Anders Malmberg

- PhD in Mathematical Statistics, Lund University, Sweden, 2005. *Space-Time Prediction of Ocean Winds*.
- Starting project *Mesoscale/Tropical Balance Constraints and Data Assimilation*. Joint work with the Mesoscale and Microscale Meteorology Division, NCAR.
- Presenting *A Stochastic Transport Model for Atmospheric Carbon Monoxide* a collaboration with David Edwards, Ave Arellano, Atmospheric Chemistry Division, NCAR, Doug Nychka, IMAGe, NCAR, and Chris Wikle, Department of Statistics, University of Missouri.

A Stochastic Transport Model for Atmospheric Carbon Monoxide

Anders Malmberg

April 27th, 2006

Outline

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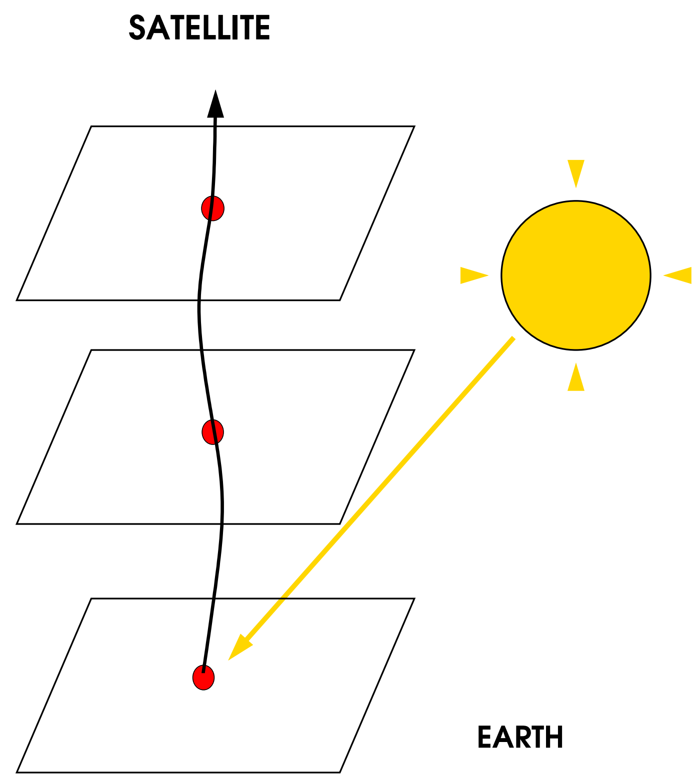
Carbon Monoxide, CO

- **Facts**
  - Anthropogenic sources accounts for a major part of the atmospheric CO. Examples are incomplete combustion in automobile engines, industrial processes, and the burning of forests to expand agriculture.
  - In the atmosphere, CO contributes to the abundance of ozone, methane, and other greenhouse gases.
  - One of six air pollutants that the US Congress and the EPA mandates regular monitoring of since it endanger public health and the environment.
- **Atmospheric Research Interests**
  - CO is used as a fingerprint for anthropogenic activities.
  - CO is important for making statements about air quality and studying climate change.

How to Remotely Sense Atmospheric CO

CO in different vertical layers in the lower portion of the atmosphere can be remotely sensed from space by measuring the surface radiance.



