

# Data Assimilation Research Testbed Tutorial



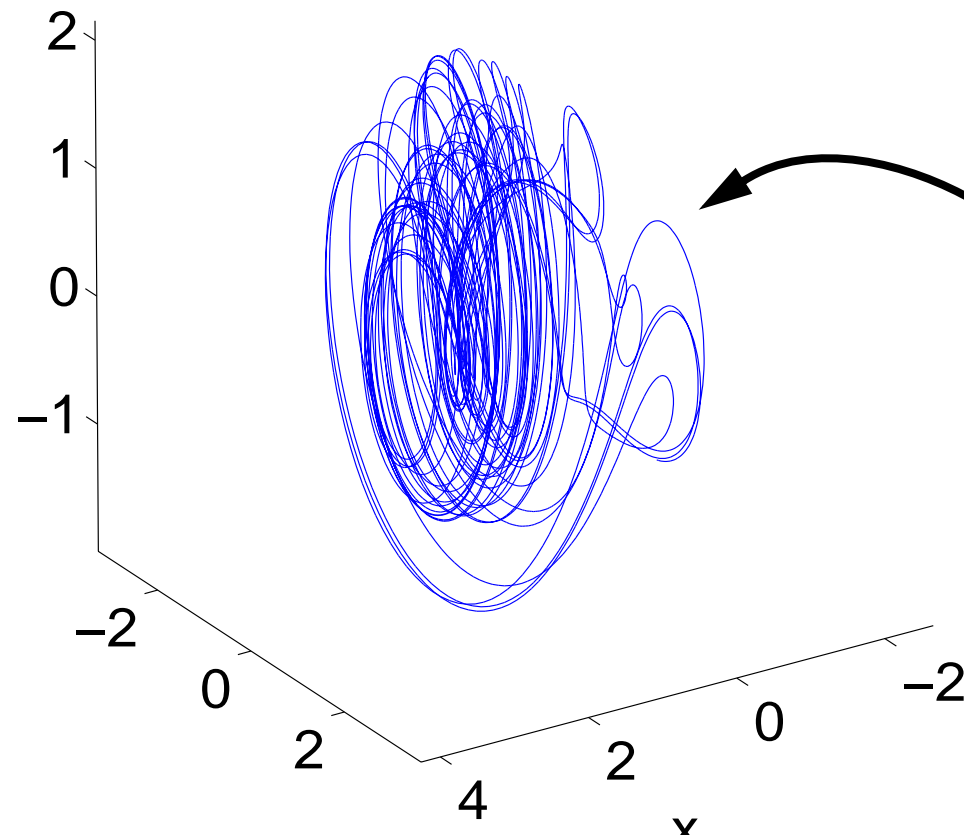
## Section 7: Some Additional Low-Order Models

Version 2.0: September, 2006

## Low-order models in DART:

Model	Size	Features
lorenz_63	3	Chaotic, nearly integral attractor, bifurcations
lorenz_84	3	More complex attractor, not as periodic
9var	9	Transient off-attractor dynamics
lorenz_96	40 (variable)	Higher dimensional system. Attractor dimension 13.
forced_lorenz_96	80 (variable)	Allows assimilation of model parameter (see Section 20).
lorenz_96_2scale	440 (variable)	Two primary interacting spatial/temporal scales.
lorenz_04	variable	Multiscale dynamics.

