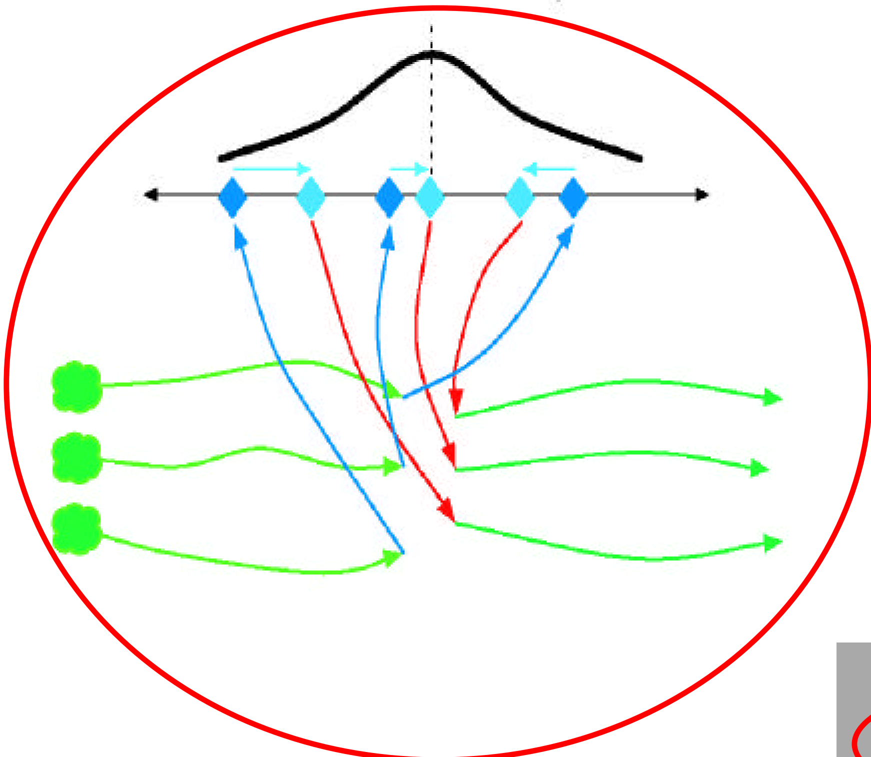
1

PLEASE POST

The Statistical and Applied Mathematical Sciences Institute
and the Institute for Mathematics Applied to Geosciences
Announce a Summer School on

Fusing Geophysical Models with Data

Boulder, CO **13-17 June 2005**



$$p\left(x, t_k \mid Y_{t_k}\right) = \frac{p\left(y_k \mid x_k\right) p\left(x, t_k \mid Y_{t_{k-1}}\right)}{\int p\left(y_k \mid \xi\right) p\left(\xi, t_k \mid Y_{t_{k-1}}\right) d\xi}$$

Data Assimilation: What is the point?

Christopher K. Wikle
Department of Statistics
University of Missouri-Columbi

- Combining Information
  - "interpolating fields for subsequent use as initial data in a model integration" (Bennett, 2002)
  - "statistical combination of observations and short-range forecasts" (Kalnay, 2003)
  - "using all the available information, to define as accurate as possible the state" (Talagrand, 1997)

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- Combining Information

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Within the Perfect Model Scenario, these two goals are likely to coincide.

They are two very different goals outside PMS

  – "using all the available information, to define as accurate as possible the state" (Talagrand, 1997)

What state?

Or perhaps better: state of what? Model-state or "reality"

PMS: There exist model(s) within the given model class which could have produced the data. (Borel "could")

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    Most useful model-state(s) for forecasting.

  - "using all the available information, to define as accurate as possible the state" (Talagrand, 1997)

    Most useful model-state(s) for now-casting.

- *Statistical Perspective:* Fusing data (observations) with prior knowledge (e.g., physical laws; model output) to get an estimate of the (distribution of) the true state of the physical system

Data Assimilation: What is the point?

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How might this be verified? falsified? ~~measured~~ known even once?
Our distributions are in model-state spaces, not the "true state space" (if such a thing even exists!)
(Do we really gain/lose anything via this neo-platonic realist belief?)

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and thus

$$p(X|Y) = \frac{p(Y|X)p(X)}{p(Y)} \qquad \text{(Bayes' Rule)}$$

provided $p(Y) \neq 0$

where Y are the observations
X is the model-state

CKW: ?true state?

1.3 Corollaries: Bayes' theorem and marginalisation

The sum and product rules of eqns (1.1) and (1.2) form the basic algebra of probability theory. Many other results can be derived from them. Amongst the most useful are two known as *Bayes' theorem* and *marginalisation*:

$$\mathrm{prob}(X\,|\,Y, I) = \frac{\mathrm{prob}(Y\,|\,X, I) \times \mathrm{prob}(X\,|\,I)}{\mathrm{prob}(Y\,|\,I)} \tag{1.3}$$

where *I* is the "relevant background information at hand".

D.S. Sivia

For *any* interesting dynamical system, outside PMS
Prob(Y|*I*) will be zero!
There is no model trajectory (stochastic or deterministic) consistent with the data.

This is ("I find this") a *very* interesting question of applied mathematics: How do we introduce "have a good idea" (aka: change ***I***) into a modelling paradigm?

Other *very* interesting maths questions include:

How to implement a Kalman Filter in a high-dimensional space with sparse observations?

How to compute the local orientation of Lyapunov vectors?

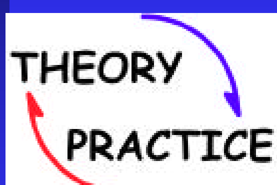
But, regarding model inadequacy and current models, I *doubt* any of these are very relevant to improving operational weathe forecasts…

When should ***you*** focus on getting a better approximation?
And when on better implementation?

eformulate Problem

Full Solution →	Approximation →	Implementation in R^m
Probabil*ity* Updating	KF	filter.c
Growth of uncertainty	Lyapunov Vector	Breeding Vector

… so I'll try to motivate this *doubt*, examine simple statistical tests of relevance, and merely aim for ad hoc methods with:



a) internally consistency
b) empirical relevance
c) operational utility

?Robust vs "Accurate"?

EDF ENERGY
SAIC
National Grid
LONDON ELECTRICITY
THE WEATHER CHANNEL
weather.com
ALSTOM
CALIFORNIA ISO
Risk Management Solutions RMS

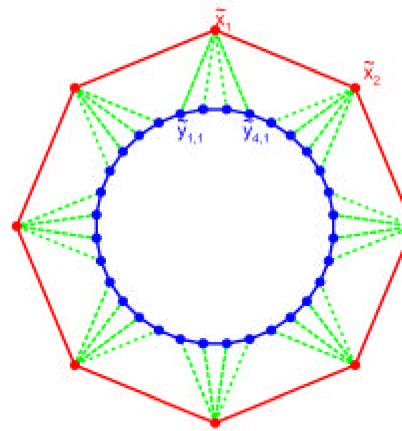
2

The system (unknown): Lorenz 95

$$\frac{d\tilde{x}_i}{dt} = -\tilde{x}_{i-2}\tilde{x}_{i-1} + \tilde{x}_{i-1}\tilde{x}_{i+1} - \tilde{x}_i + F - \frac{h_{\tilde{x}}c}{b}\sum_{j=1}^{n} \tilde{y}_{j,i} \qquad (2.1)$$

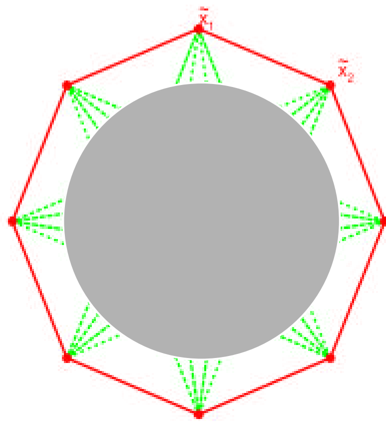
$$\frac{d\tilde{y}_{j,i}}{dt} = cb\tilde{y}_{j+1,i}\left(\tilde{y}_{j-1,i} - \tilde{y}_{j+2,i}\right) - c\tilde{y}_{j,i} + \frac{h_{\tilde{y}}c}{b}\tilde{x}_i. \qquad (2.2)$$

where $i = 1, \ldots, m$ and $j = 1, \ldots, n$ and with cyclic boundary conditions on both the $\tilde{x}_i$ and the $\tilde{y}_{j,i}$ (that is $\tilde{x}_{m+1} = \tilde{x}_1$, $\tilde{y}_{(n+1,i)} = \tilde{y}_{(1,i)}$ and so on). In the computations below $F = 10$, $m = 8$ and $n = 4$. The constants b and c are both equal to 10, so the small-scale dynamics are 10 time faster



The model:

$$\frac{dx_i}{dt} = -x_{i-2}x_{i-1} + x_{i-1}x_{i+1} - x_i + F$$



From Smith (2001)

x and x_tilde live in different state spaces!
What is meant by the uncertainty in "F"?

Is there a "correct" parameter value?
Or a meaningful P(x | s, I) ?

$$\tilde{\alpha} \equiv \left\langle F - \frac{h_{\tilde{x}} c}{b} \sum_{j=1}^{J} \tilde{y}_{j,i} \right\rangle_{\tilde{\mathbf{x}}}$$

(No)

From Smith (2001):

$$\frac{dx_i}{dt} = -x_{i-2}x_{i-1} + x_{i-1}x_{i+1} - x_i + P_i(\mathbf{x}, t), \qquad i{=}1, m \tag{2.3}$$

These equations for the model variables $\mathbf{x}$ are structurally similar to Equations 1 which determined the large scale $\tilde{\mathbf{x}}$ dynamics of the system,

options we have explored for $P_i(\mathbf{x}, t)$ include:

$$P_i(\mathbf{x}, t) \quad = \quad \begin{cases} \alpha_0 & \text{constant} \\ \alpha_0 + \alpha_1 x_i & \text{linear} \\ \alpha_0 + \boldsymbol{\alpha} \cdot \boldsymbol{x} & \text{m-linear} \\ H_1(\mathbf{x}) & \text{nonlocal1} \\ H_2(\mathbf{x}, \frac{\Delta \mathbf{x}}{\Delta t}) & \text{nonlocal2} \\ \text{I.I.D}_{obs} & \text{IID} \\ \gamma_1 P_i(\mathbf{x}, t-1) + N(0, \gamma_0) & \text{AR(1)} \end{cases}$$

(Discrete tim

Ignoring H_1 and H_2, each model class is imperfect:

The most appropriate form of P(x,t) depends on the user.

None provide accountable probability forecasts.

3

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THEORY
PRACTICE

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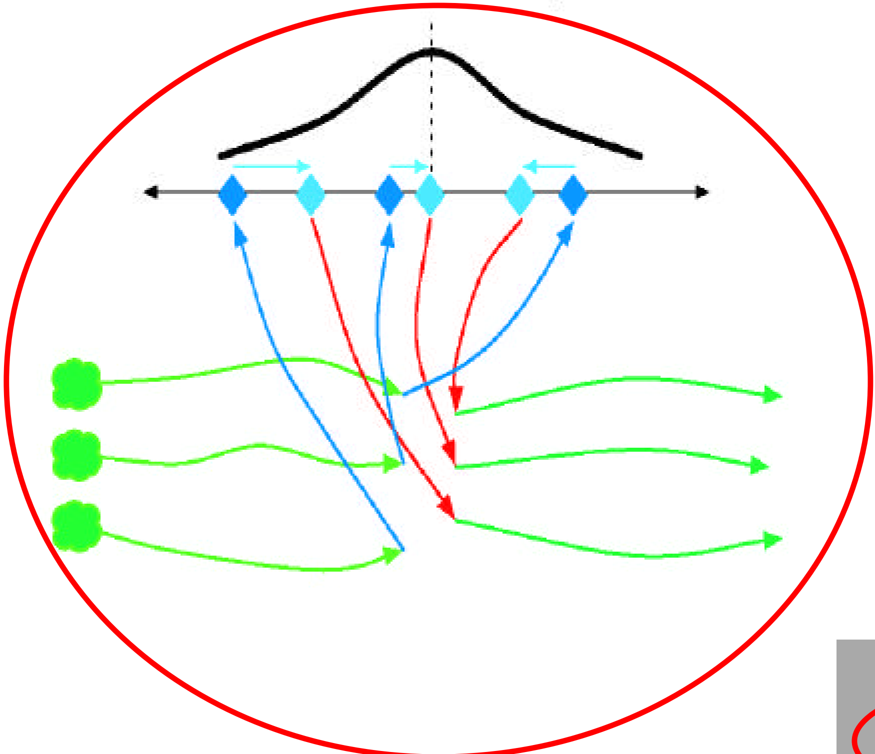

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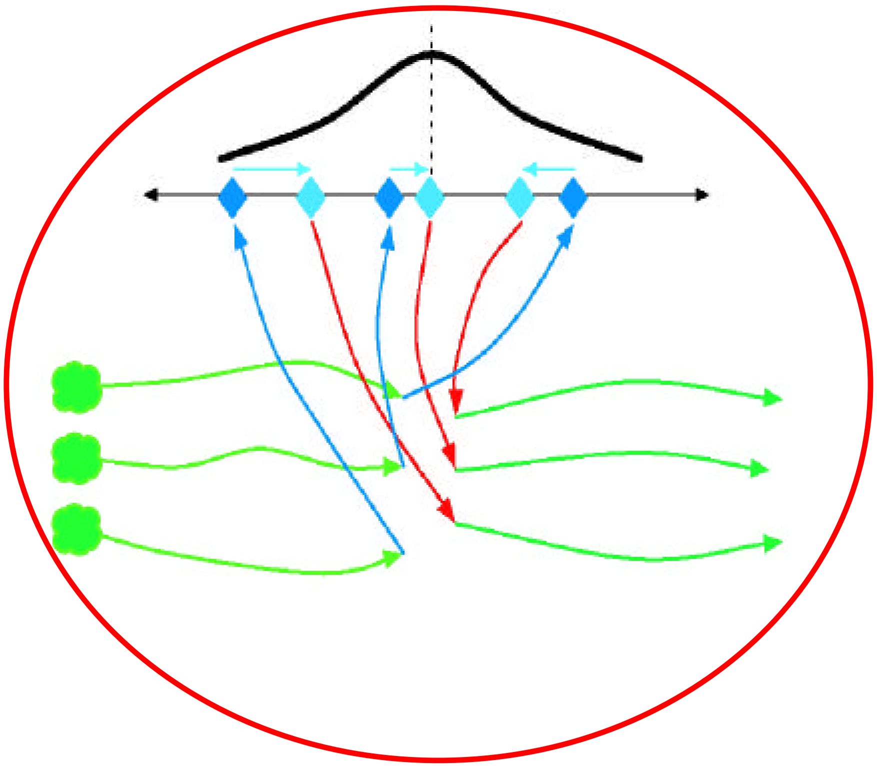


Utter and Senseless Destruction of Dynamical Information

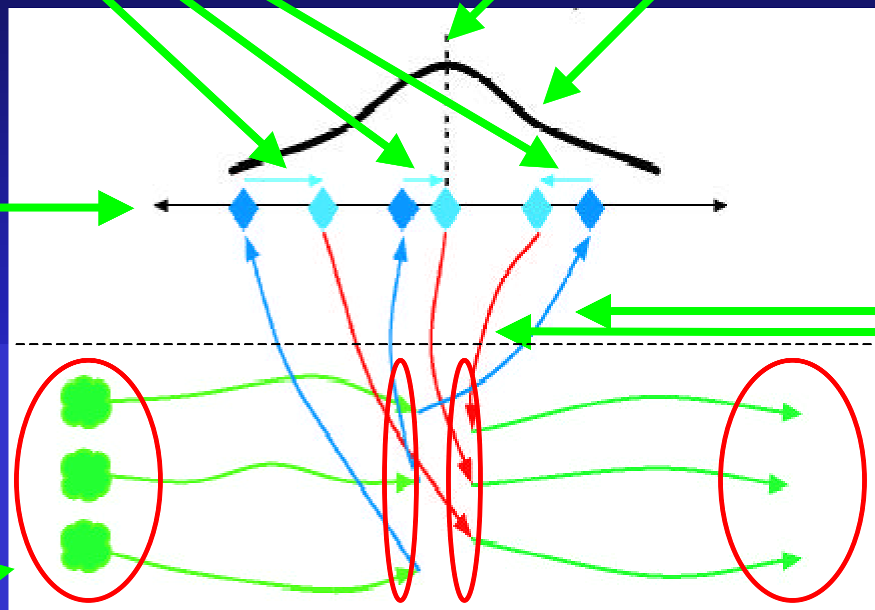
What is this?

