Radar observations in DART

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1. Introduction

The purpose of this document is to link theory on radar measurements and how these measurements are represented for the assimilation in the Data Assimilation Research Testbed (DART).

2. Radar observation location

What is usually known about radar observation is the position of the radar (λ_r, ϕ_r, h_r) , the length of the path *r* between the target and the radar (also referred to as the range), and the azimuth and elevation angles of the electromagnetic beam as it leaves the radar (α, θ_e) . This information has to be translated into longitude, latitude, and height at the observation location. In the case of Doppler radial velocities, the orientation of the beam at the observation location also has to be estimated. This is the purpose of what follows.

Doviak and Zrnic (1993) give in their (2.28b-c) approximations for the height of the observation *h* (above sea level) and the great circle distance *s* (see Fig. 1):

$$h = \left[r^2 + (k_e a)^2 + 2 r k_e a \sin(\theta_e)\right]^{1/2} - k_e a + h_r \quad (1)$$

$$s = k_e a \sin^{-1}\left(\frac{r \cos(\theta_e)}{k_e a + h}\right) \quad (2)$$

where r is the range, k_e = 4/3 assumes that the vertical gradient of the refractive index is constant and equal to -1/4a, a is the radius of the earth, and θ_e is the elevation angle of the beam as it leaves the radar. Note that Fig. 1 depicts the particular case $h_r = 0$, h_r being the elevation of the radar above sea level. Let (λ_r, ϕ_r) be the longitude and latitude at the radar location and (λ_o, ϕ_o) be the longitude and latitude of the observation. For $s << a$, the following approximations hold:

$$a\cos\left(\frac{(\phi_r+\phi_o)}{2}\right)(\lambda_o-\lambda_r)\approx s\sin(\alpha) \tag{3}$$

$$a(\phi_o-\phi_r)\approx s\cos(\alpha) \tag{4}$$

where α is the azimuth angle of the beam as it leaves the radar. The azimuth angle here is zero northward and positive clockwise. These expressions can be easily inverted to give (λ_o, ϕ_o) .

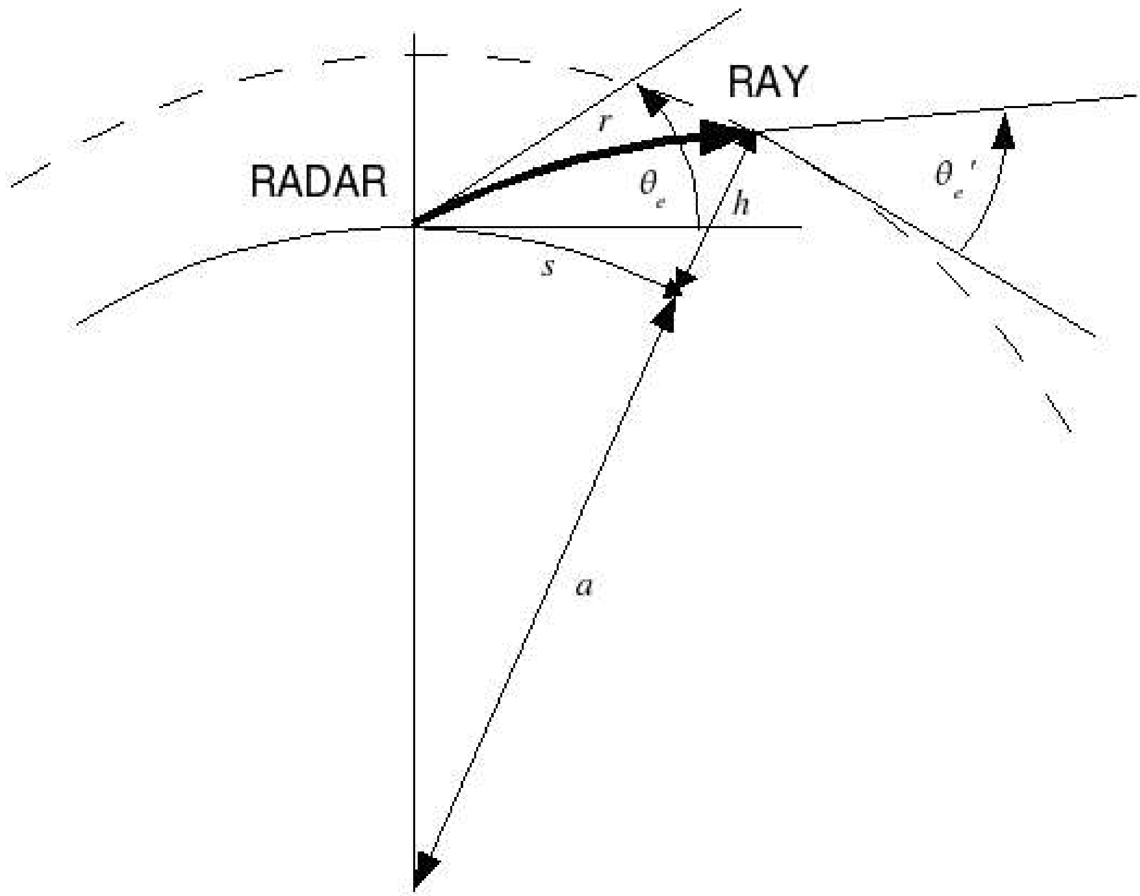

Figure 1 Circular path of a ray in an atmosphere in which the refractive index is linearly dependent on height.