Satellite Retrievals

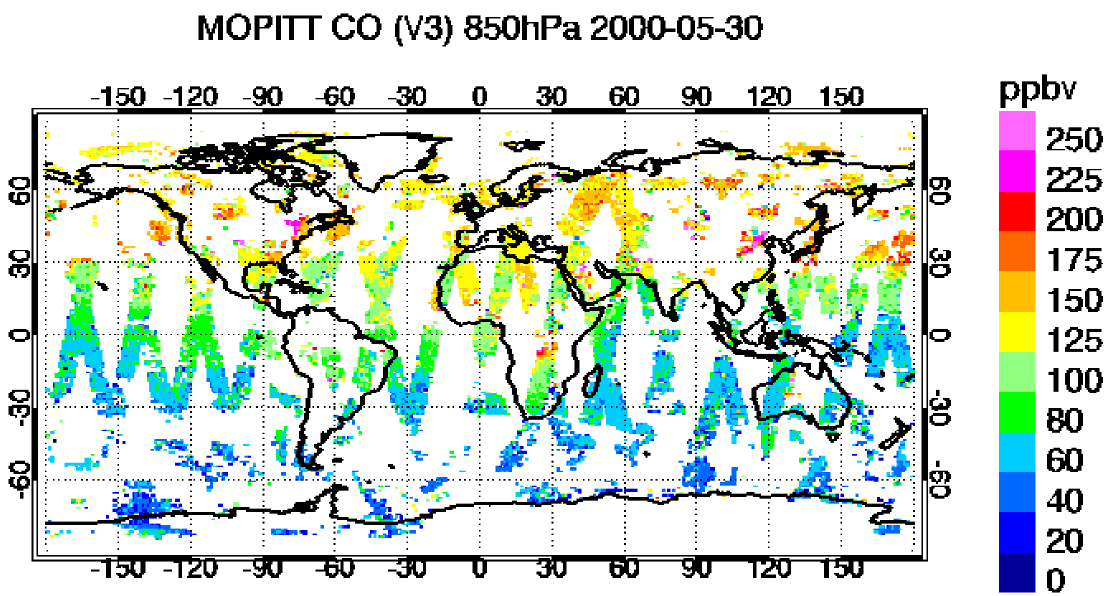
Solving the inverse problem, the column of retrieved concentrations at a longitude/latitude location s and time t, can be approximated with

$$\begin{pmatrix} Z^1(s,t) \\ Z^2(s,t) \\ \vdots \\ Z^7(s,t) \end{pmatrix} = A \begin{pmatrix} \alpha^1(s,t) \\ \alpha^2(s,t) \\ \vdots \\ \alpha^7(s,t) \end{pmatrix} + (\mathbf{I} - A)\alpha_{prior} + \begin{pmatrix} \epsilon^1(s,t) \\ \epsilon^2(s,t) \\ \vdots \\ \epsilon^7(s,t) \end{pmatrix}$$

where A is a 7 by 7 averaging matrix, $\alpha(s,t) = (\alpha^1(s,t), \ldots, \alpha^7(s,t))^T$ is the unknown true concentration that we want to estimate, and ϵ is assumed to be additive white noise. The inverse problem is constrained by α_{prior}.

Satellite Retrievals

The Terra satellite circles the earth in 99 minutes and a near global coverage in 5 days. The plot shows daily satellite retrievals, at 850 hPa, gridded to 1 degree horizontal resolution.



Gridded at 1x1deg from MOP02-20000530-L2V5.7.1.prov.hdf (apriori fraction < 50%)

Statistical Challenges

- **We have:**
  - sparse data in space and time,
  - data contaminated with observation noise.
- **We want to:**
  - validate satellite data,
  - have a complete space-time data set and a quantified measure of uncertainty,
  - compare satellite data to chemical transport model data.
- **Statistical challenges:**
  - estimate a dynamical model in four dimensions that allows us to fill in spatial and temporal gaps in the data,
  - merge different data sets in an objective fashion while accounting for their different uncertainties.

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Bayes' Theorem and Hierarchical Modeling

Assuming we want to estimate a process and some parameters given data, $[process, parameters | data]$, we use Bayes' Theorem to rewrite this in densities that we can specify:

$$\frac{[data|process, parameters][process|parameters][parameters]}{[process, parameters]}$$

- data stage, likelihood of data given process,
- process stage, specifying spatial and temporal dynamics,
- parameter stage, with hyper parameters.

