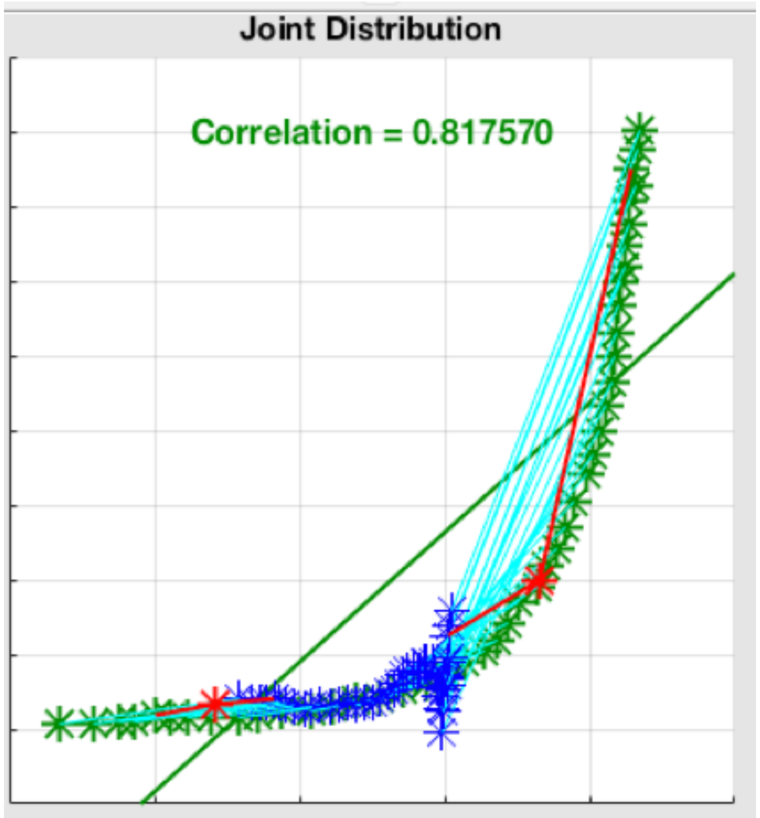


This is an RHF update with 4 increments. Individual increments highlighted for two ensemble members.

For an EAKF, posterior would be identical to machine precision.

Nearly identical for RHF.

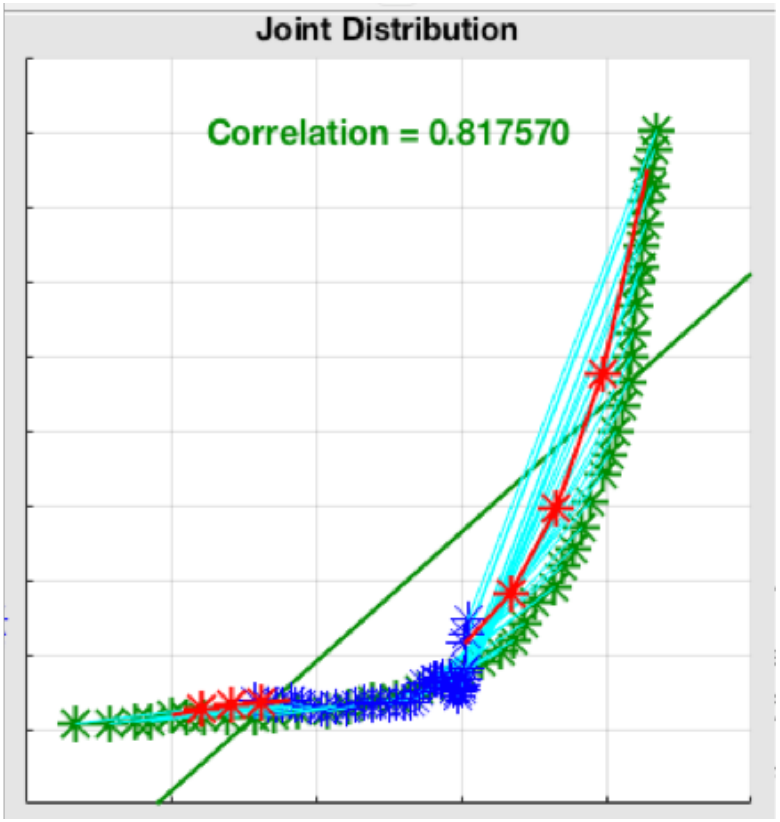
Local Linear Regression with Incremental Update



2 increments with subsets ½ ensemble.

Posterior for state qualitatively improving.