Where did this come from

Obs Space

Projection Operator(s)
(?one-to-one?)

Model-State Space



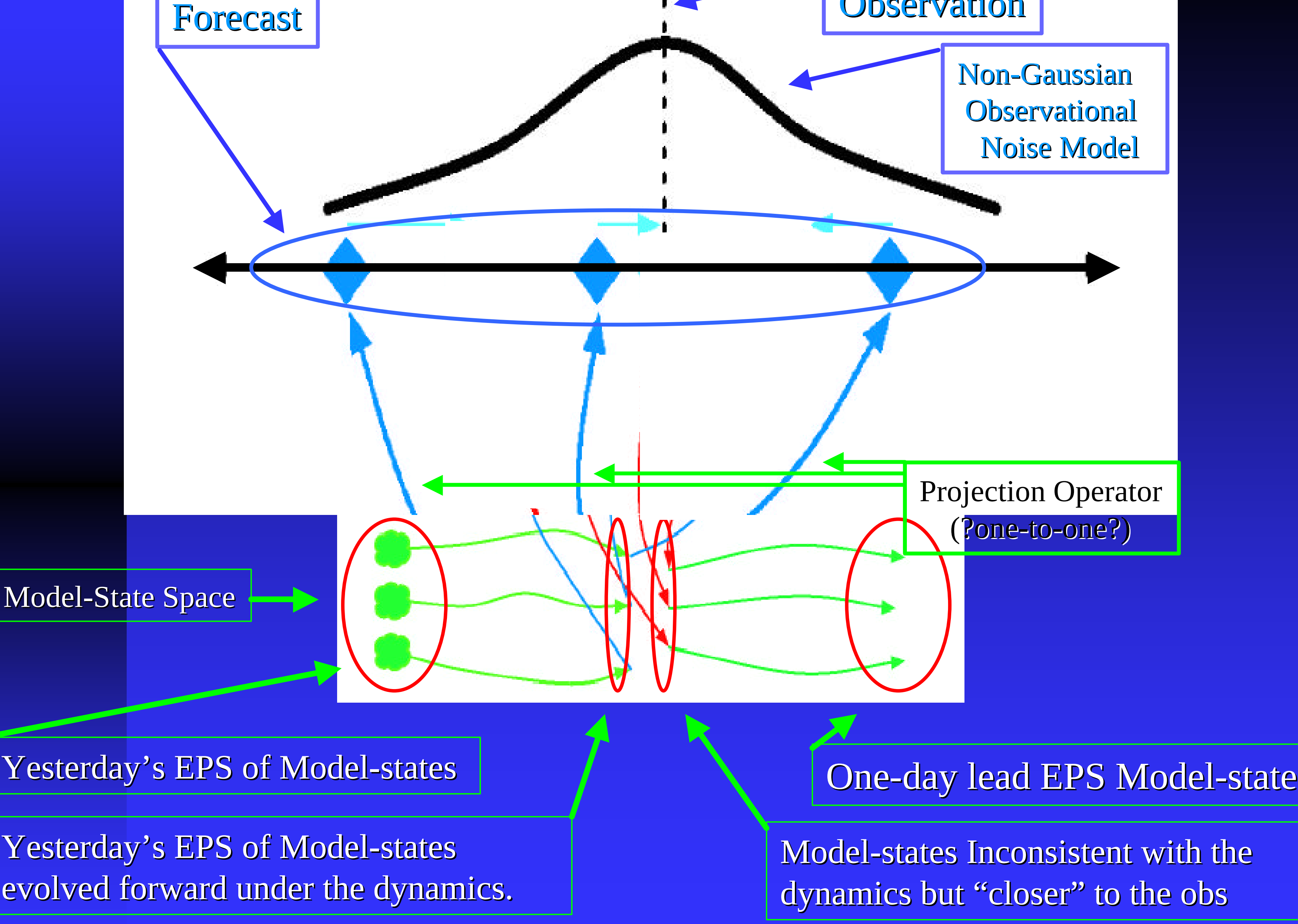
Yesterday’s EPS of Model-states

Yesterday’s EPS of Model-states
evolved forward under the dynamics.

One-day lead EPS Model-state

Model-states Inconsistent with the
dynamics but “closer” to the obs

Where is the forecast?



Forecast
Observation
Non-Gaussian Observational Noise Model
Projection Operator (?one-to-one?)
Model-State Space
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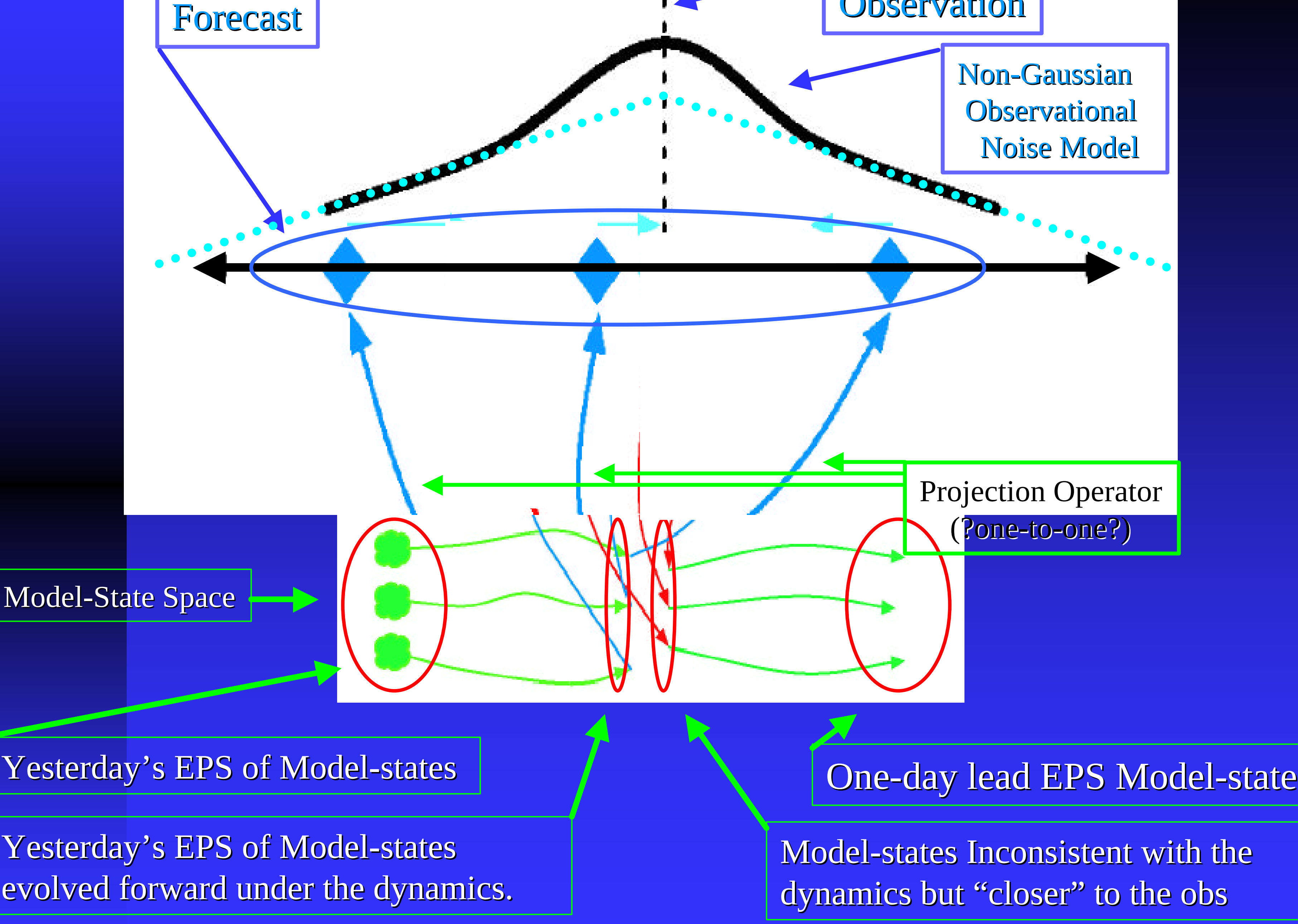
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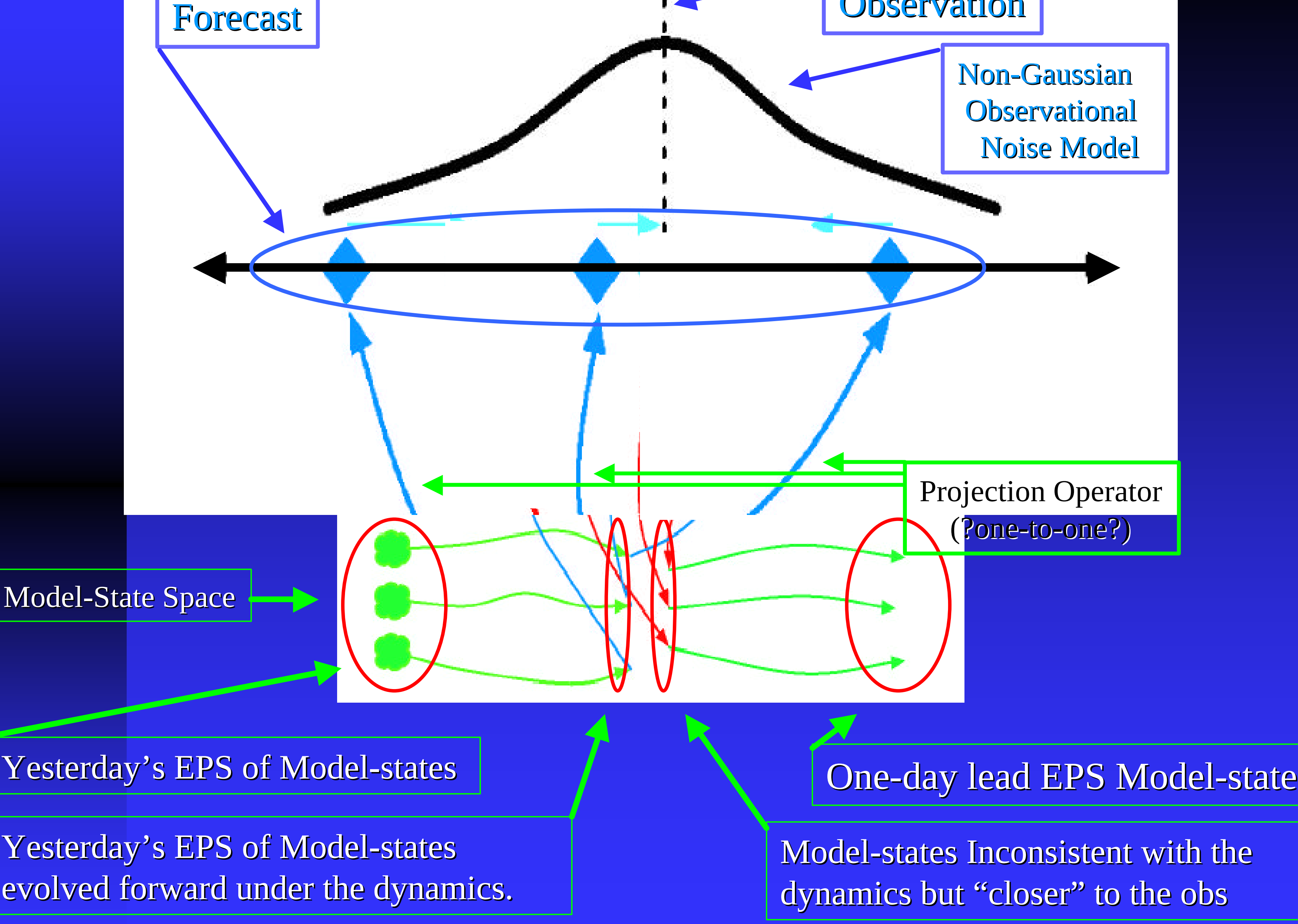
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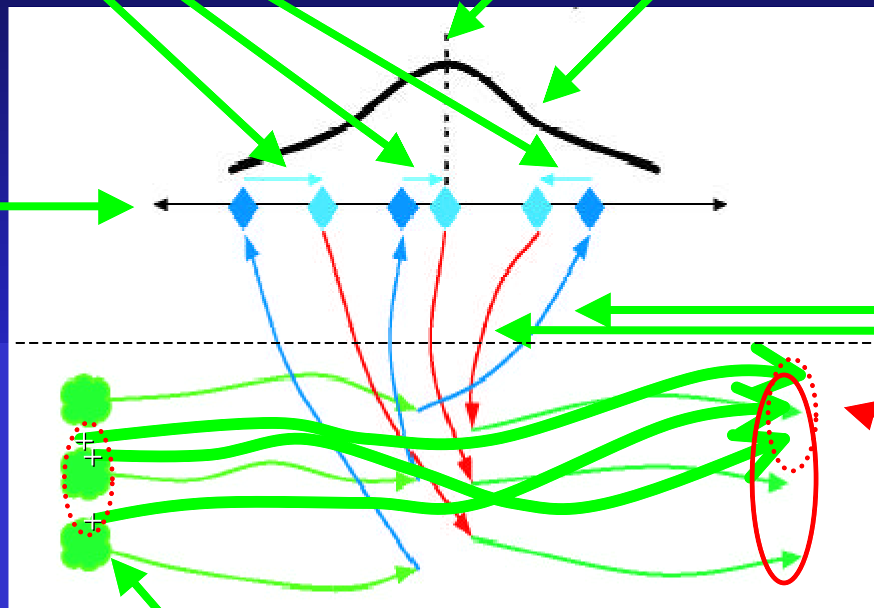
Utter and Senseless Destruction of Dynamical Information

What is this?

Where did this come from

Obs Space



Projection Operator(s)
(?one-to-one?)

Model-State Space

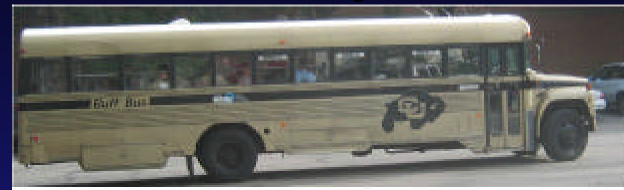
Model-states consistent with the dynamics,
“closer” to the obs,
tighter EPS of model-states at day 2

Can I use my knowledge of the dynamics to find more relevant states?