## Lorenz 84 model:



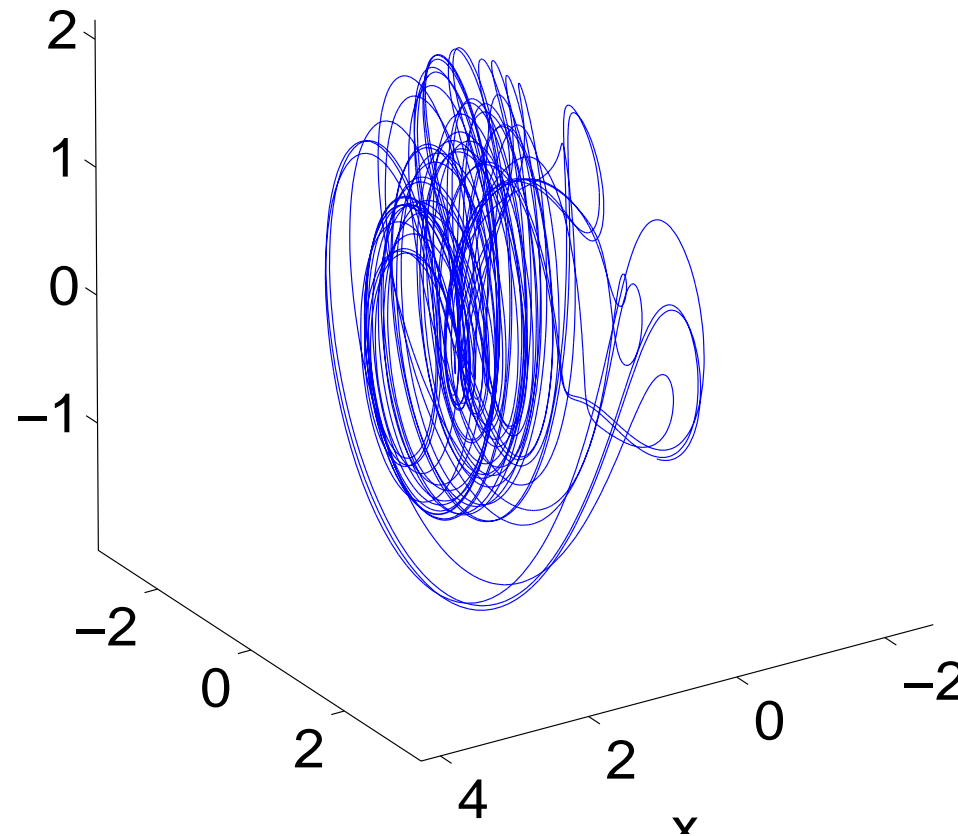
Attractor not sheet-like.

Rare significant deviations.

Trajectories along deviations  
don't 'mesh' back up with rest of  
attractor.

This behavior can be challenging  
for certain filter variants.

## Lorenz 84 model:



3-variables:

$$\frac{dx_1}{dt} = -x_2^2 - x_3^2 - ax_1 + af$$

$$\frac{dx_2}{dt} = x_1x_2 - bx_1x_3 - x_2 + g$$

$$\frac{dx_3}{dt} = bx_1x_2 + x_1x_3 - x_3$$

Parameters:  $a = 0.25$ ,  $b = 4$ ,  
 $f = 8$ ,  $g = 1.25$   
can be set from model\_nml.

## Lorenz 84 model:

Run *csh workshop\_setup.csh* in directory *models/lorenz\_84/work*.

Each state variable is observed every once every hour.  
Observational error variance is 1.

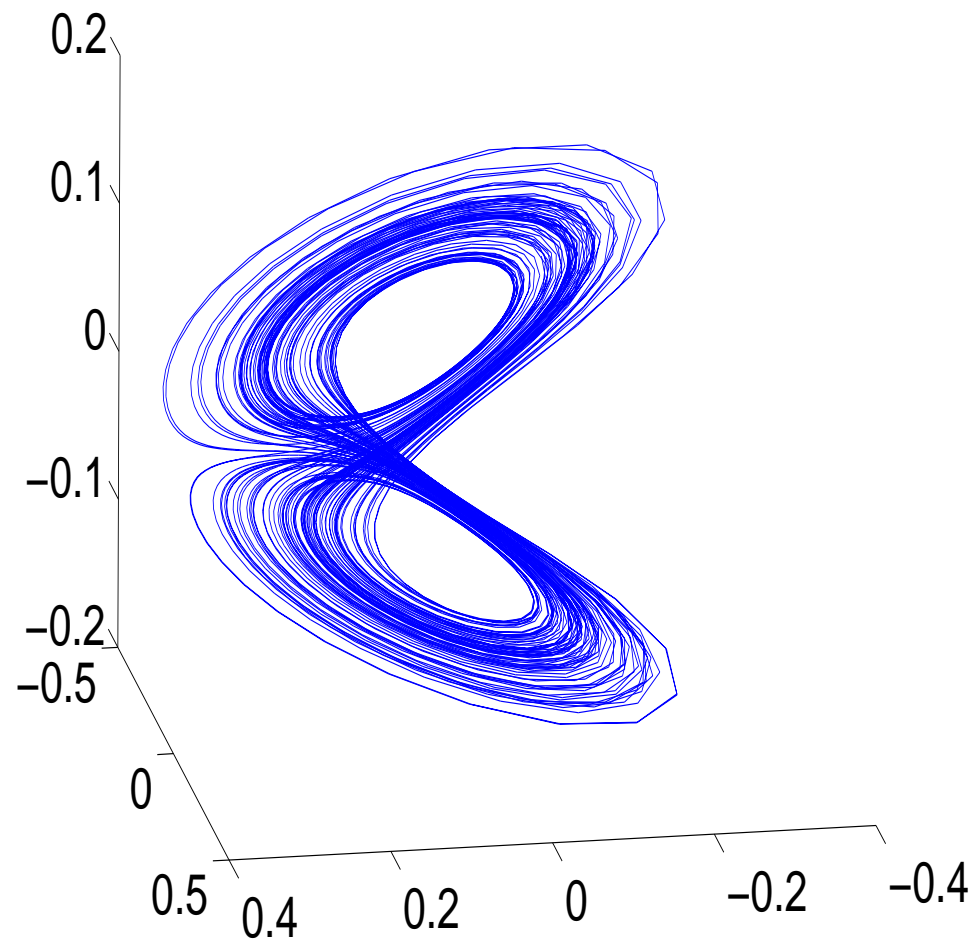
Use matlab to examine the output.