3. Radar observation operators

To assimilate observations, a model is required to represent the observations in terms of state variables of the system. In the present section, the observation operators for radar reflectivity and Doppler velocity are described. It is assumed that radar data are point measurements but one has to keep in mind that real radar data represents measurements over a finite volume, weighted by the radar antenna pattern.

a. Radar reflectivity

The reflectivity factor for each hydrometeor category is calculated separately. The total reflectivity is simply the sum of the contributions from each hydrometeor category. The reflectivity contributed by ice crystals and clouds is assumed negligible. The reflectivity factors here are appropriate when the radar wavelength is much larger than the particle sizes.

The size distribution of the i^{th} hydrometeor class is assumed to be well approximated by an exponential function:

$$n_i(D) = n_{0i} \exp(-\lambda_i D) \tag{5}$$

where D is the particle diameter and the intercept parameter n_{0i} and the slope parameter of the size distribution λ_i are related to the mixing ratio of the species q_i by:

$$\lambda_i = \left(\frac{\pi \rho_i n_{0i}}{\rho q_i} \right)^{0.25} \tag{6}$$

where ρ_i is the density of the species and ρ is the air density. There are large uncertainties on ρ_i and the intercept parameters n_{0i} values. See Gilmore et al. (2004) for a discussion on this topic. Table 1 gives specific values for ρ_i and n_{0i}.

The effective reflectivity factor (the observed quantity) is

$$Z_e = C r^2 \frac{P_r}{P_t} \tag{7}$$

where P_t is the transmitted power, P_r is the received power, r is the range to the target, and C is the radar calibration coefficient. The effective reflectivity factor is modeled by a constant times the 6th moment of the size distribution:

$$Z_e \approx \sum_i c_i \int_0^\infty n_i(D) D^6 \, dD \tag{8}$$

According to theory, the backscattered energy is proportional to the 6th moment of the size distribution when all scatterers are spherical and the scattering is in the Rayleigh regime (radar wavelength >> particle diameter). If all scatterers are spherical, then c_i = 1. In general, the measurement Z_e is associated with a mixture of hydrometeor types. In what follows, explicit expressions for reflectivity factor are given for rain, dry and wet graupel/hail, and dry and wet snow.

i. Rain:

$$c_r = 1\,; Z_r = \frac{7.2\times 10^{20} (\rho q_r)^{1.75}}{\pi^{1.75} n_{0r}^{0.75} \rho_r^{1.75}} \tag{9}$$

ii. Wet snow:

$$c_{s,wet} = 1\,; Z_{s,wet} = \frac{7.2\times 10^{20} (\rho q_s)^{1.75}}{\pi^{1.75} n_{0s}^{0.75} \rho_s^{1.75}} \tag{10}$$

iii. Dry snow:

$$c_{s,dry} = \left(\frac{\left| K_{ice} \right|^2}{\left| K_w \right|^2} \right) \left(\frac{\rho_s^2}{\rho_r^2} \right); Z_{s,dry} = \frac{c_{s,dry} 7.2 \times 10^{20} (\rho q_s)^{1.75}}{\pi^{1.75} n_{0s}^{0.75} \rho_s^{1.75}} \tag{11}$$

iv. Wet graupel/hail:

$$c_{g,wet} = \frac{(7.2 \times 10^{20})^{0.95}}{\Gamma(7)} (n_{0g})^{0.0375} \left(\frac{\pi \rho_g}{\rho q_g} \right)^{0.0875} ; Z_{g,wet} = \left(\frac{7.2 \times 10^{20} (\rho q_g)^{1.75}}{\pi^{1.75} n_{0g}^{0.75} \rho_g^{1.75}} \right)^{0.95} \tag{12}$$

v. Dry graupel/hail:

$$c_{g,dry} = \left(\frac{\left| K_{ice} \right|^2}{\left| K_w \right|^2} \right) \left(\frac{\rho_g^2}{\rho_r^2} \right); Z_{g,dry} = \frac{c_{g,dry} 7.2 \times 10^{20} (\rho q_g)^{1.75}}{\pi^{1.75} n_{0g}^{0.75} \rho_g^{1.75}} \tag{13}$$