4

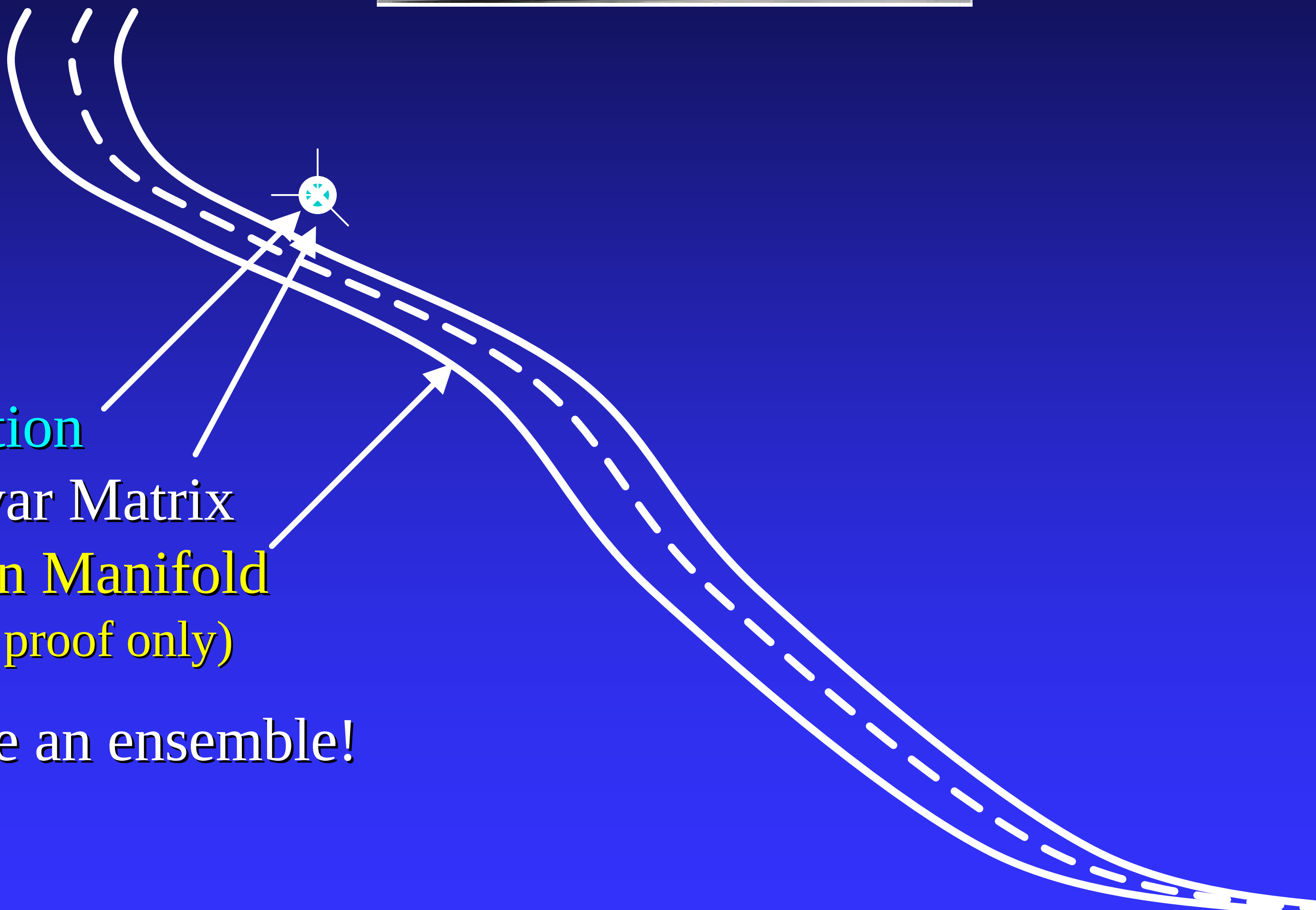
What is a manifold?

"Utter and Senseless Destruction of Dynamical Information?"



M=11

(x,y,z,u,w,v…)



Observation

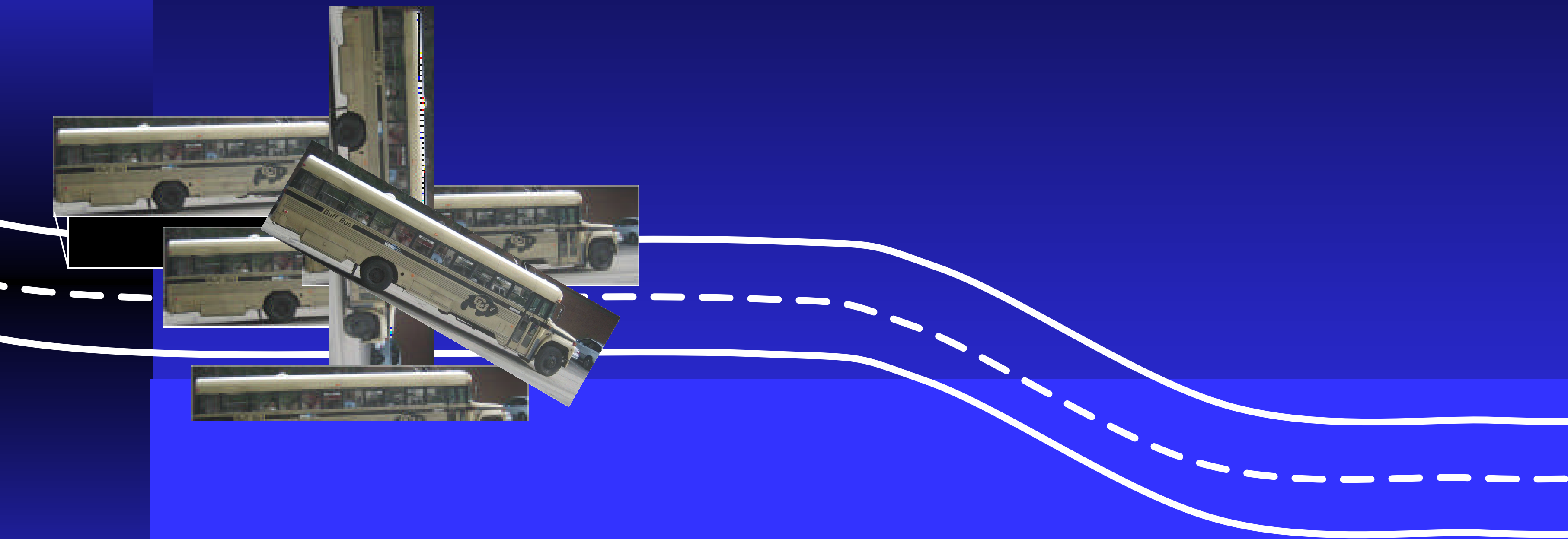
Obs-Covar Matrix

Unknown Manifold

(existence proof only)

Lets make an ensemble!

Now evolve the ensemble under the (perfect) model:

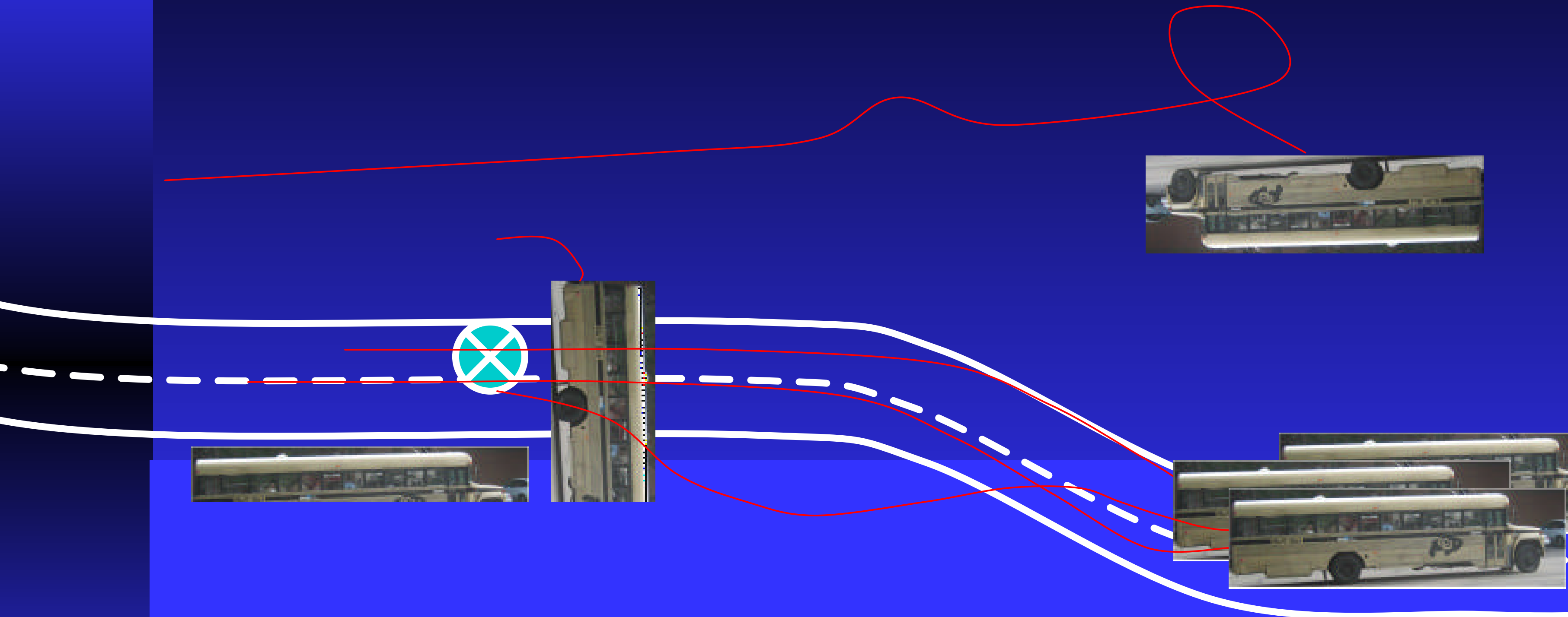


t=0

Lets make an ensemble!

Now evolve the ensemble under the (perfect) model:

And get a new observation…



Do I really want to make a KF update?

-or-

Can I use the fact that the model dynamics
(stochastic or deterministic) trace out the manifold
I know exists but cannot sample directly?!?

t=1

5

Shree's movie is a nice illustration that with a perfect model, simple nonlinear filters can outperform ***his*** n+1st variant KF.

Is this surprising?

"Of course, in general these tasks (prediction, separation, detection) may be done better by nonlinear filters."

(Kalman, 1960; first substantial footnote)

But this talk is on model error, not ISIS (Indistinguishable States Importance Sampling)…

De-fusing Perfect Model Expectations
(in real world data and systems)

Leonard Smith
Centre for the Analysis of Time Series
London School of Economics
Pembroke College, Oxford
lenny@maths.ox.ac.uk

Kevin Judd , Jochen Broecher, Liam Clarke
(Shree and Neil [and Jim and Chris])
www.lsecats.org

This Galton Board is a mathematical model.

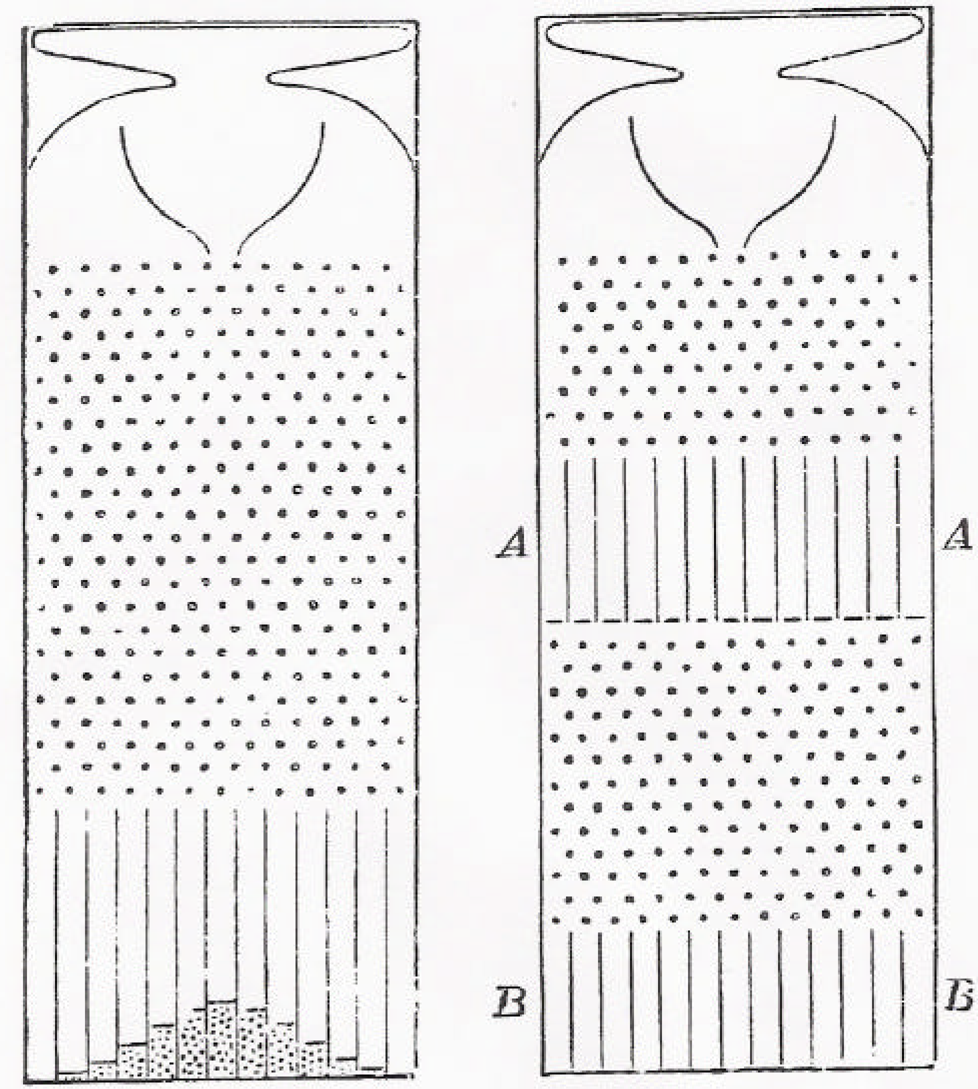


Figure 9.2 A schematic drawing of Galton's Quincunx, from Galton (1889a, p. 63).

I term this a thought experiment because, while Galton clearly in several places described the variant of the Quincunx that performed the experiment, there is no indication that he actually built the apparatus. And having tried to build such a machine, I can testify that it is exceedingly difficult to make one that will accomplish the task in a satisfactory manner.

(our first hint of model error)

{and a typical theoretician's response}

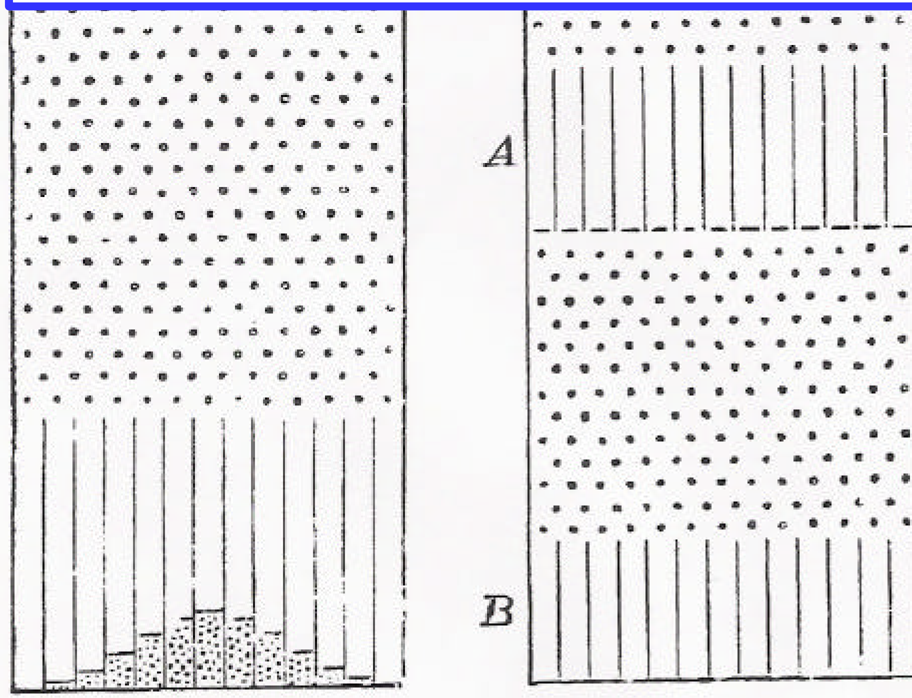
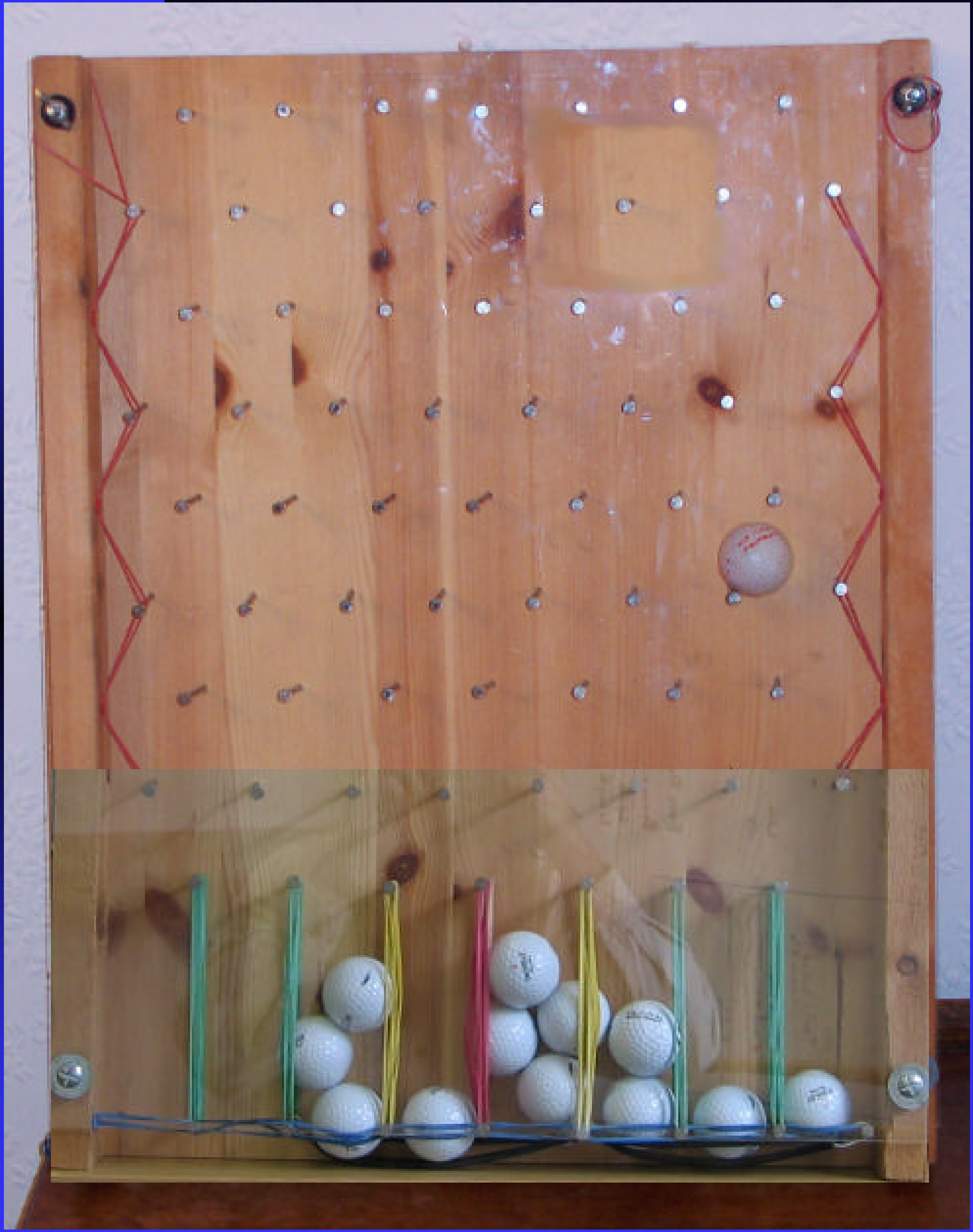