Data Stage, $[data|process, parameters]$

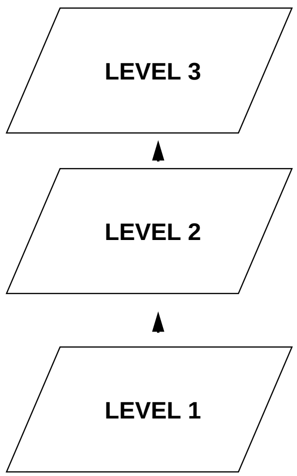
Using the approximate solution of the inverse model, we assume that the likelihood of a column of satellite data, Z, given the true process, α, is,

$$\begin{pmatrix} Z^1(s,t) \\ Z^2(s,t) \\ \vdots \\ Z^7(s,t) \end{pmatrix} \Bigg| \begin{pmatrix} \alpha^1(s,t) \\ \alpha^2(s,t) \\ \vdots \\ \alpha^7(s,t) \end{pmatrix} \sim MVN\Bigg(\mathrm{A} \begin{pmatrix} \alpha^1(s,t) \\ \alpha^2(s,t) \\ \vdots \\ \alpha^7(s,t) \end{pmatrix} + (\mathbf{I} - \mathrm{A})\alpha_{prior}, \Sigma_\epsilon\Bigg),$$

where Σ_ϵ is the variance-covariance matrix of the observations.

In the vertical, we assume a Markov property. At each time we assume the vertical levels are conditionally independent,

$$[\alpha^3(t), \alpha^2(t), \alpha^1(t)] = [\alpha^3(t)|\alpha^2(t)][\alpha^2(t)|\alpha^1(t)][\alpha^1(t)]$$

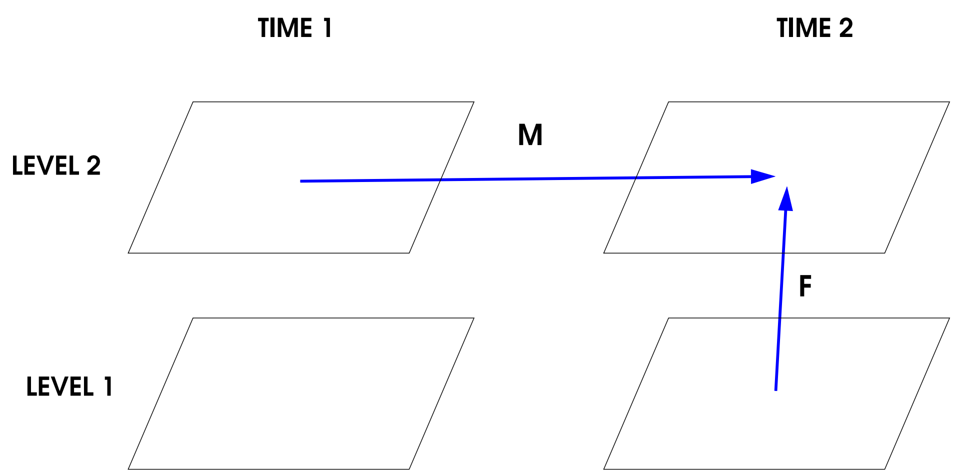


Process Stage, [*process*|*parameters*]

The temporal dynamics are assumed to follow an advection diffusion equation,

$$\alpha^2(t_2)|\alpha^1(t_2), \alpha^2(t_1) \sim MVN\left(\mathrm{M}(t_1)\alpha^2(t_1) + \mathrm{F}\alpha^1(t_2), \Sigma_\eta\right),$$

where M is the advection diffusion matrix, and the forcing matrix F, origins in the vertical Markov assumption.



Outline

Observing Systems Simulation Experiment

General Setup:

- 15×15 longitude latitude domain in the North Pacific.
- For the **vertical** model we assume that,