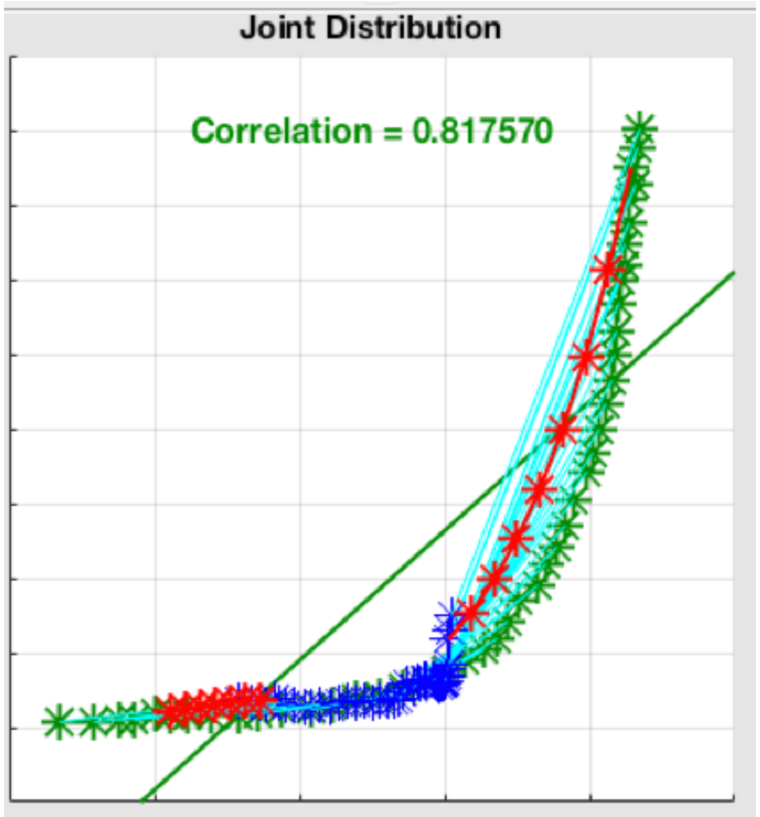
Local Linear Regression with Incremental Update



4 increments with subsets ½ ensemble.

Posterior for state qualitatively improving.

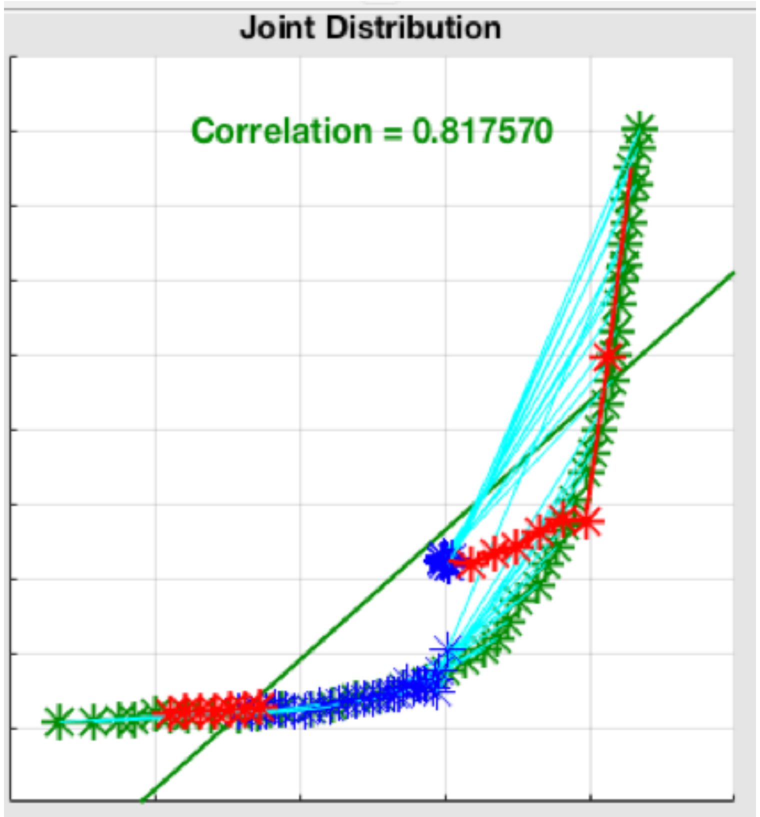
Local Linear Regression with Incremental Update



8 increments with subsets ½ ensemble.

Posterior for state qualitatively improving.

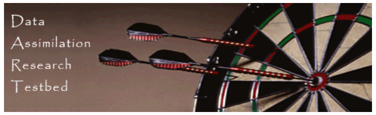
Local Linear Regression with Incremental Update



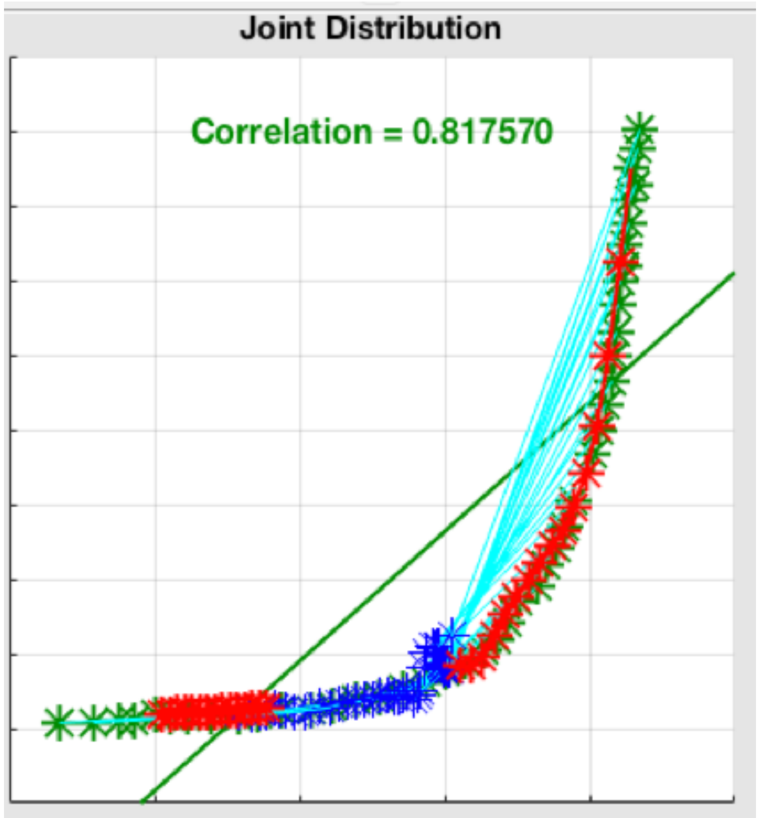
8 increments with subset 1/4 ensemble.