Figure 9.2 A schematic drawing of Galton's Quincunx, from Galton (1889a, p. 63)

While the Galton Board is a mathematical model

… this is Not A Galton (NAG) Board.

It is neither stochastic or chaotic; but at least it i



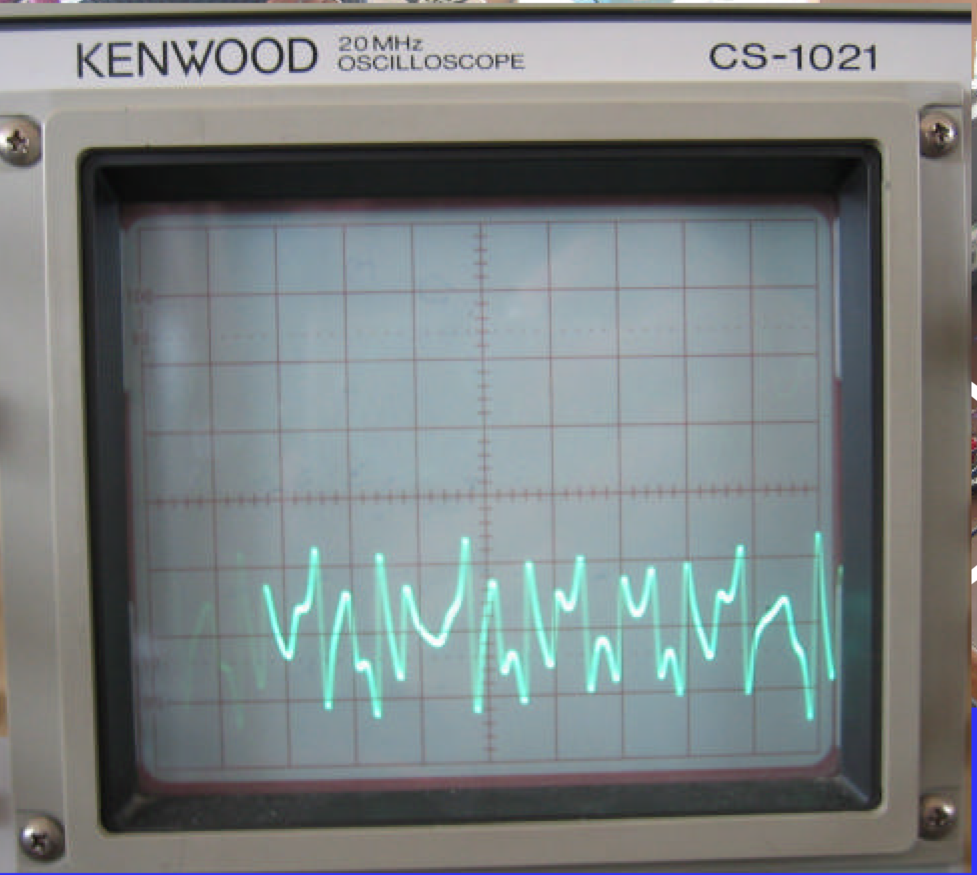
Reality needn't be complex,
it merely needs to be real.

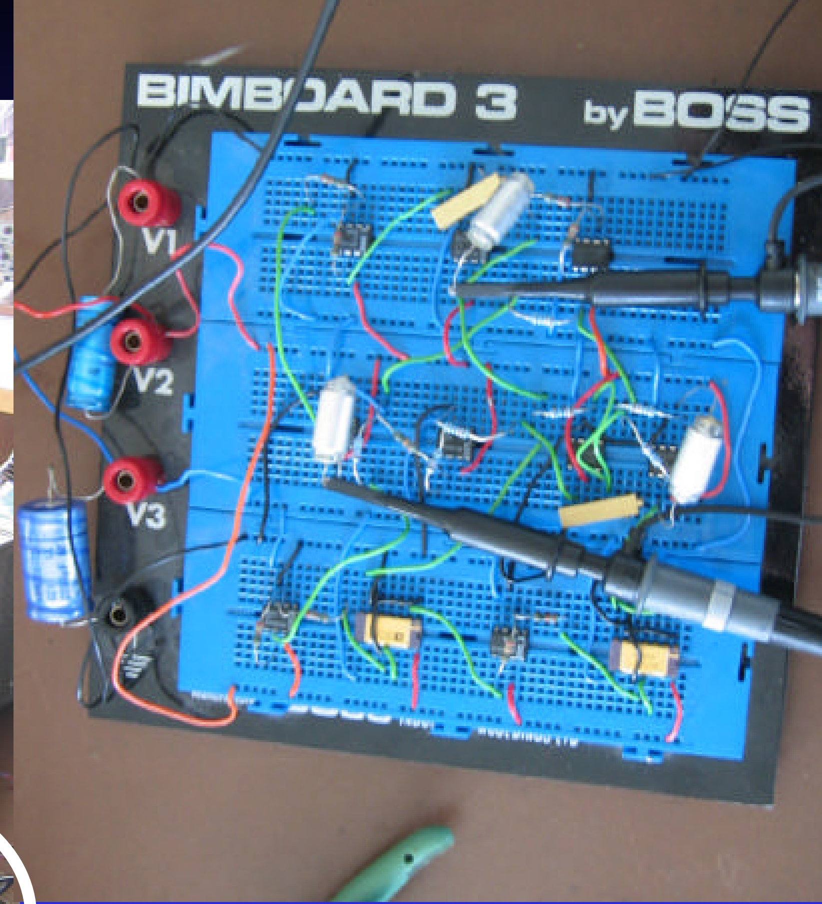
What do you see when you look at an ensemble predictio system?

In the NAG board, this corresponds to predicting wit a collection (ensemble) of gol balls… but if reality is not a golf ball, then how do we interpret these distribtions?

THAT IS ***THE*** QUESTION FROM MODEL ERROR!

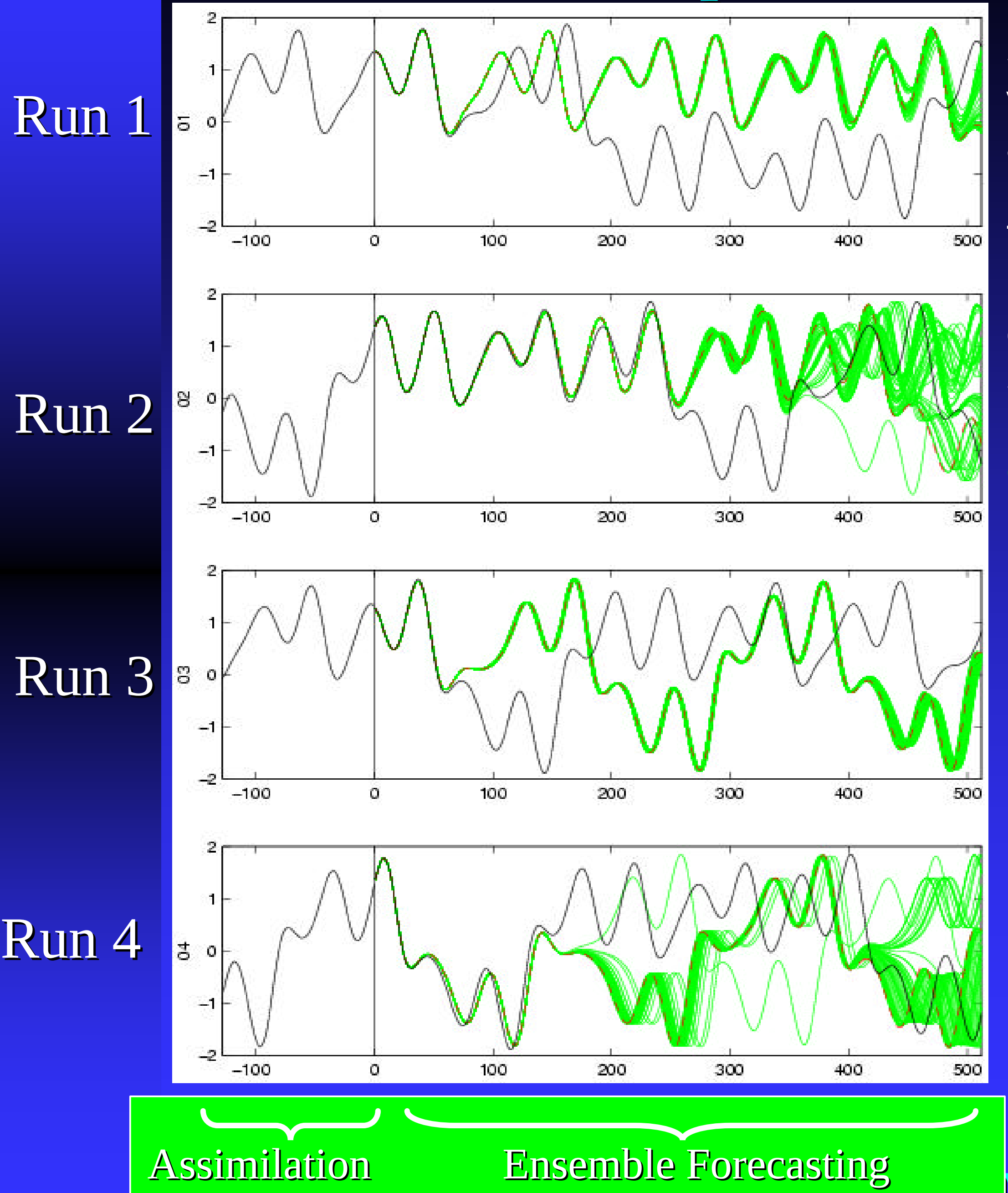
6





This is a real physical system
It's interest here lies in the fact that I *cannot* forecast it!

Recurrent example: Chaotic Circuit



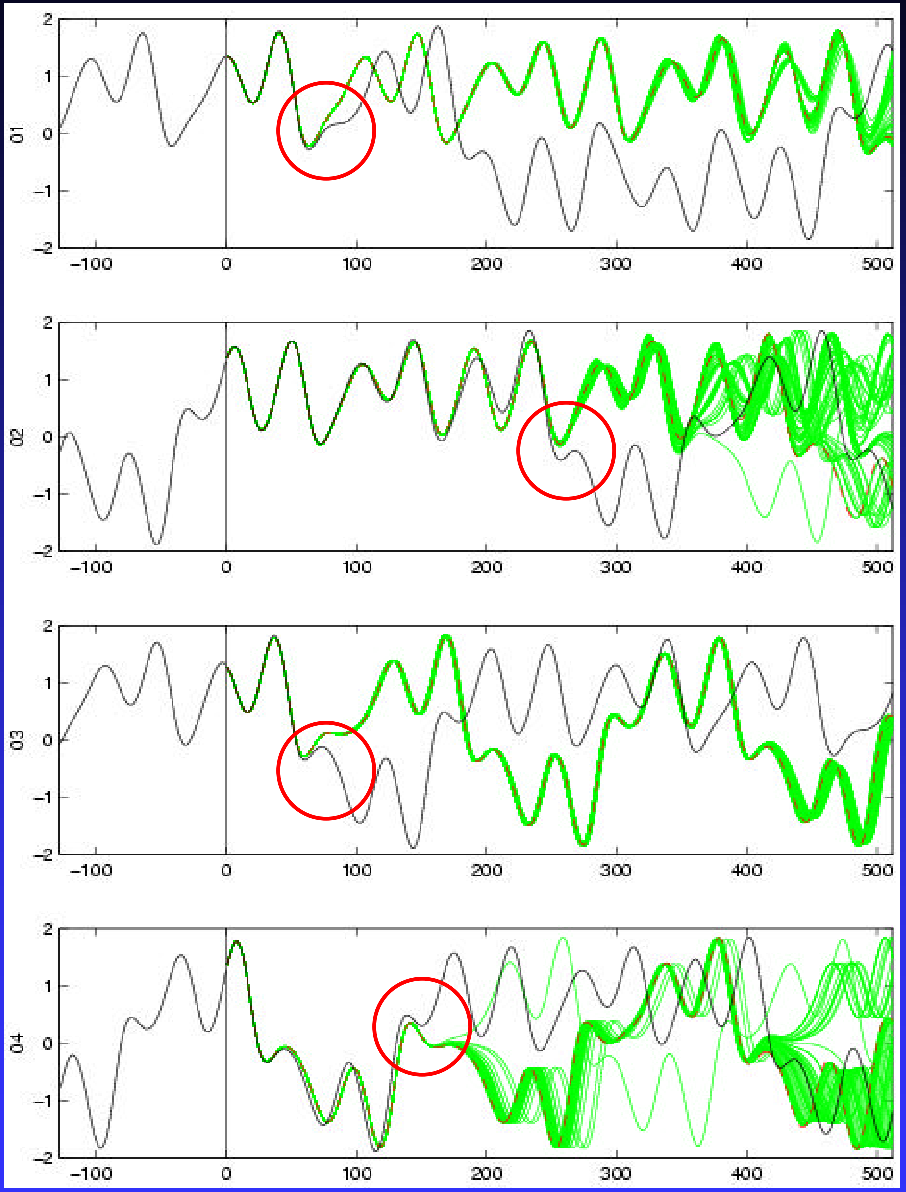
Short term (weather) forecasts are very skilful both for statistical models and for simulation models.

The best models I can find are Local Polynomial Models (Smith 1994) generalized from the Farmer and Sidorovich (1987) local linear models.

Great several step rms error,
Informative Ensemble Information
Poor probability forecasts.