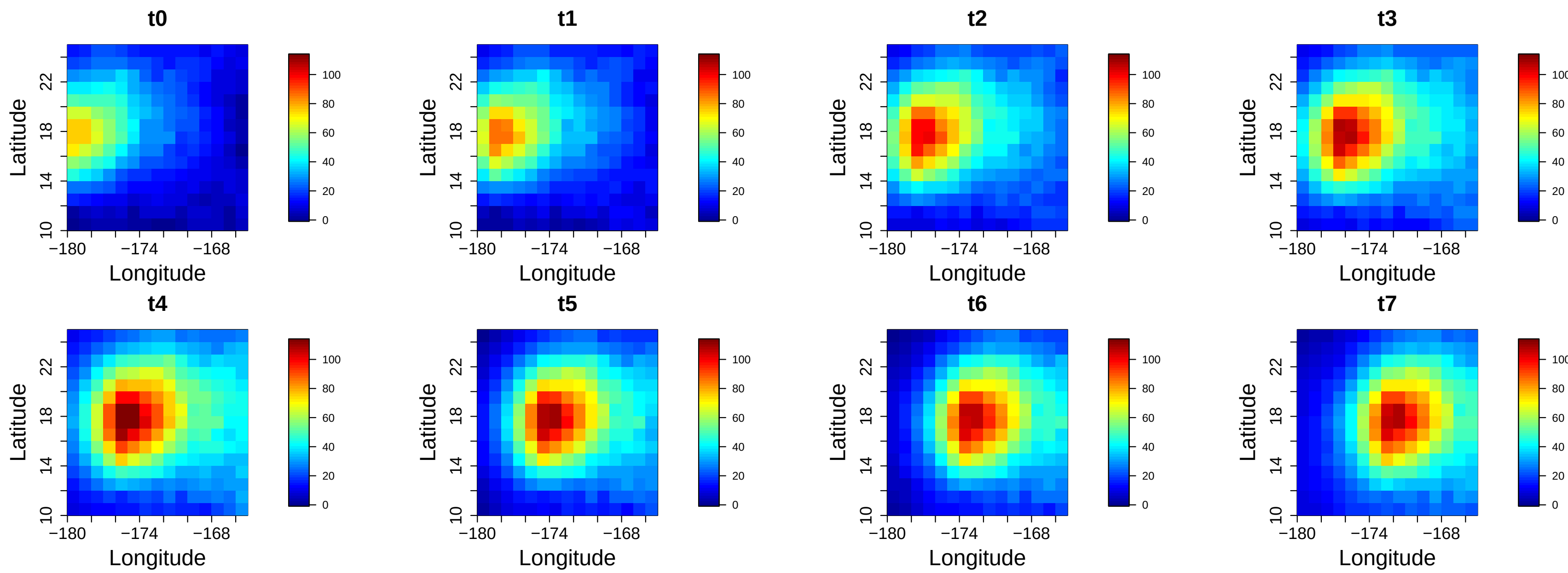
$$\alpha^j(s_i, t) = f^j \alpha^{j-1}(s_i, t).$$

- For the **horizontal** model we assume a simple translation,

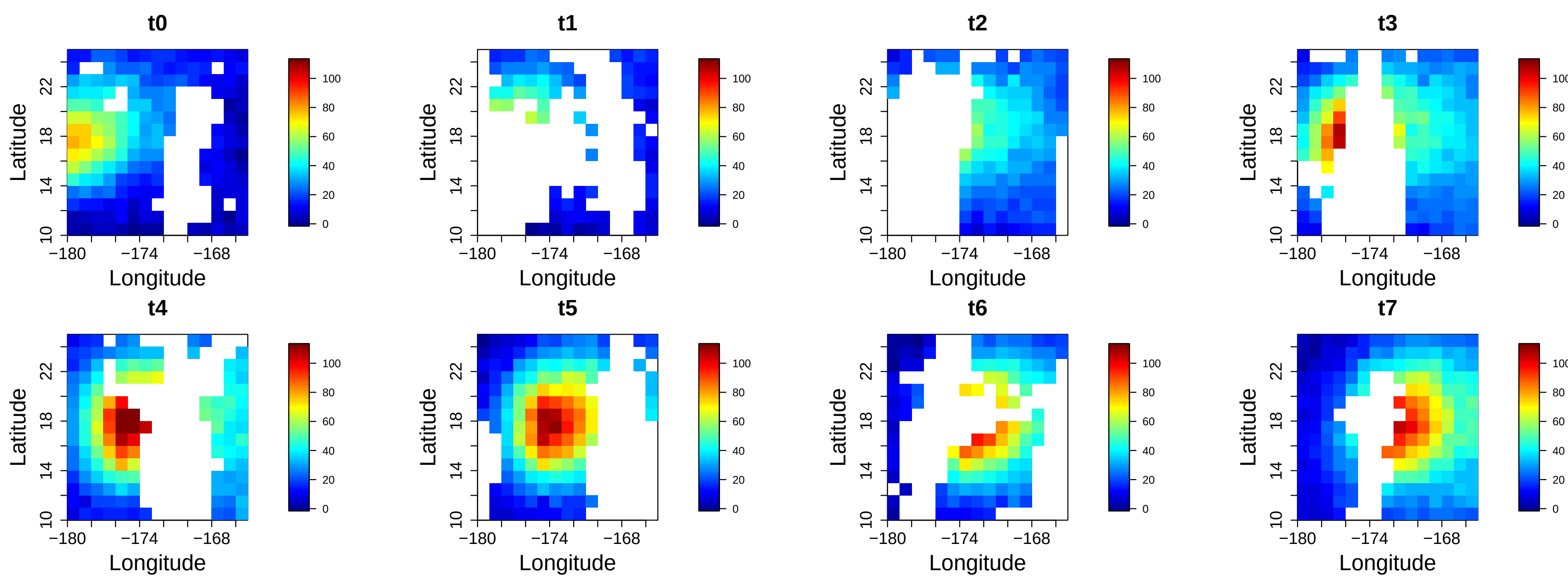
$$\alpha^j(s_i, t) = m^j \alpha^j(s_i - \Delta_x, t - 1).$$