There's a new type of filter challenge represented here.

Can you identify it?

Can you propose ways to address it with techniques learned to date?

## 9 Variable model:



Three groups of variables

Variables 1-3: Divergence

Variables 4-6: Vorticity.

Variables 7-9: Height.

In general, divergence is small.

Height and pressure similar.

Height and pressure have  
attractor similar to Lorenz\_63.

## 9 Variable model:

$$\dot{X}_i = U_j U_k + V_j V_k - v_0 a_i X_i + Y_i + a_i z_i \quad (1)$$

$$\dot{Y}_i = U_j Y_k + Y_j V_k - X_i - v_0 a_i Y_i \quad (2)$$

$$\dot{z}_i = U_j (z_k - h_k) + (z_j - h_j) V_k - g_0 X_i - K_0 a_i z_i + F_i \quad (3)$$

$$U_i = -b_j x_i + c y_i \quad (4)$$

$$V_i = -b_k x_i - c y_i \quad (5)$$

$$X_i = -a_i x_i \quad (6)$$

$$Y_i = -a_i y_i \quad (7)$$

X=> Divergence,                      Y=>Vorticity,                      z=> height

Parameters can be adjusted from model\_nml.

## 9 Variable model:

When perturbed off the attractor, mimics ‘gravity waves’.  
Transient, high frequency oscillations dominate divergence variables.  
Can also appear in height and pressure variables.

Run *csh workshop\_setup.csh* in directory *models/9var/work*.

Y1, Y2, Y3 (the ‘vorticity’ variables) are observed once every 6 hours  
Observational error variance is 0.4.

Use matlab to examine the output.