5-dim delay space,
delay of ~three steps,
locally optimised neighbourhood
512 member ensembles

Forecasts busts in a Chaotic Circuit



Why do good models go bad

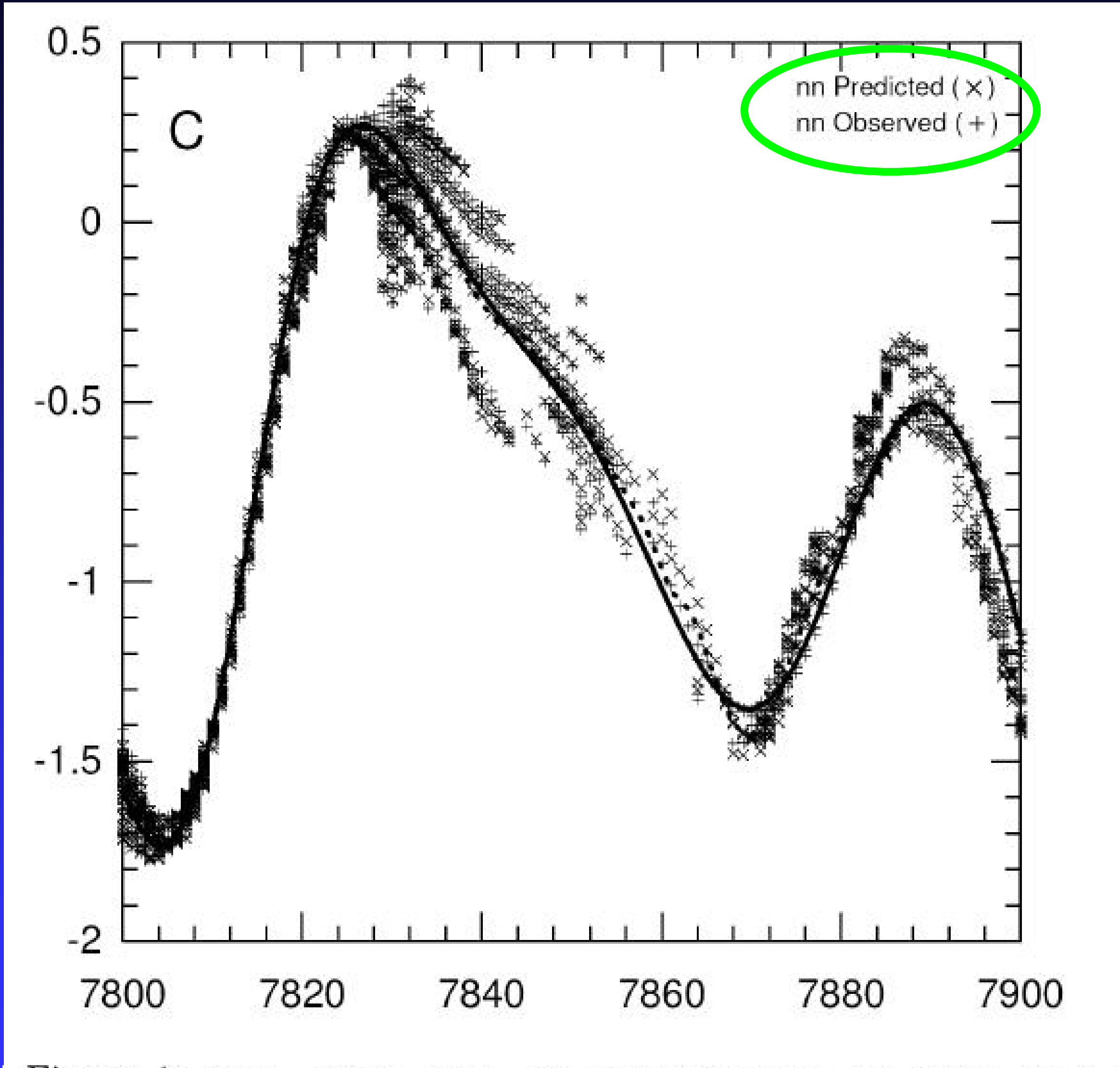
Why these fairly common ensemble busts?

And what is noise in this context, really?

(Outside PMS, there is no “true model-state with which to define this realisation of the noise model)

Consider many out-of-sample analogue forecasts.

(Lorenz 1963)

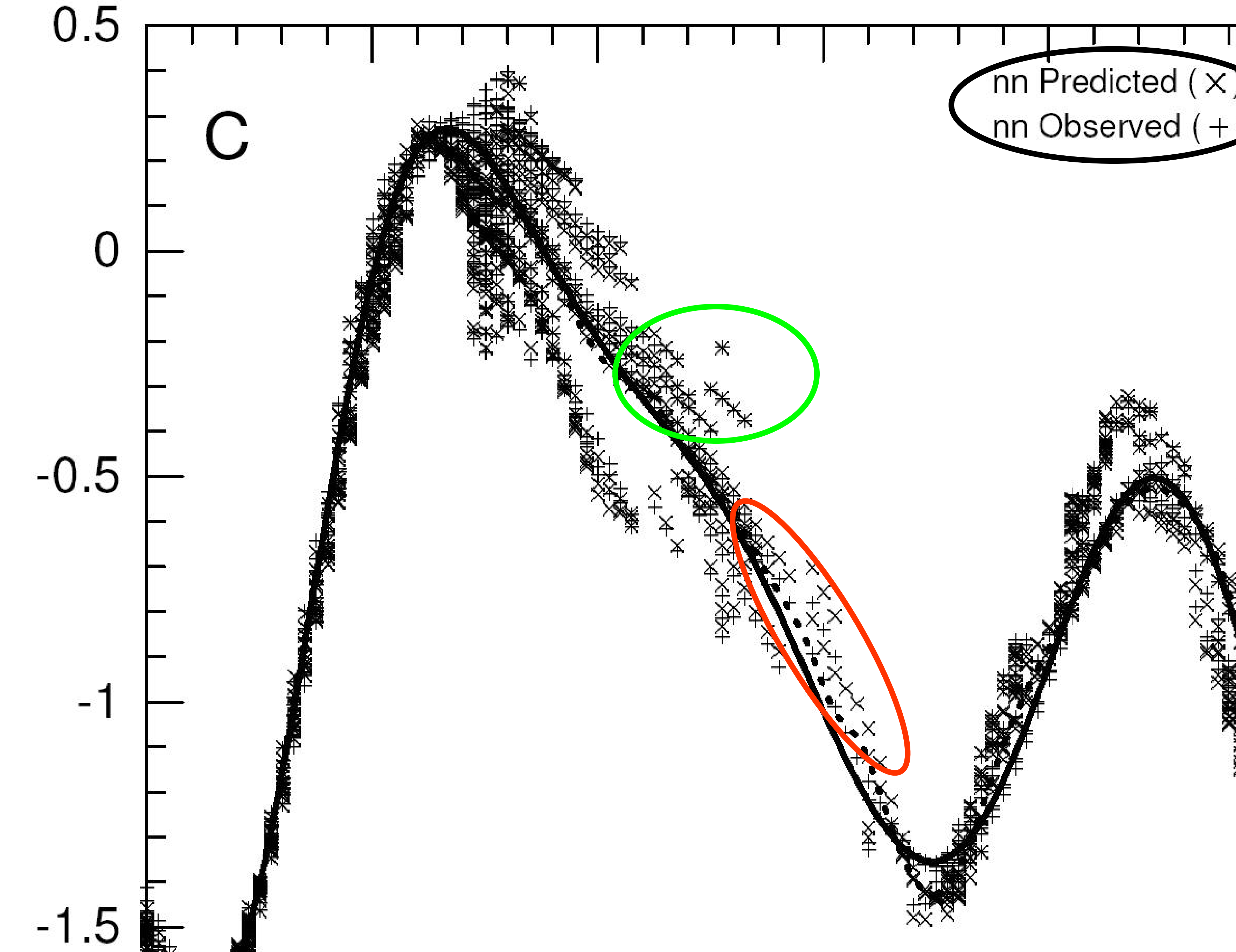


Each forecast is a
a fixed lead time
(15 steps).

An “x” over a “+
is good!

Base points are
chosen from near
returns in the
model-state space

Figure 1: Four voltage series with related forecasts, for details see the text. A) A sample of the observations. B) Iterated ensemble forecasts using two different models. C) 15 step ahead forecasts of near recurrences. D) Zoom of C showing state-dependent systematic model error.



0.5
0
-0.5
-1
-1.5
C
nn Predicted (×)
nn Observed (+)

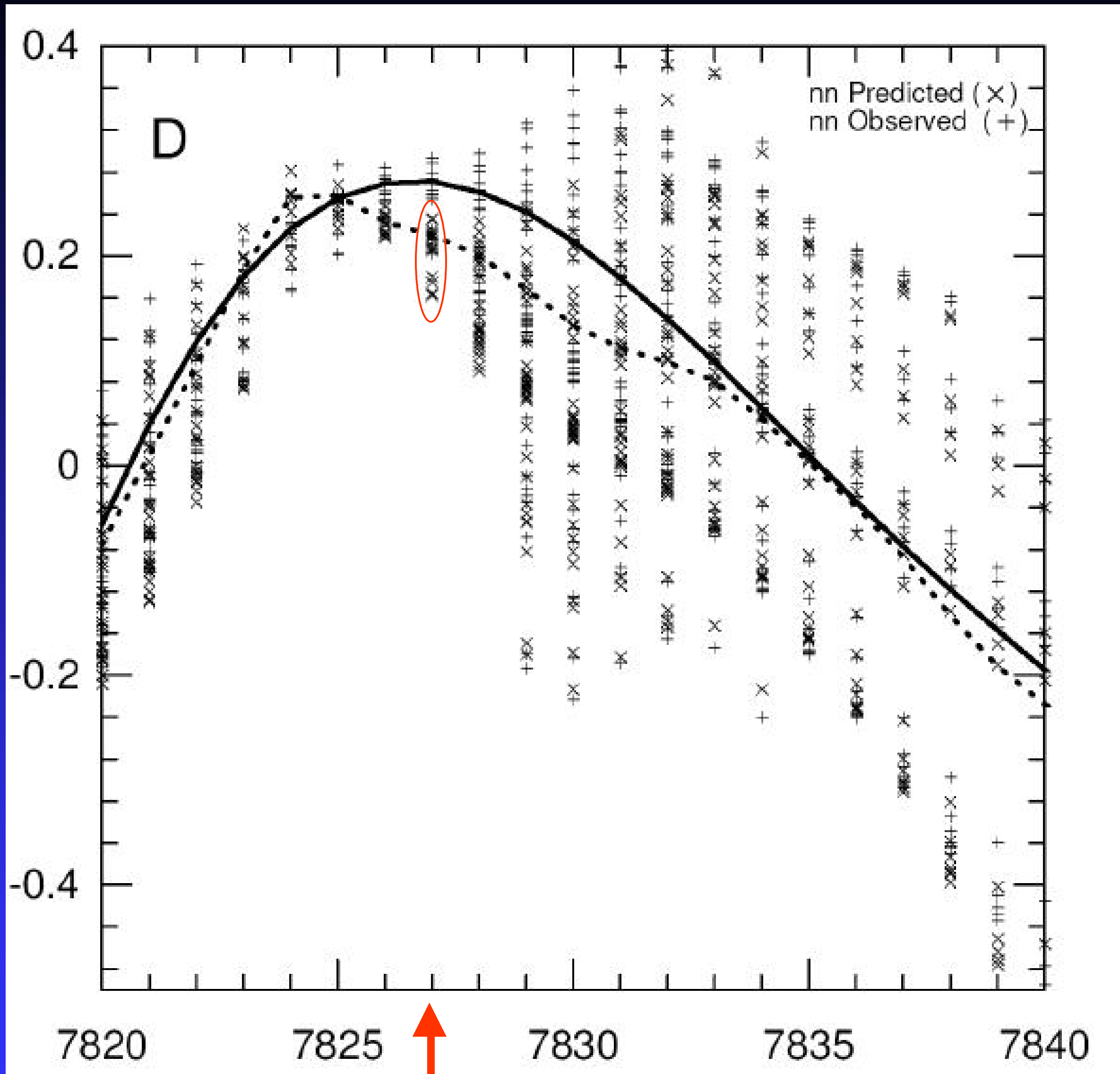
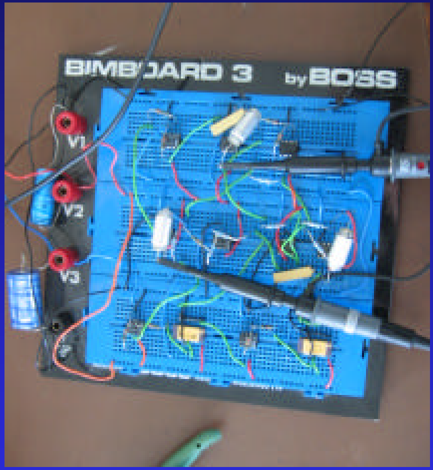


Figure 1: Four voltage series with related forecasts, for details see the text. A) A sample of the observations. B) Iterated ensemble forecasts using two different models. C) 15 step ahead forecasts of near recurrences. D) Zoom of C showing state-dependent systematic model error.

The model dynamics look like *a circuit*, but not *this circuit*…

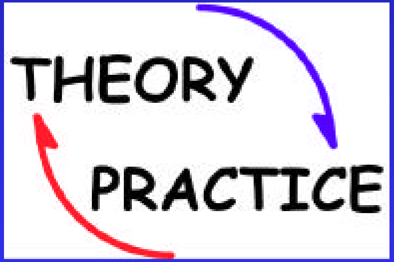


This suggests that that model inadequacy keeps "perfect" PDF forecasts beyond our reach in a manner analogous to the way observational noise rules out definitive RMS forecasts.

Is this a pessimistic view?

Yes: in exactly the same sense that accepting that the square root of 2 was an irrational number was a pessimistic choice for the Pythagoreans!

It meant a lovely [rational] mathematical dream was merely a dream; and opened up huge possibilities for the advancement of maths and applied maths.

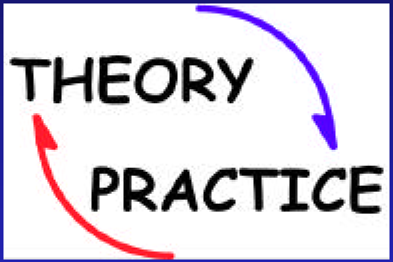


Nevertheless, it has proven more useful!