- Using these parameters and dynamical noise a “true” atmospheric plume is simulated.
- Apply Gibbs sampler to estimate f^j, m^j, and α^j.

Simulated Truth 700 hPa

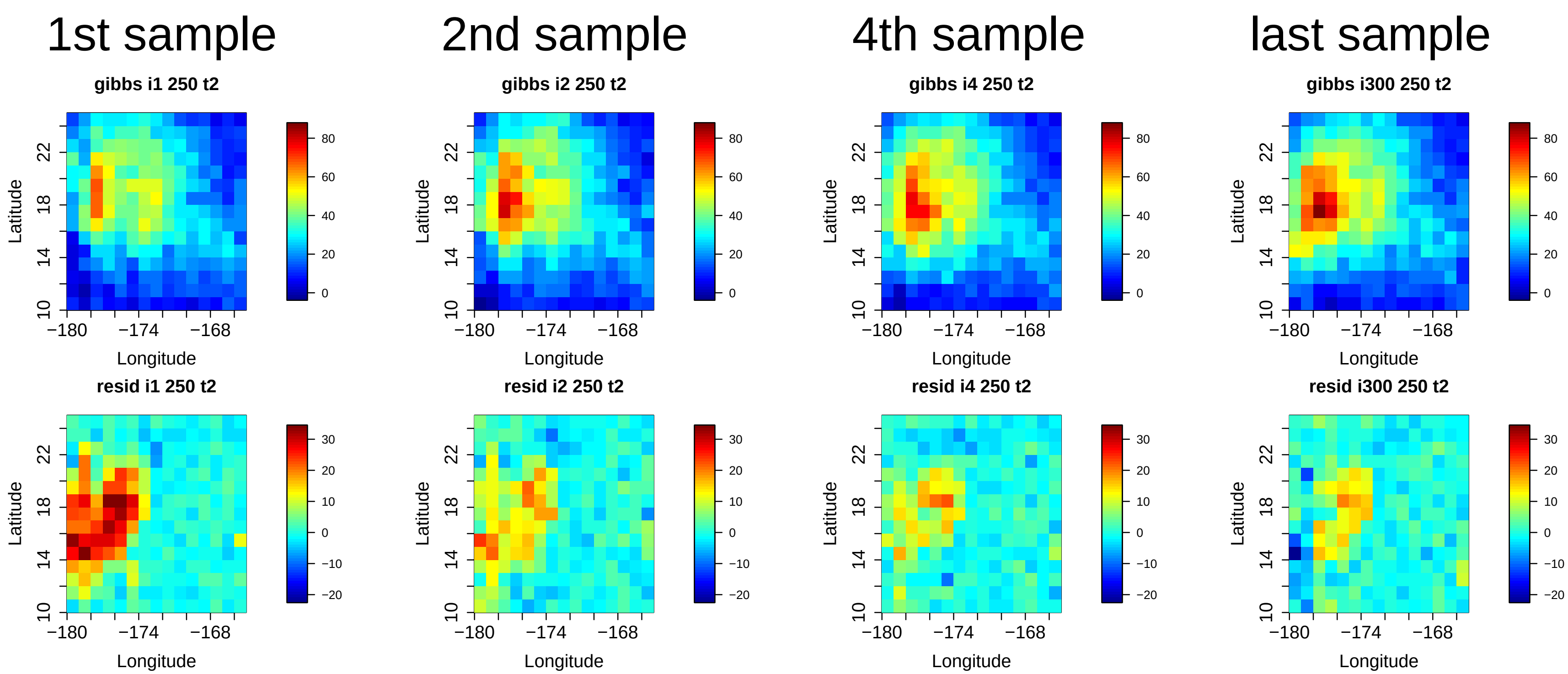


Simulated Satellite Data 700 hPa



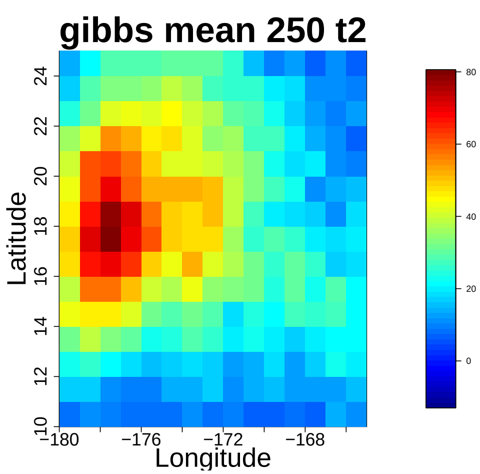
Gibbs Samples 250 hPa, time t_2, with strong prior