Posterior for state degraded.

Increment is moving outside of local linear validity.



Local Linear Regression with Incremental Update



16 increments with subset 1/4 ensemble.

Posterior for state improved.

Local Linear Regression with Incremental Update

If relation between observation and state is locally a continuous, smooth (first two derivatives continuous) function:
Then, in the limit of a large ensemble, fixed local subset size, and large number of increments:

The local linear regression with incremental update converges to the correct posterior distribution.

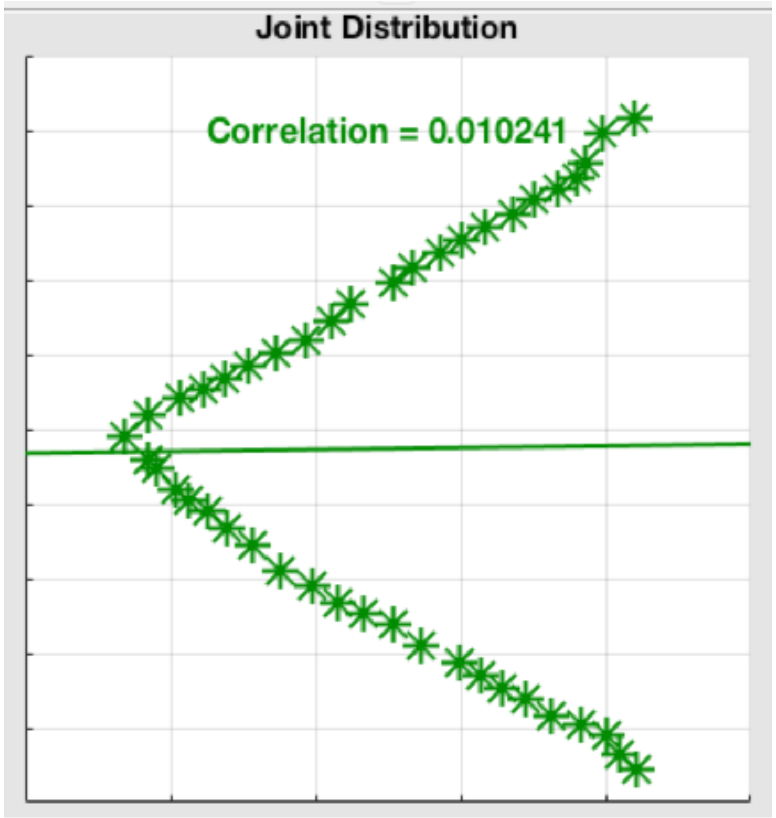
Local Linear Regression with Incremental Update

If relation between observation and state is locally a continuous, smooth (first two derivatives continuous) function:
Then, in the limit of a large ensemble, fixed local subset size, and large number of increments:

The local linear regression with incremental update converges to the correct posterior distribution.

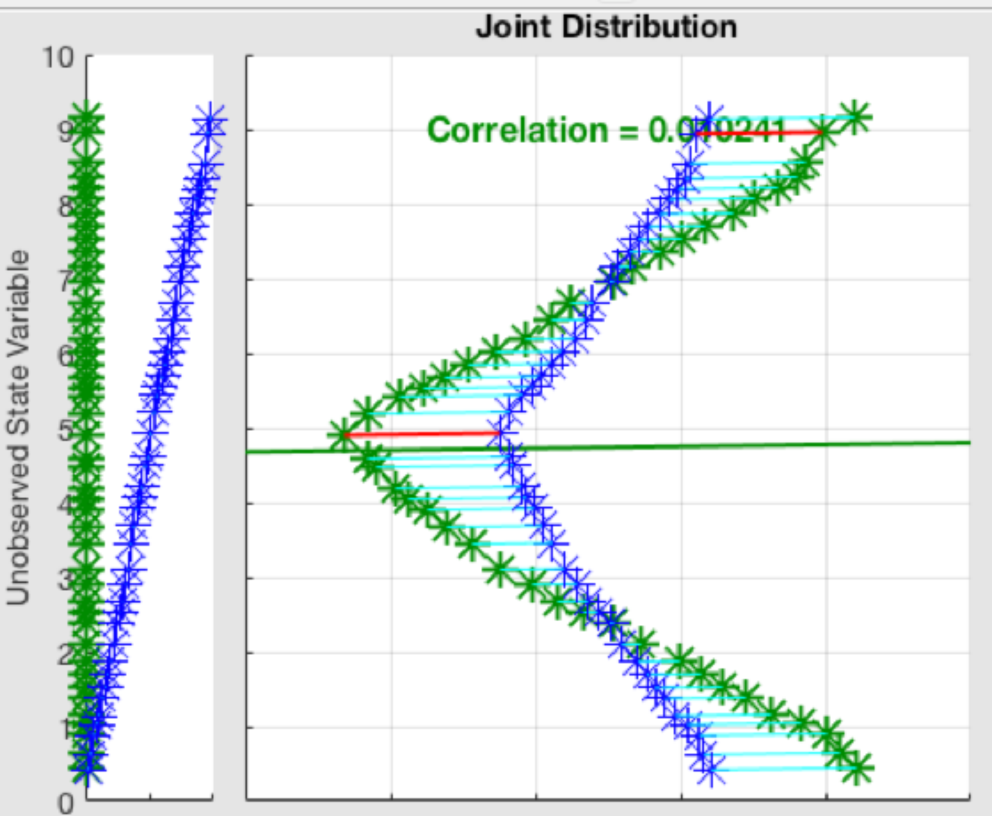
This could be very expensive,
No guarantees about what goes on in the presence of noise.