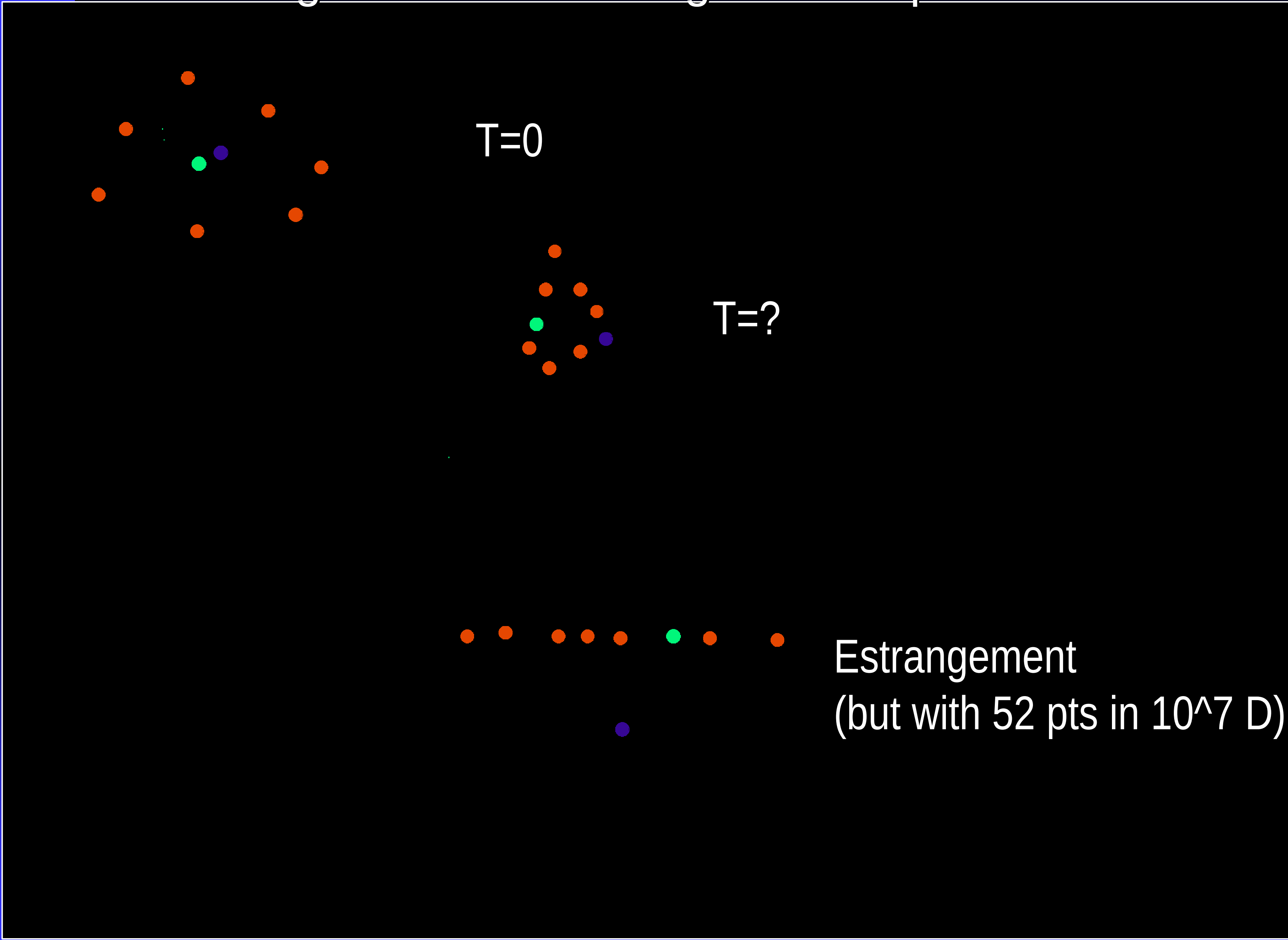
7

So we have blatant inconsistency between theory and practice.

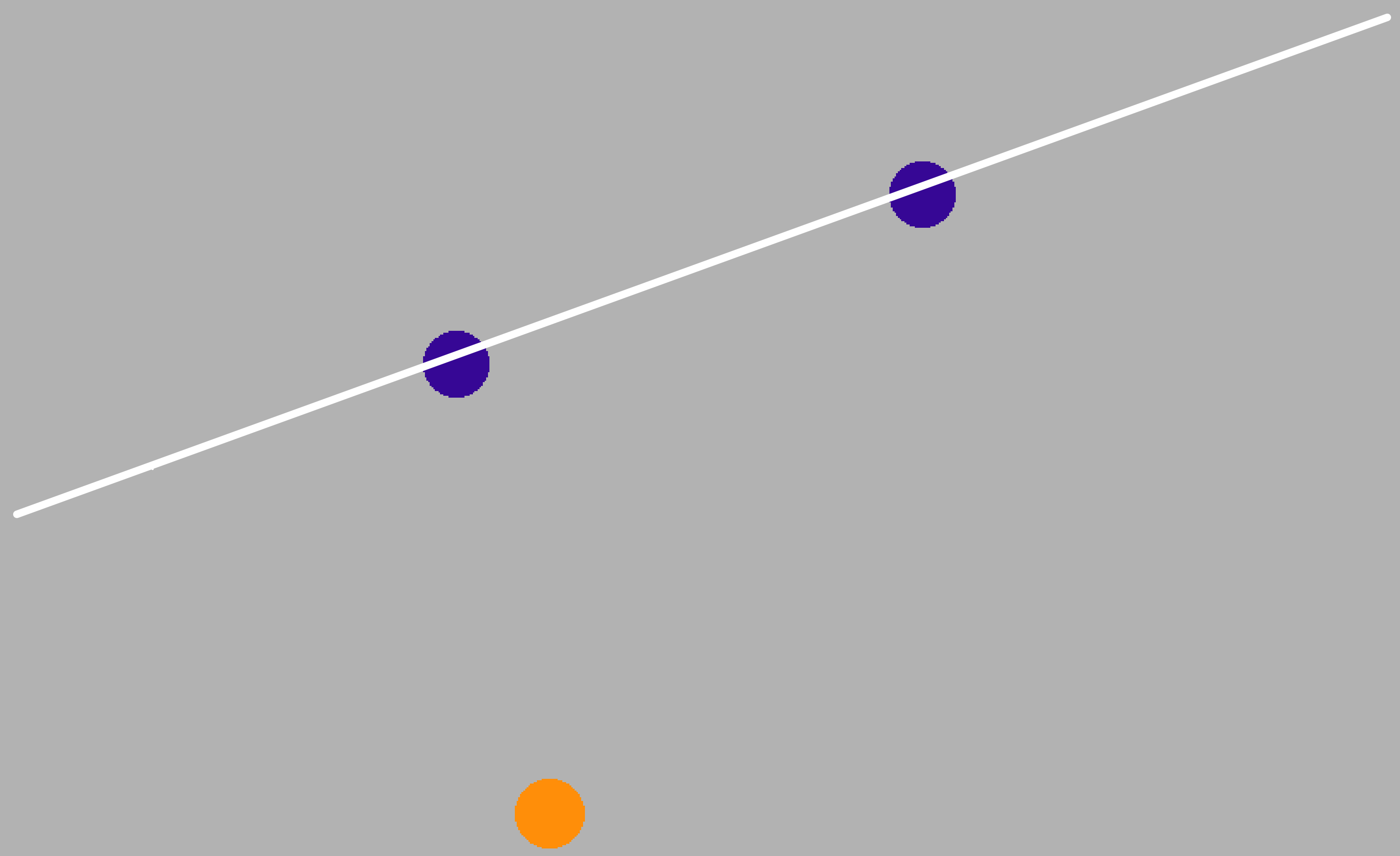
How might we see this in a 10^7 dimensional model?

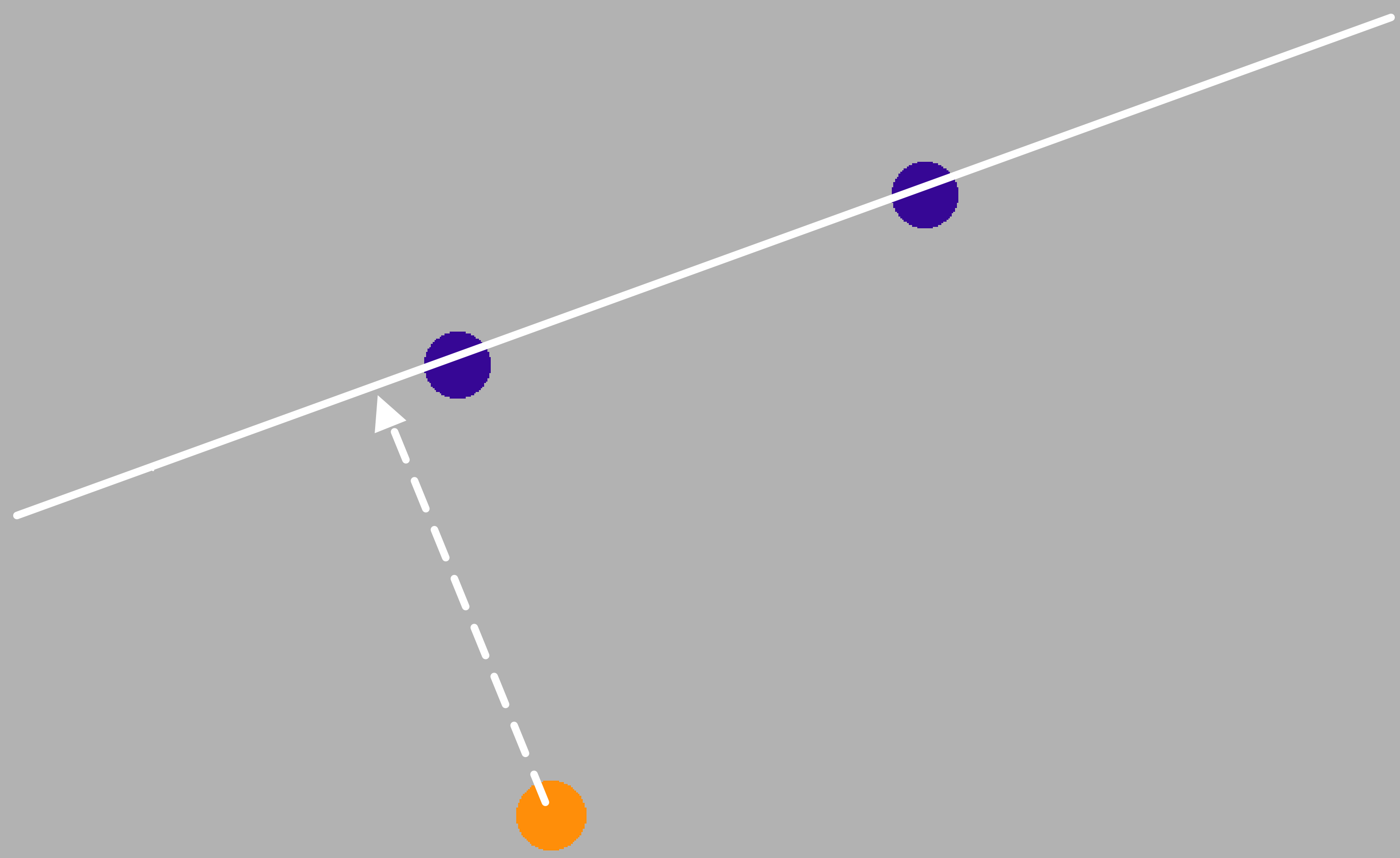


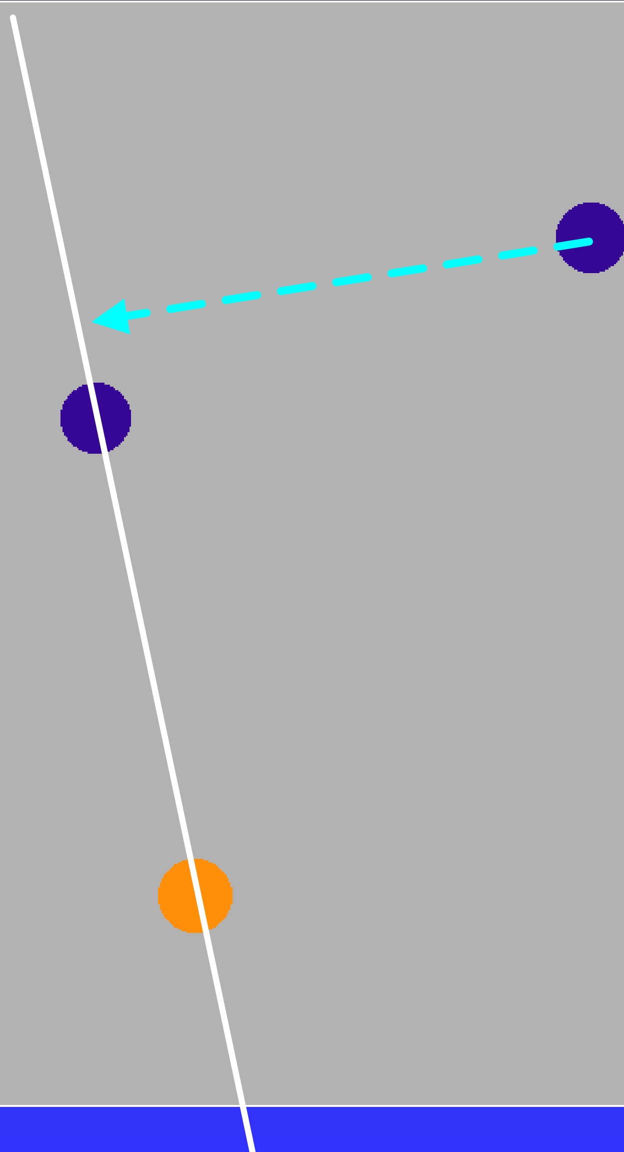
Detecting Ensemble Estrangement in practice

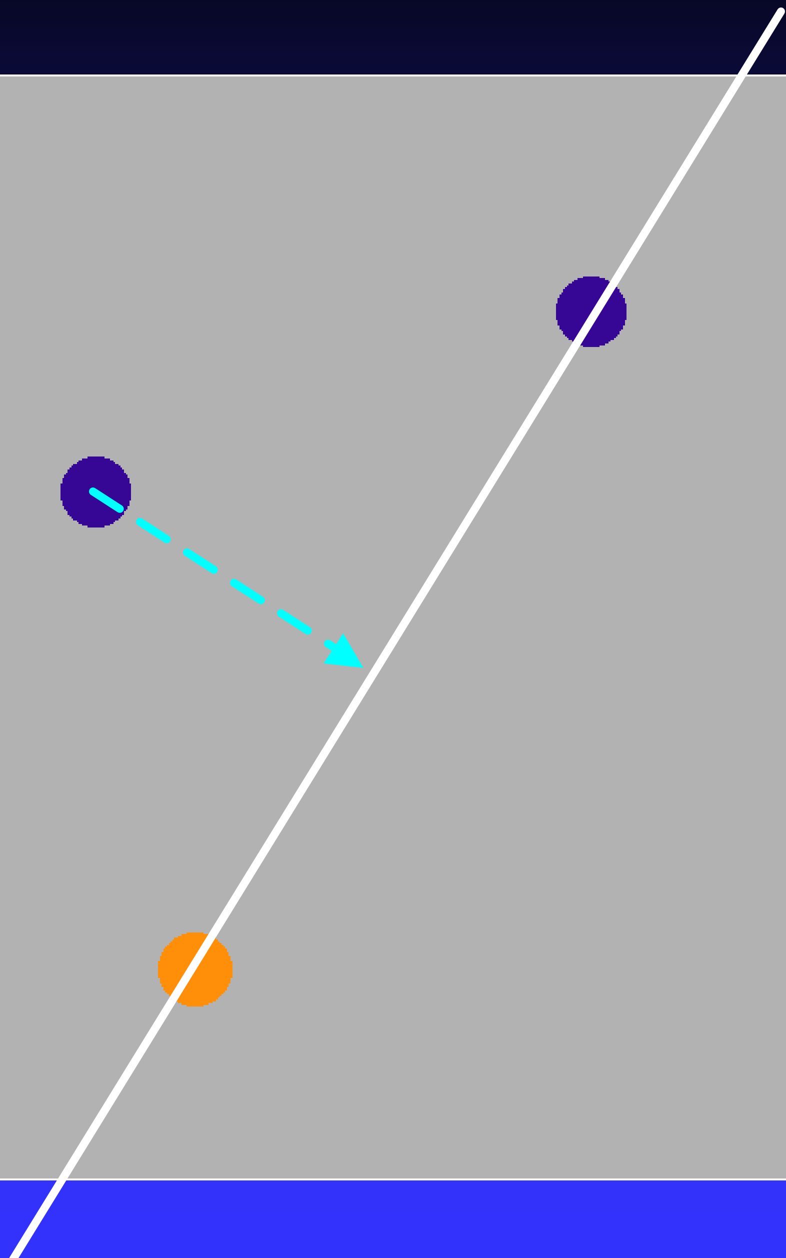


Any 51 points in a 10^7 space will lie in the same 'line'.