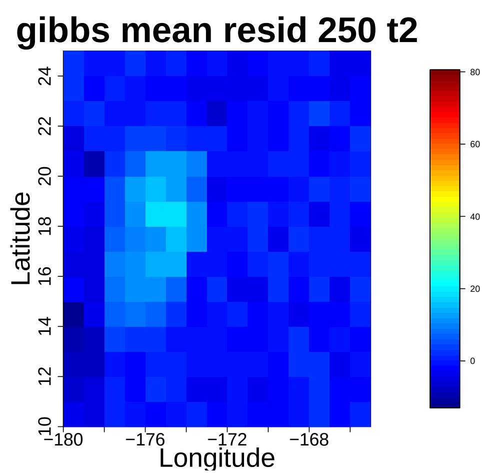
Upper levels has proved to be hard to estimate in this model.

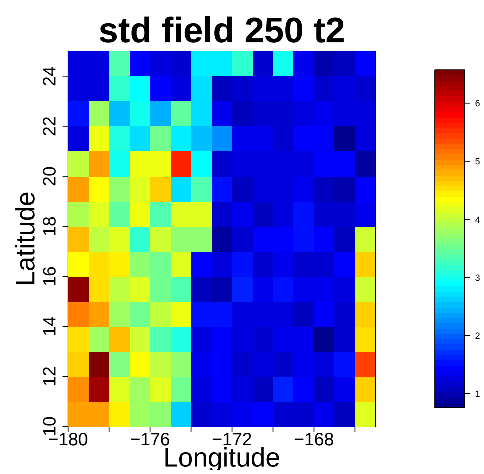


Gibbs Statistics 250 hPa, time t_2, with strong prior

Using close to true parameters for f and m the plume is traceable.
Locally, the data from the satellite decreases the standard deviation.