Multi-valued, not smooth example.



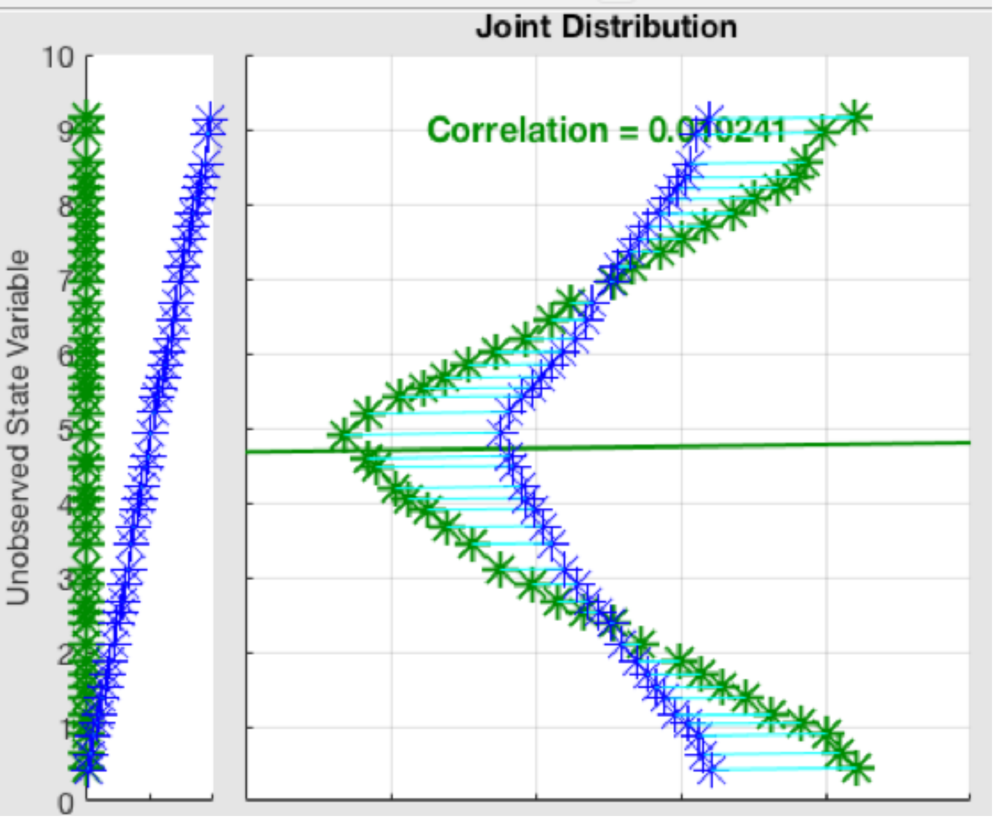
Similar in form to a wind speed observation with state velocity component.

Multi-valued, not smooth example.



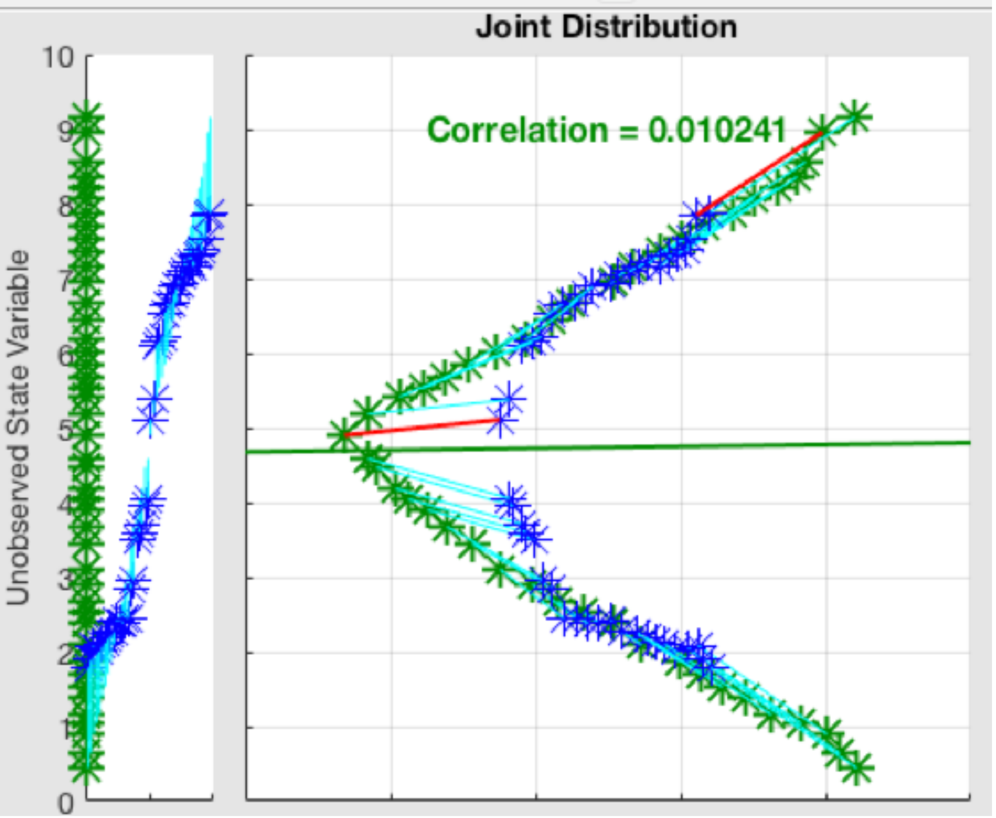
Standard regression does not capture bimodality of state posterior.