Here $\left| K^2 \right|$ is the dielectric factor. Expressions for dry particles are applied for temperatures below 0°C and expressions for wet particles are applied for temperatures above 0°C. Hence, a vertical discontinuity in reflectivity is expected at the melting layer. To avoid this problem, one can define a transition zone where the amount of dry and wet particles changes continuously, from dry particles only to wet particles only as the temperature increases and crosses the melting point. The expression for wet graupel/hail (12) is elevated to the 0.95 power to take into account Mie effect (Smith, 1984) and is appropriate for 10-cm radars. For details about these 5 expressions, see Smith et al. (1975) and Smith (1984). Additional references can be found in Ferrier (1994).

b. Doppler velocity

The radial wind sample from the model is computed as follows:

$$v_r = d_x u + d_y v + d_z (w - w_t) \quad (14)$$

where (u, v, w) are the zonal, meridional, and vertical wind components, w_t is the terminal fall speed of the radar scatterers, and

$$d_x \equiv \sin(\alpha')\cos(\theta_e') \quad (15)$$

$$d_y \equiv \cos(\alpha')\cos(\theta_e') \quad (16)$$

$$d_z \equiv \sin(\theta_e') \quad (17)$$

where the primes refer to the azimuth and elevation angles at the observation location. The direction vector d is calculated before the assimilation for each observation and stored in the observation file. This strategy is employed to avoid superfluous calculation during the assimilation. The azimuth angle of the observed velocity component is approximated to the azimuth angle of the beam as it leaves the radar $(\alpha' = \alpha)$. For the elevation angle of the observed velocity component, we follow Battan's (1973) equation (3.18a) and assume the same vertical gradient of the refractive index as in section 2:

$$\theta_e' \approx \left[\frac{1.5h}{a} + \theta_e^2 \right]^{1/2} \quad (18)$$

This approximation is appropriate for small elevation angles.

The terminal fall speeds for precipitating particle of diameter D for rain, snow, and graupel are, respectively,

$$U_r = aD^b \left(\frac{\rho_0}{\rho} \right)^{1/2} \tag{19}$$

$$U_s = cD^d \left(\frac{\rho_0}{\rho} \right)^{1/2} \tag{20}$$

$$U_g = \left(\frac{4 g \rho_g}{3 C_D \rho} \right)^{1/2} D^{1/2} \quad . \tag{21}$$

For references, see Lin et al, (1983). We define w_t the reflectivity-weighted mean terminal speeds as

$$w_t = \frac{\sum_i c_i \int_0^\infty U_i n_i(D) D^6 dD}{\sum_i Z_i} = \frac{\sum_i \omega_i}{\sum_i Z_i} \tag{22}$$

where the ω_i are the reflectivity-convoluted mean terminal speeds for each hydrometeror category. The 5 terms in the denominator are given by equations (9) to (13). For the numerator in (22), we have

i. Rain:

$$\omega_r = 10^{18} n_{0r} a \left(\frac{\rho_0}{\rho} \right)^{1/2} \Gamma(b+7) \left(\frac{\rho q_r}{\pi n_{0r} \rho_r} \right)^{\left(\frac{b+7}{4} \right)} \tag{23}$$

ii. Snow:

$$\omega_{s,dry/wet}=10^{18}c_{s,dry/wet}n_{0s}c\left(\frac{\rho_0}{\rho}\right)^{1/2}\Gamma(d+7)\left(\frac{\rho q_s}{\pi n_{0s}\rho_s}\right)^{\left(\frac{d+7}{4}\right)} \tag{24}$$

iii. Graupel / Hail:

$$\omega_{g,dry/wet}=10^{18}c_{g,dry/wet}n_{0g}\left(\frac{4g\rho_g}{3C_D\rho}\right)^{1/2}\Gamma(7.5)\left(\frac{\rho q_g}{\pi n_{0g}\rho_g}\right)^{(1.875)} \tag{25}$$

where the factor 10^{18} takes into account that the reflectivity in the denominator has mm^6 m^{-3} units. The reflectivity-weighted mean terminal speeds as functions of precipitation content are shown in Fig. 2. For references on reflectivity-weighted mean terminal speeds derivations, see Doviak and Zrnic (1993) p.275, Hauser and Amayenc (1981), and Conway and Zrnic (1993) in their appendix.