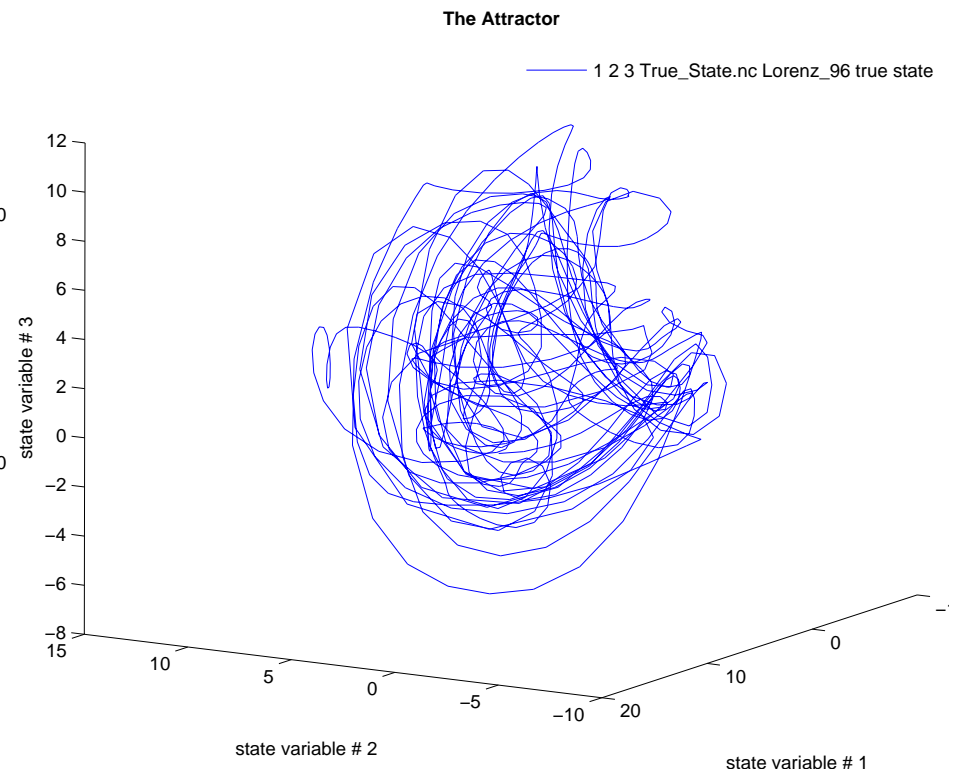
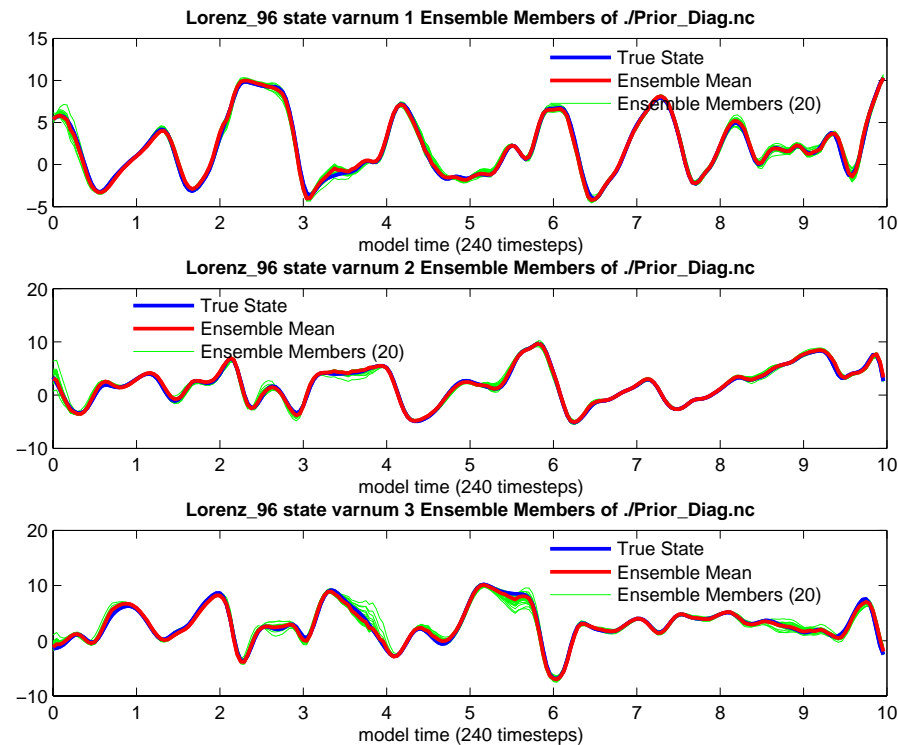
How do different filter kinds interact with ‘gravity’ waves?



## Lorenz\_96 (40-variable) model:

One dimensional cyclic domain  $[0.0, 1.0]$ .

Acts something like synoptic scale weather around mid-latitude circle.



## Lorenz\_96 (40-variable) model:

Attractor dimension 13 by some measures.

Start to explore model sizes closer to ensemble size.

Can examine possible degeneracy issues with sample covariance.

Naive application of small ensembles diverges in many cases.

Run *csh workshop\_setup.csh* in directory *models/lorenz\_96/work*.

40 observations, randomly located in time but fixed in space.

Observed once an hour; Observational error variance is 1.0.

Use matlab to examine the output.

Need new techniques to fix this.