Multi-valued, not smooth example.



Rank regression nearly identical in this case.

Multi-valued, not smooth example.

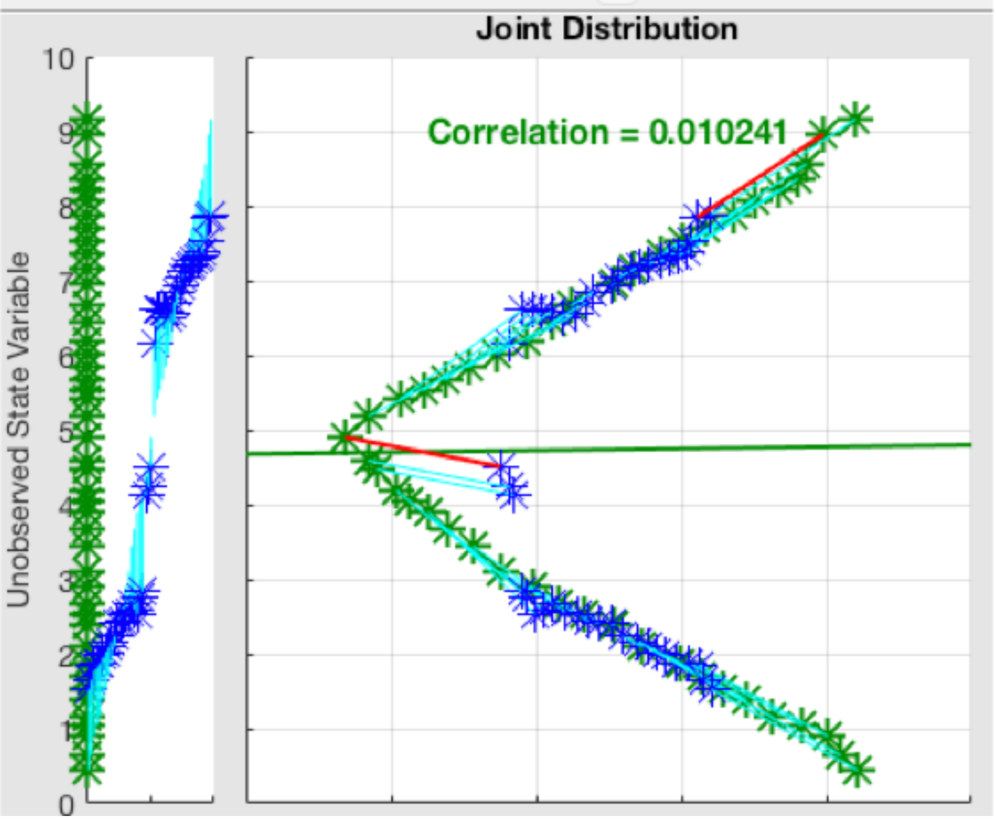


Local regression with ½ of the ensemble does much better.

Captures bimodal posterior.

Note problems where relation is not smooth.

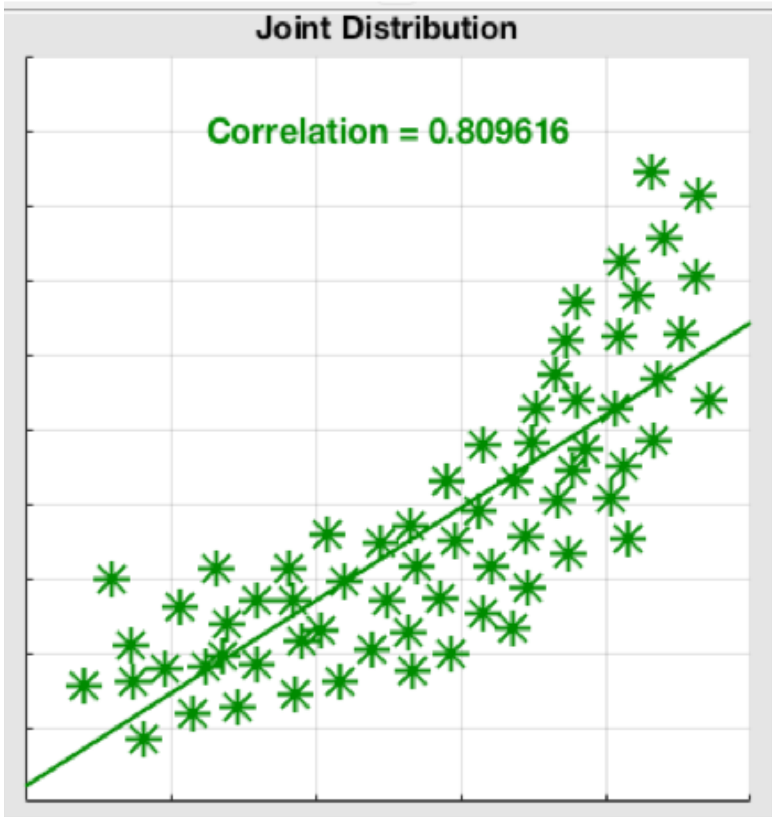
Multi-valued, not smooth example.