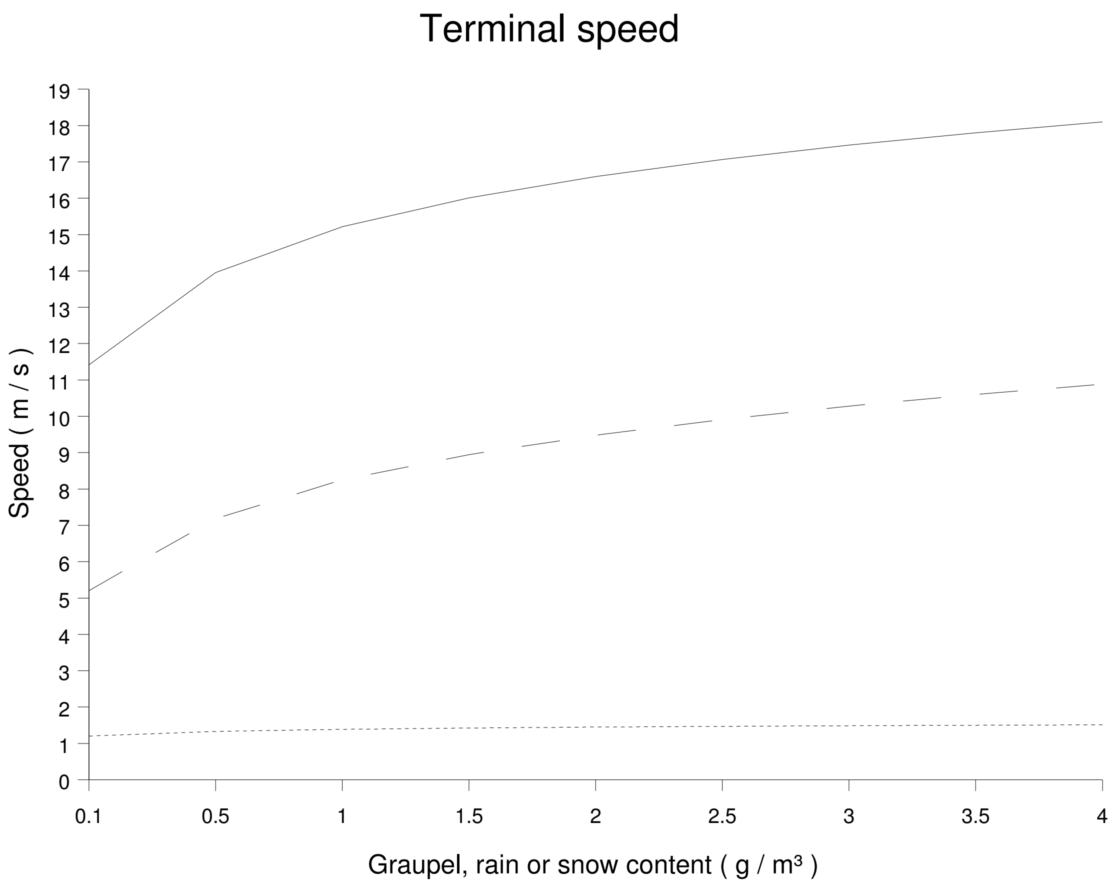

Figure 2 Reflectivity-weighted mean terminal speeds for graupel (full line), rain (dashed line) and snow (dotted line).

LIST OF SYMBOLS

Notation	***Description***	***DART value [range]***	***Units***
a	constant in empirical formula for U_r	842	$m^{(1-b)}$ s^{-1}
b	constant in empirical formula for U_r	0.8	
c	constant in empirical formula for U_s	4.84	$m^{(1-d)}$ s^{-1}

Notation	***Description***	***DART value** [range]*	***Units***
C_D	drag coefficient for hailstone	0.6	
d	constant in empirical formula for U_s	0.25	
g	gravitational acceleration	9.81	m s^{-2}
K_{ice}^2 / K_w^2	dielectric factor ratio	0.224 (0.189 or 0.224)†	
n_{0r}	intercept parameter of the raindrop size distribution	8×10^6	m^{-4}
n_{0g}	intercept parameter of the graup size distribution	4×10^4 ‡ $[4\times10^3, 4\times10^6]$	m^{-4}
n_{0s}	intercept parameter of the snow size distribution	3×10^6	m^{-4}
q	mixing ratio		kg kg^{-1}
Z_e	effective reflectivity factor		mm^6 m^{-3}
ρ	air density		kg m^{-3}
ρ_0	surface air density	1	kg m^{-3}
ρ_g	density of graupel	917‡ [400, 900]	kg m^{-3}
ρ_s	density of snow	100	kg m^{-3}
ρ_r	density of water	1000	kg m^{-3}

† According to Smith (1984), there are two choices for the dielectric factor, depending on how the snow particle sizes are specified. If melted raindrop diameters are used, then the factor is 0.224. If equivalent ice sphere diameters are used, then the factor is 0.189.

‡ From Lin et al. (1983).

APPENDIX

$$\int_0^\infty x^{\nu-1} e^{-\mu x} dx = \frac{1}{\mu^\nu} \Gamma(\nu),\ R(\mu) > 0, R(\nu) > 0 \tag{A1}$$

The gamma function Γ is related to factorials when the argument is an integer:

$$\Gamma(n) = (n-1)! \tag{A2}$$

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