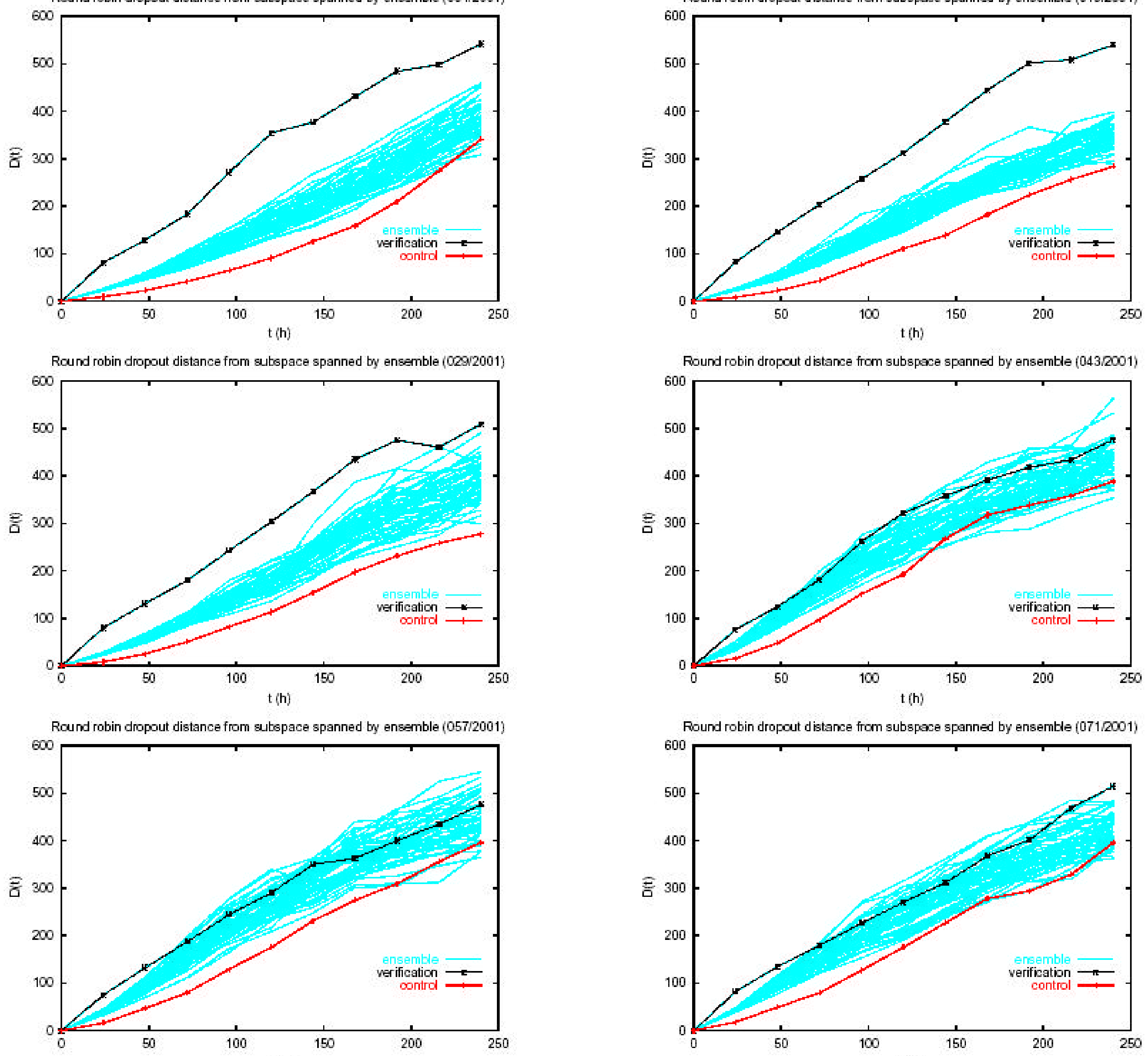




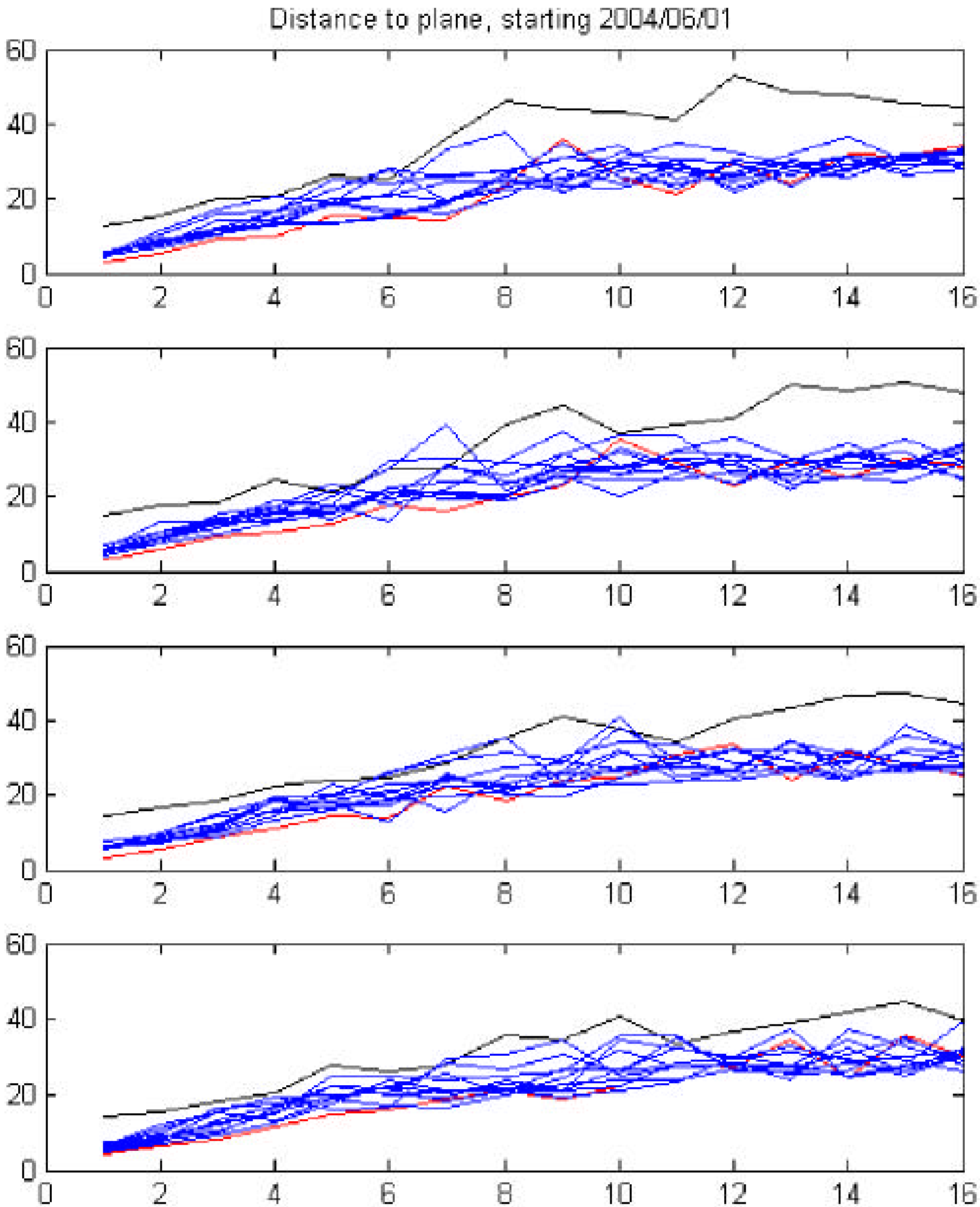




ad Spread
with the
correct
magnitude)

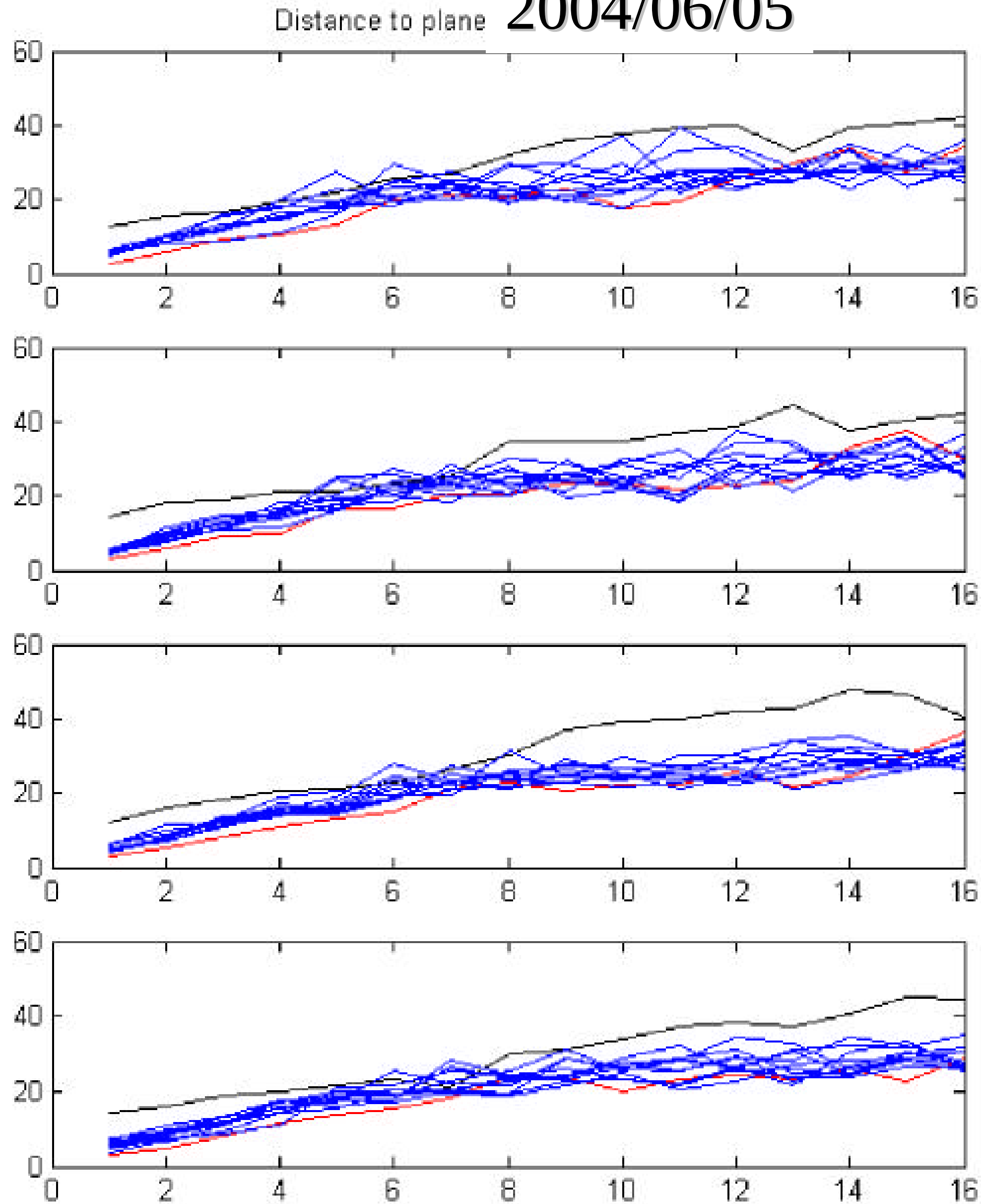


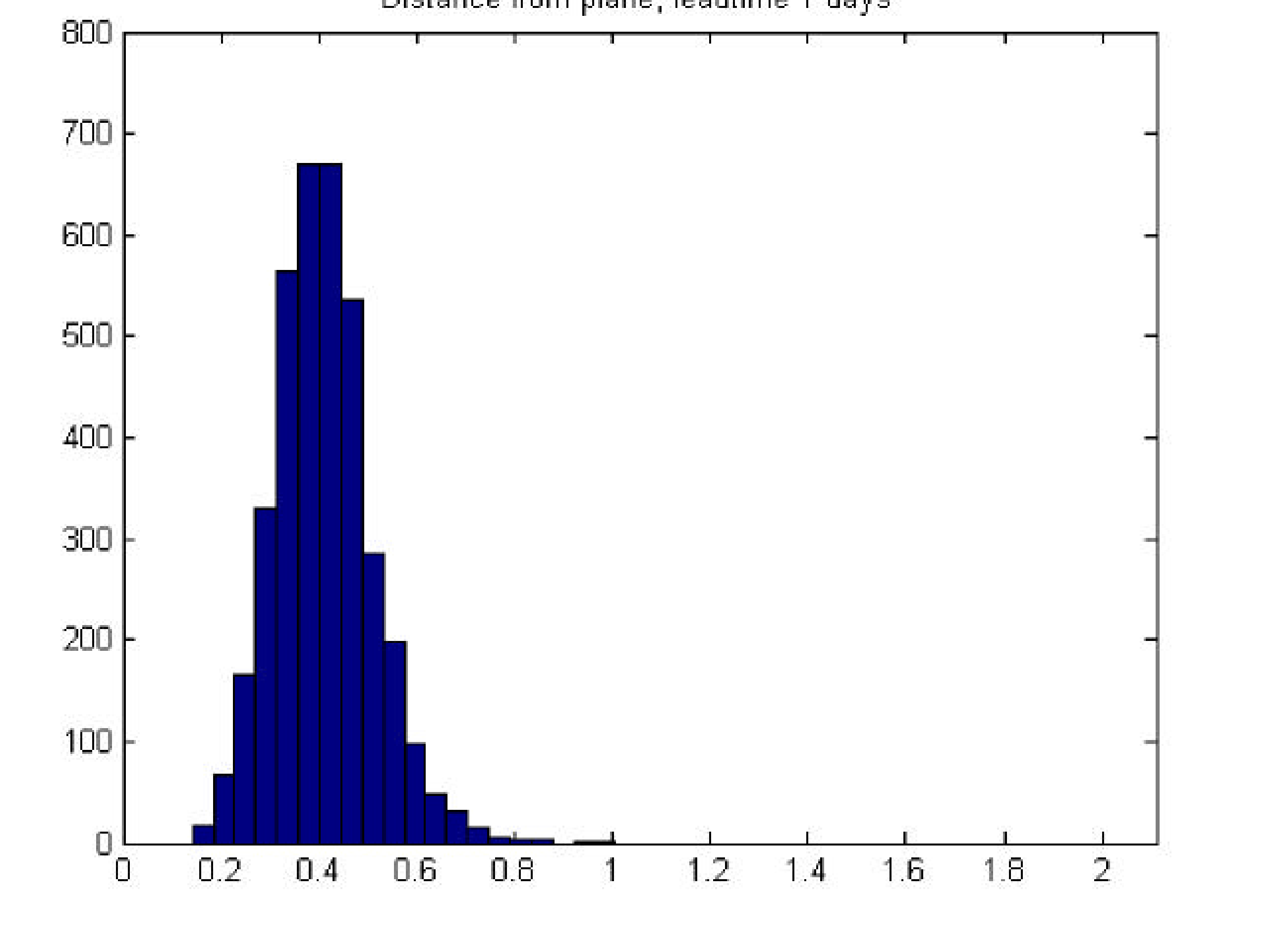
ECMWF
2001



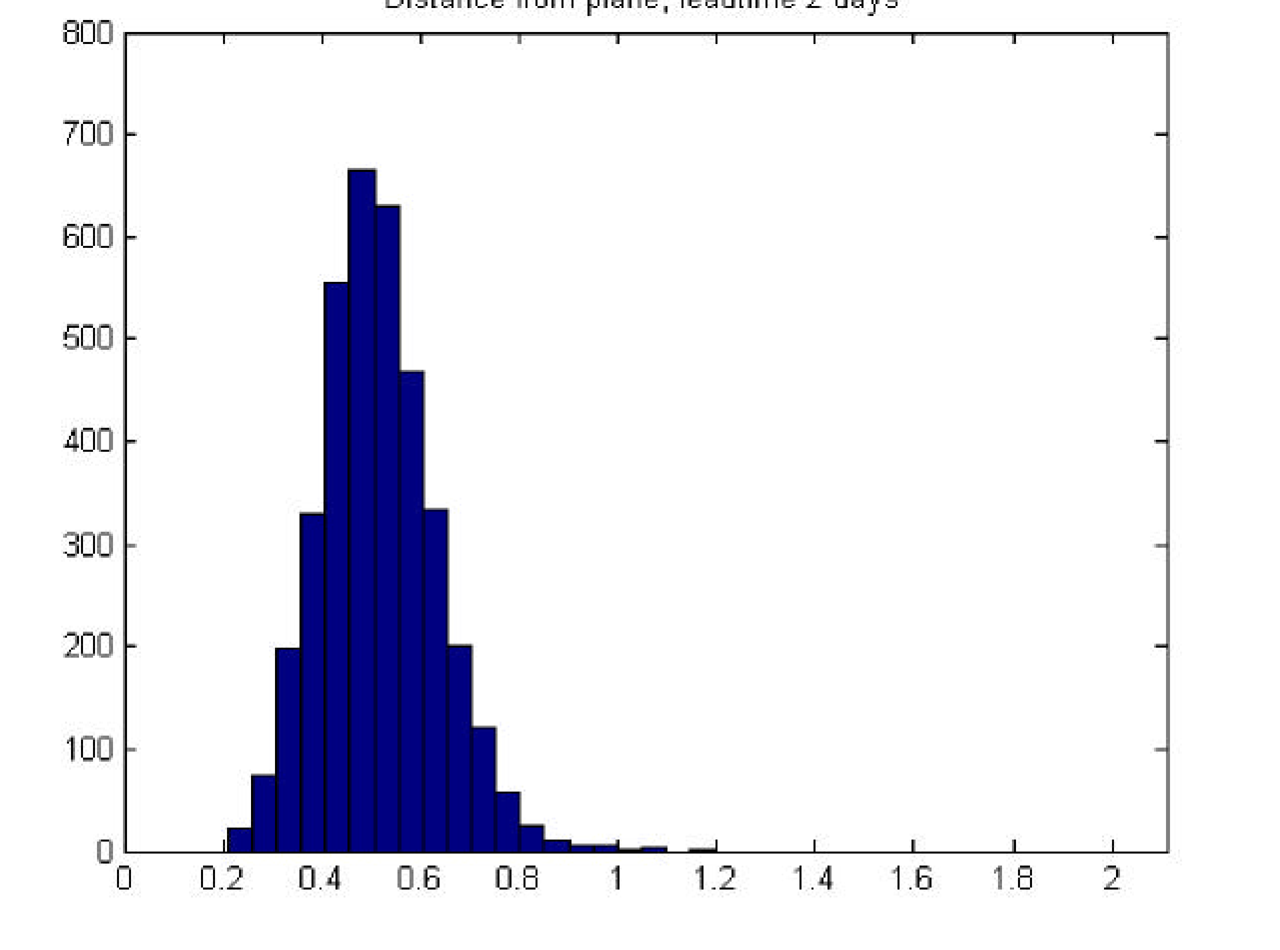
NCEP
June 2004 – June 2005
T2m (US and EU)
D = 213 (6^o x 6^o)