Local regression with1/4 of the ensemble does even better.

No need for incremental updates here.

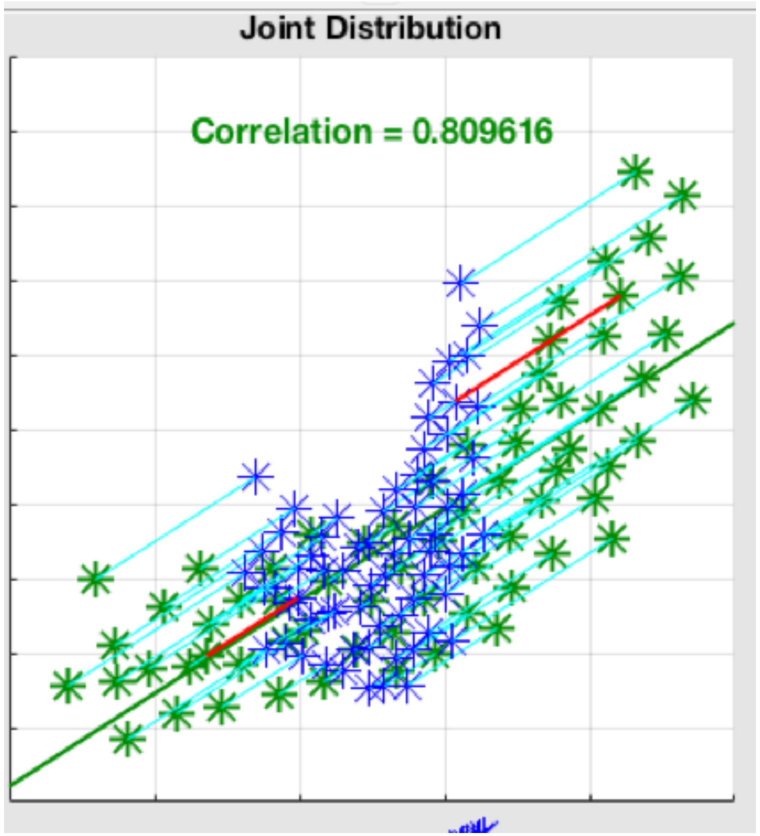
Regression with Noisy Priors



Most geophysical applications have noisy bivariate priors.

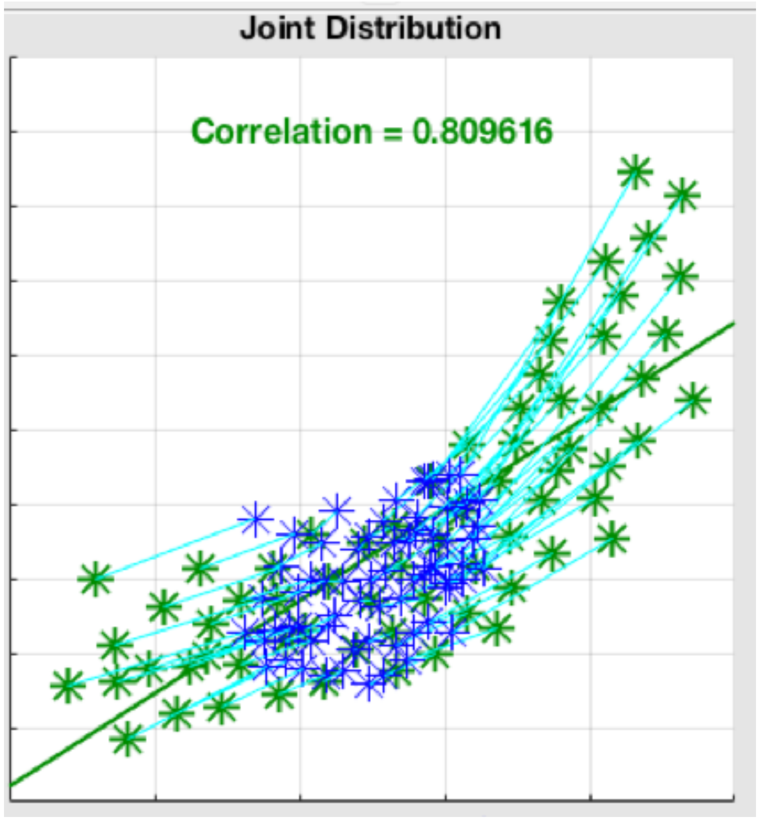
Usually hard to detect nonlinearity (even this example is still pretty extreme).