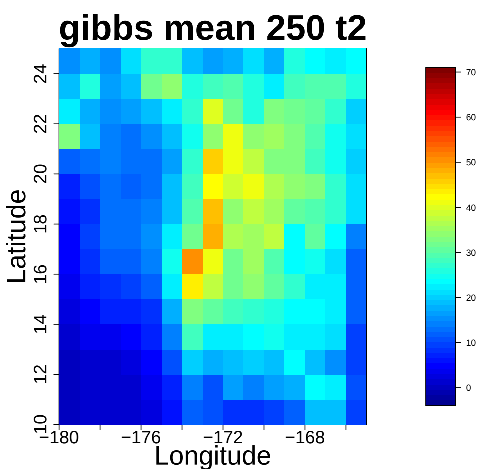


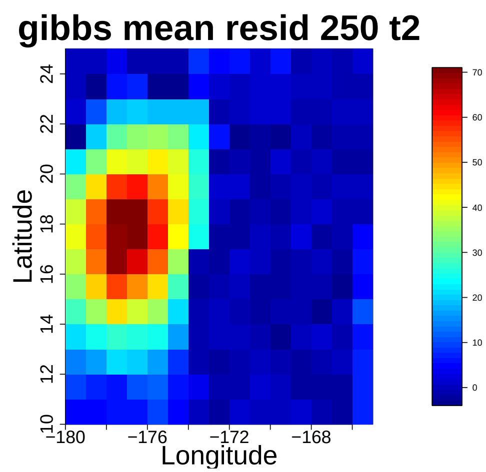


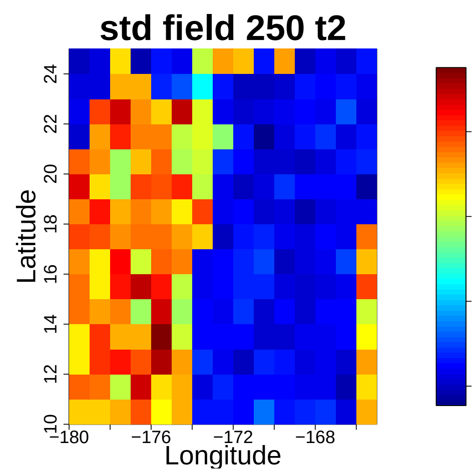


Gibbs Statistics 250 hPa, time t_2

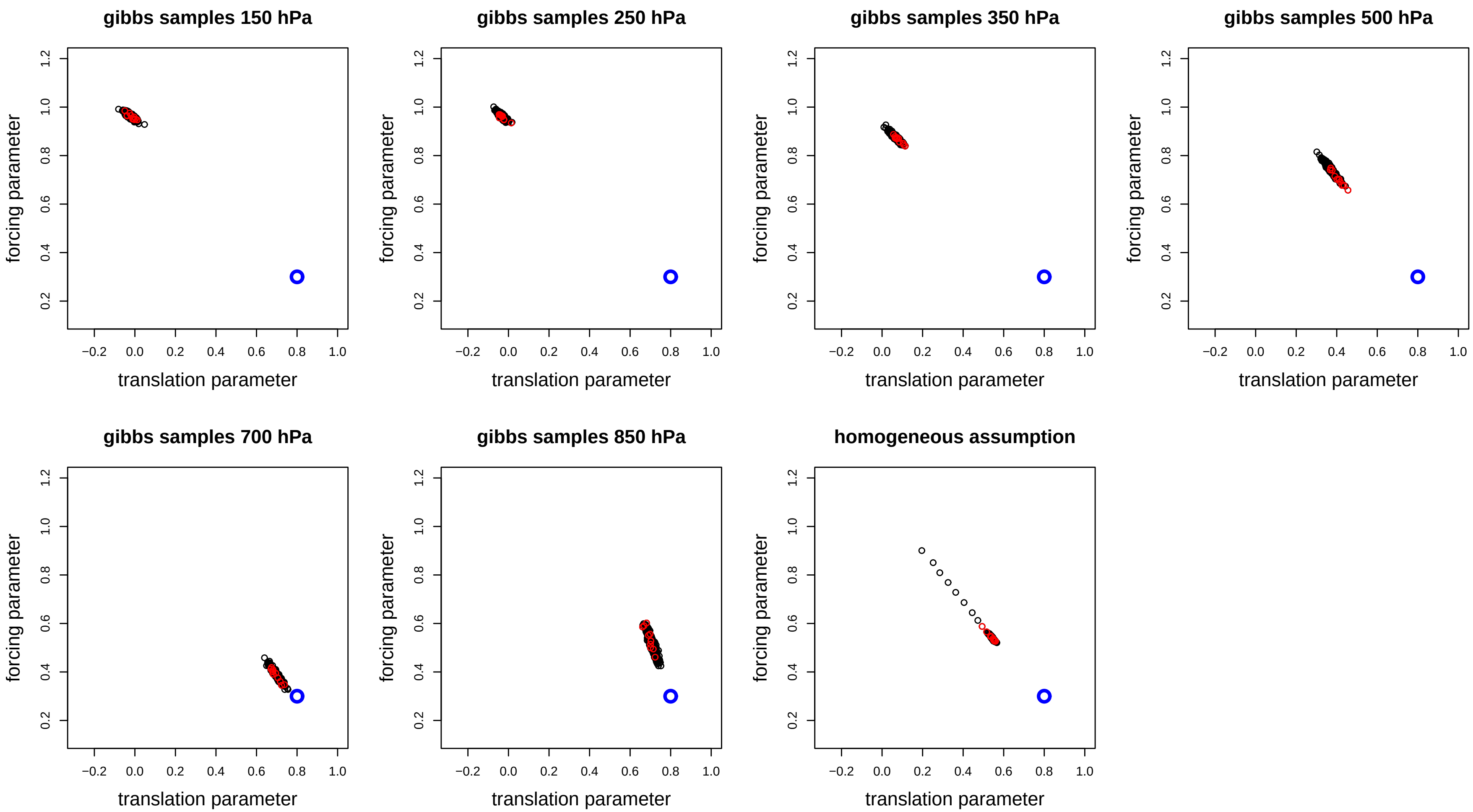
When we use a flat prior it does not work as well. The translation parameter for this level is close to zero and information is not conveyed over time.







Samples of *f* versus Samples of *m*



Full Conditional Means of m and f

$$
\begin{aligned}
E(m|\cdot) \propto & \sum_{k=1}^{T} \alpha^{j}(t_{k-1})^{T} \Sigma_{\eta}^{-1} \alpha^{j}(t_k) - \sum_{k=1}^{T} \alpha^{j}(t_{k-1})^{T} \Sigma_{\eta}^{-1} \alpha^{j-1}(t_k) \cdot f + \sigma_m^{-2} \mu_m, \\
E(f|\cdot) = & \Big(\sum_{k=1}^{T} \alpha^{j-1}(t_k)^{T} \Sigma_{\eta}^{-1} (\alpha^{j-1}(t_k)) \Big)^{-1}. \\
& \sum_{k=1}^{T} \alpha^{j-1}(t_k)^{T} \Sigma_{\eta}^{-1} (\alpha^{j}(t_k) - \alpha^{j}(t_{k-1}) \cdot m) + \sigma_f^{-2} \mu_f.
\end{aligned}
$$

- If $f \to 1$ and $\alpha^{j}(t_k) \sim \alpha^{j-1}(t_k)$ then $(\alpha^{j}(t_k) - \alpha^{j-1}(t_k) f) \to 0$.
- If $m \to 0$ then $E(f|\cdot) \to 1$.

Outline

Background
- Satellite Data
- Statistical Challenges

Statistical Model
- Bayes' Theorem and Hierarchical Modeling
- Hierarchical Stages

Observing Systems Simulation Experiment
- Experiment Setup
- Gibbs Estimations

Future Work

Future Challenges

- Apply full advection/diffusion model to output from chemical transport model. To what extent can a simple stochastic model emulate complex dynamics?
- What impact on parameter estimates and spatial prediction do the sparse sampling have? Will the dynamical model be informative enough to do the spatial interpolation?
- When we apply the true averaging kernel, how much can we resolve in the vertical?