Regression with Noisy Priors



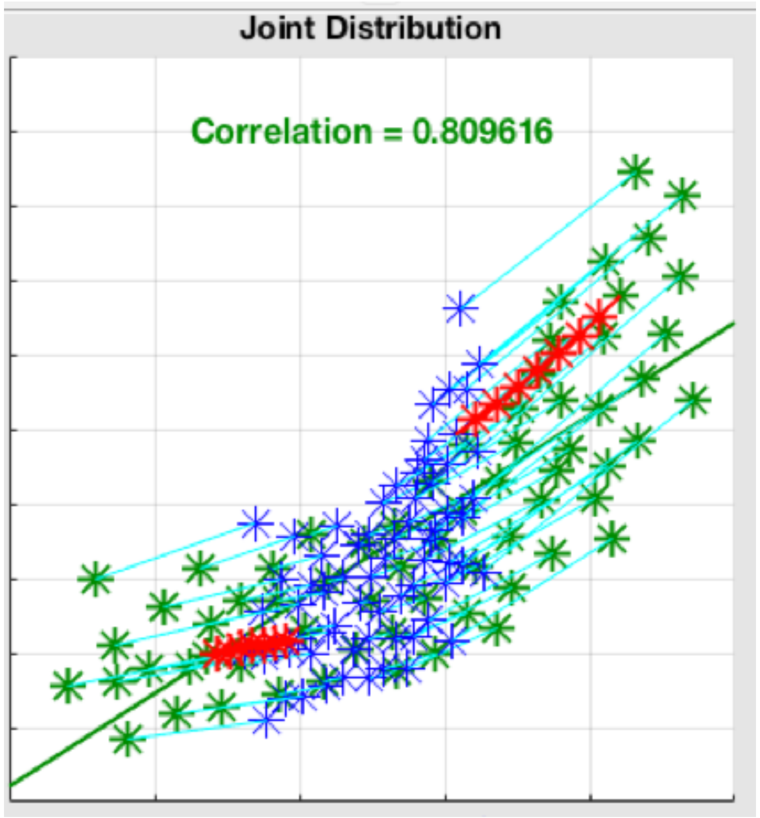
Standard regression
EAKF places many posterior members outside of the prior bivariate distribution.

Regression with Noisy Priors



Rank regression does a significantly better job.

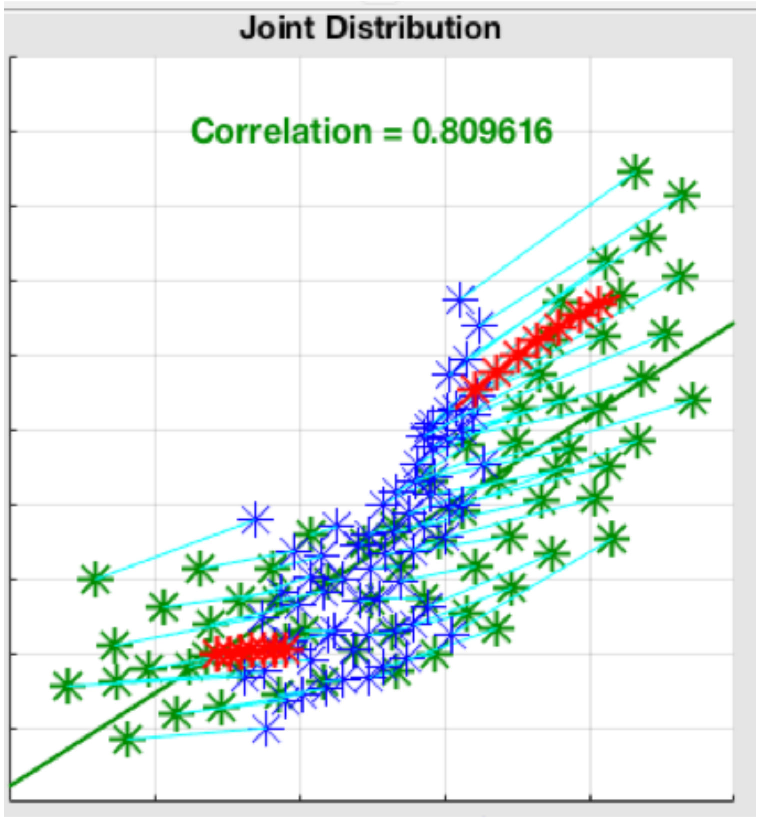
Regression with Noisy Priors



Local incremental regression. This result is for local ensemble with nearest ½ of ensemble and 8 increments.

Need bigger local ensembles to reduce sampling errors.

Regression with Noisy Priors



Local incremental regression. This result is for local ensemble with nearest 1/4 of ensemble and 8 increments.

The small ensemble subsets lead to large sampling error. Probably worse than standard RHF.

Computational Cost

Computational cost for the state variable update:

Base regression: $O(m^2n)$
Rank regression: $O(m^2n\log n)$
Local regression: $O(m^2n^2\log n)$

m: sum of state size plus number of observations,
n: ensemble size.

Latter two can be made less on average with some work.

Good for GPUs (more computation per byte).

Results: Lorenz96

Standard model configuration, perfect model, three cases.

1. Identity observations, error variance 1, every 12 hours,
2. Identity observations, error variance 16, every hour,
3. 40 random observing locations, observation is log(state), error variance 1024, every 12 hours.

Fixed multiplicative inflation, fixed Gaspari-Cohn localization.
Search through 100 pairs of inflation/localization for each case.
Results for best case.

Results: Local Incremental Updates

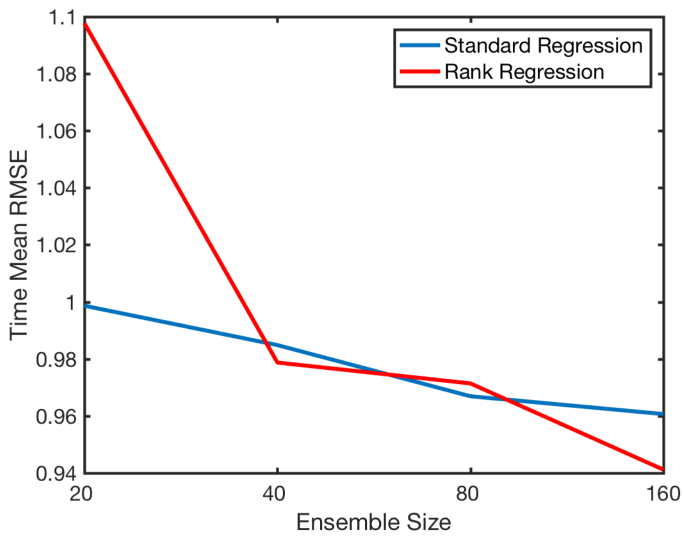
Checked subsets of 1/2, 1/4,1/8 of state variables.
Number of increments 1, 2, 4, 8.

Always did at least as well as other methods.

But, very expensive…

Results: Standard and Rank Regression

Error variance 16, every hour



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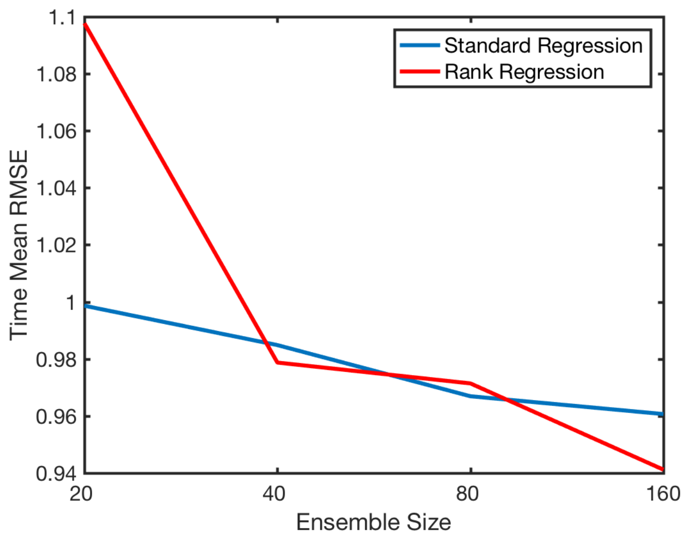
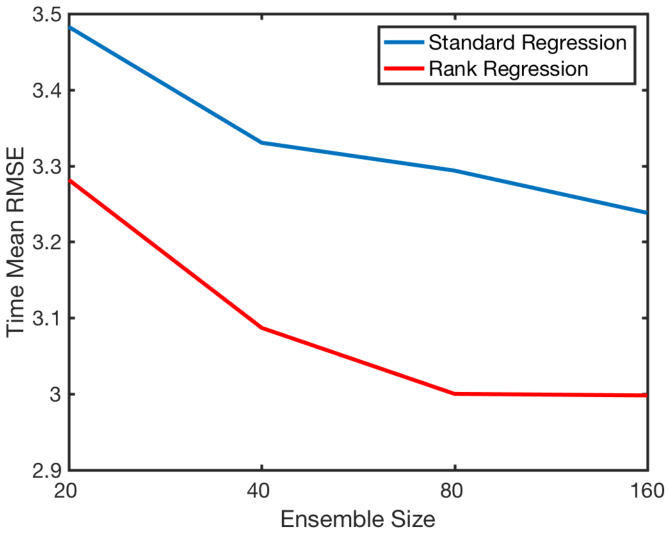
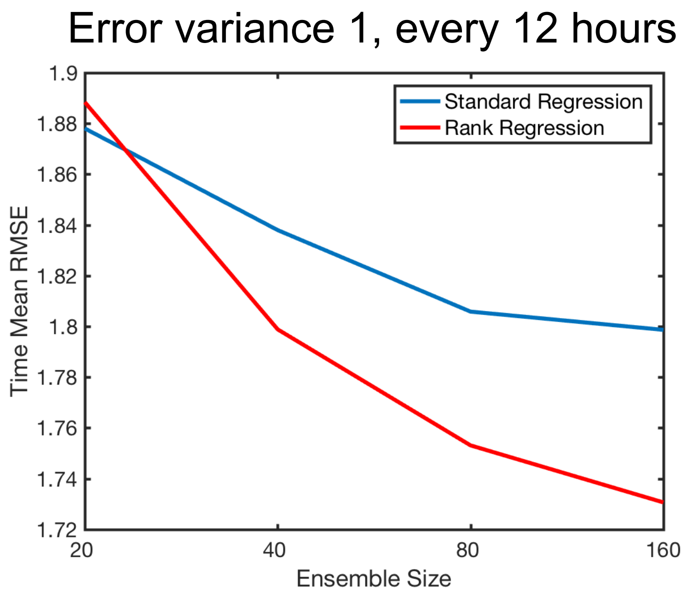


Log, error variance 1024, every 12 hours



Results: Standard and Rank Regression



Conclusions

Sequential ensemble filters can:

- Apply non-Gaussian methods in observation space,
- Nonlinear methods for bivariate regression.
- Lots of things to explore in this context.

Conclusions

Local regression with incremental update can be effective for locally smooth, continuous relations.

Can be expensive for 'noisy' bivariate priors:
- Requires large subsets (hence large ensembles),
- Subsets can be found efficiently,
- Incremental update is a multiplicative cost.

Can provide lower bounds for accuracy in some cases.

Conclusions

Rank regression effective for monotonic bivariate relations.

May be effective for:

- Nonlinear forward operators,
- Transformed state variables (log, anamorphosis, …).

Surprisingly effective for some more standard cases.

Moderate increase in cost.

Should be studied further.

All results here with DARTLAB tools freely available in DART.

www.image.ucar.edu/DAReS/DART

Anderson, J., Hoar, T., Raeder, K., Liu, H., Collins, N., Torn, R., Arellano, A., 2009: *The Data Assimilation Research Testbed: A community facility.* BAMS, **90**, 1283—1296, doi: 10.1175/2009BAMS2618.1