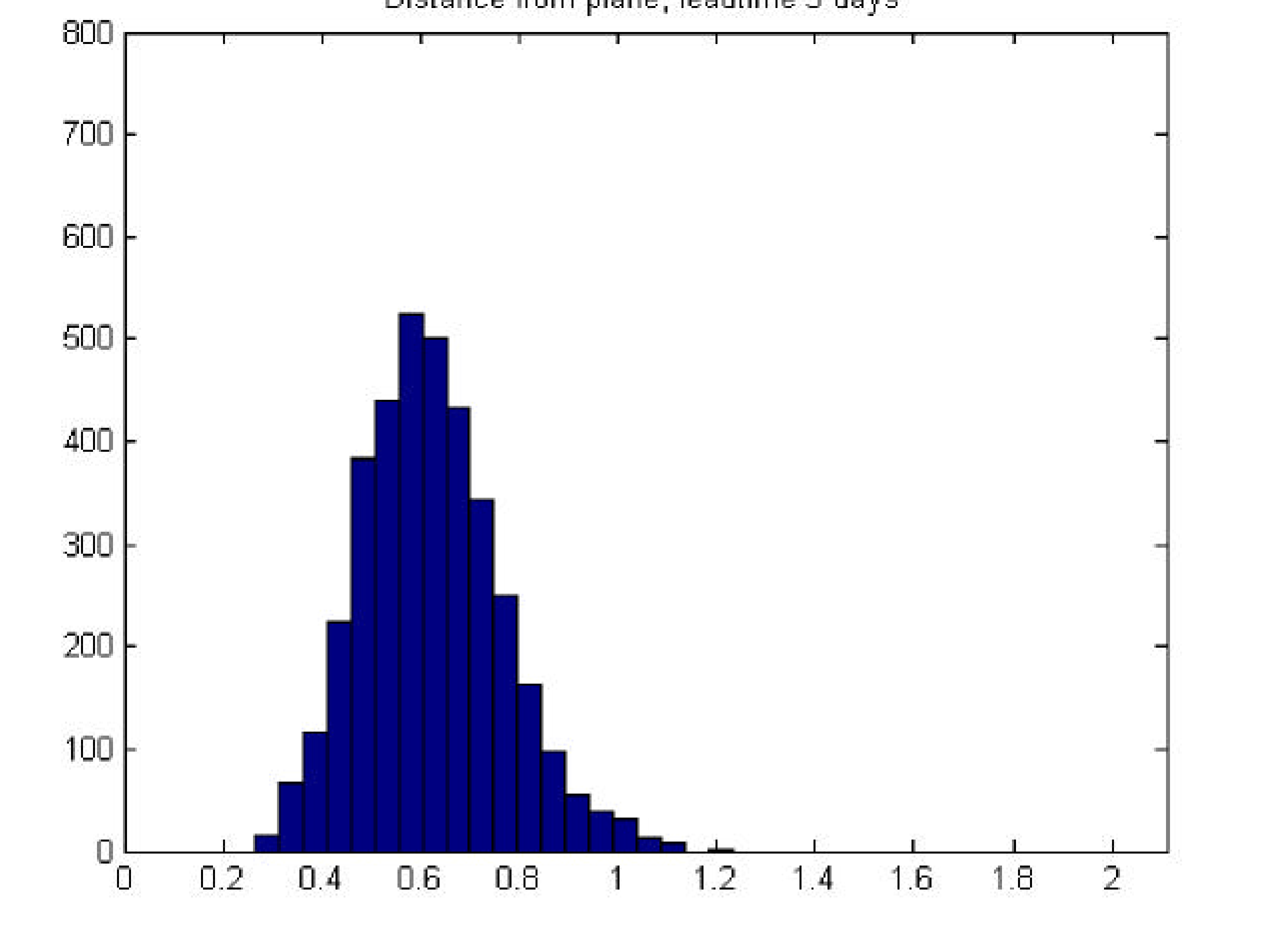
2004/06/05

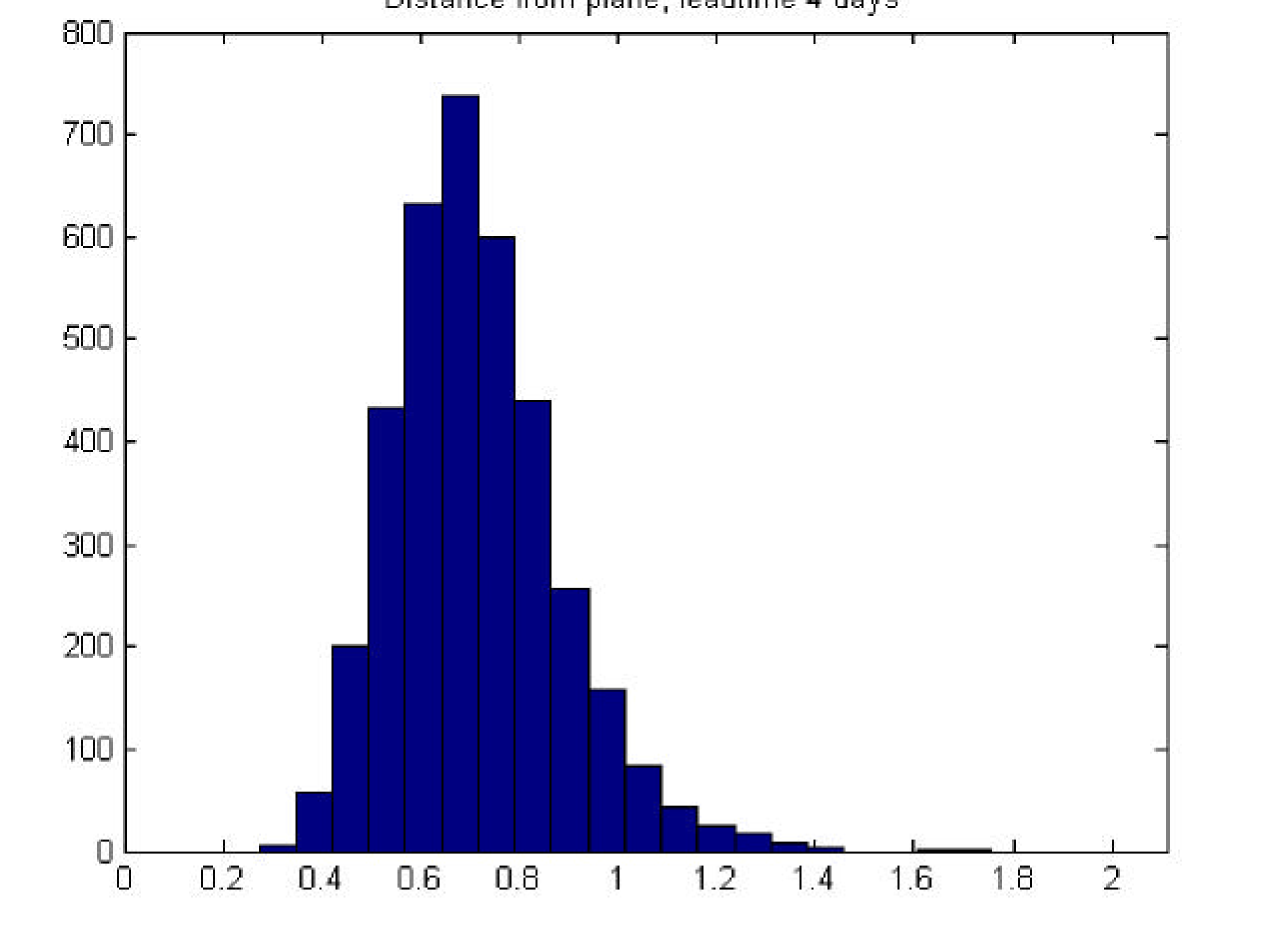


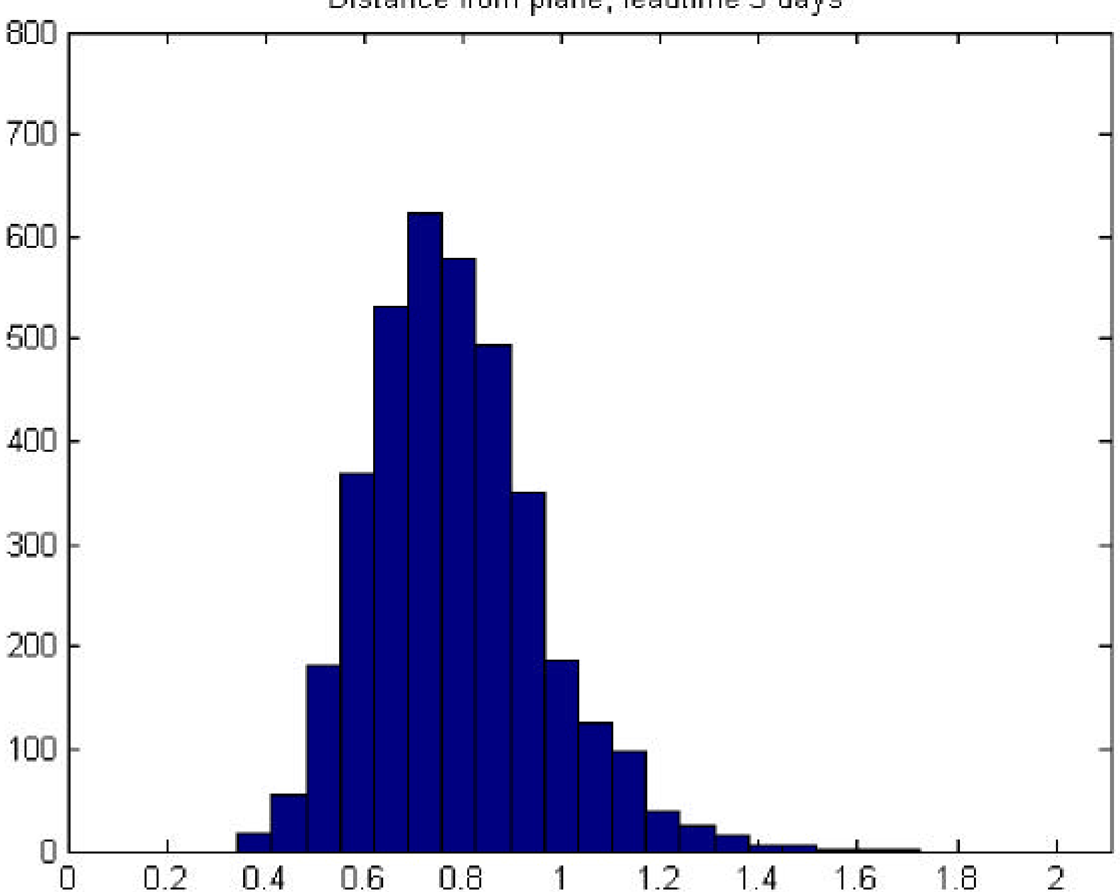


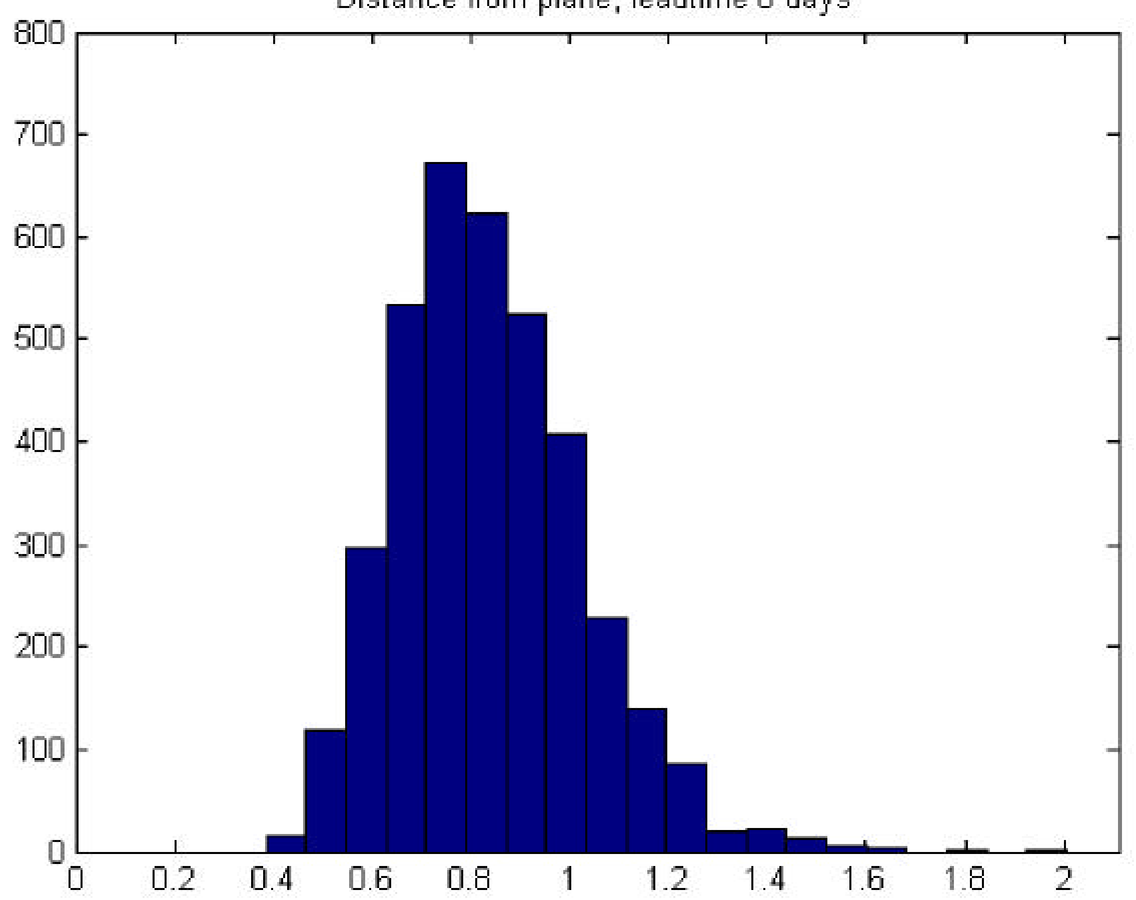
Histogram of one year of statistics.
For each day, the distance of target from the plane defines the unit distance.

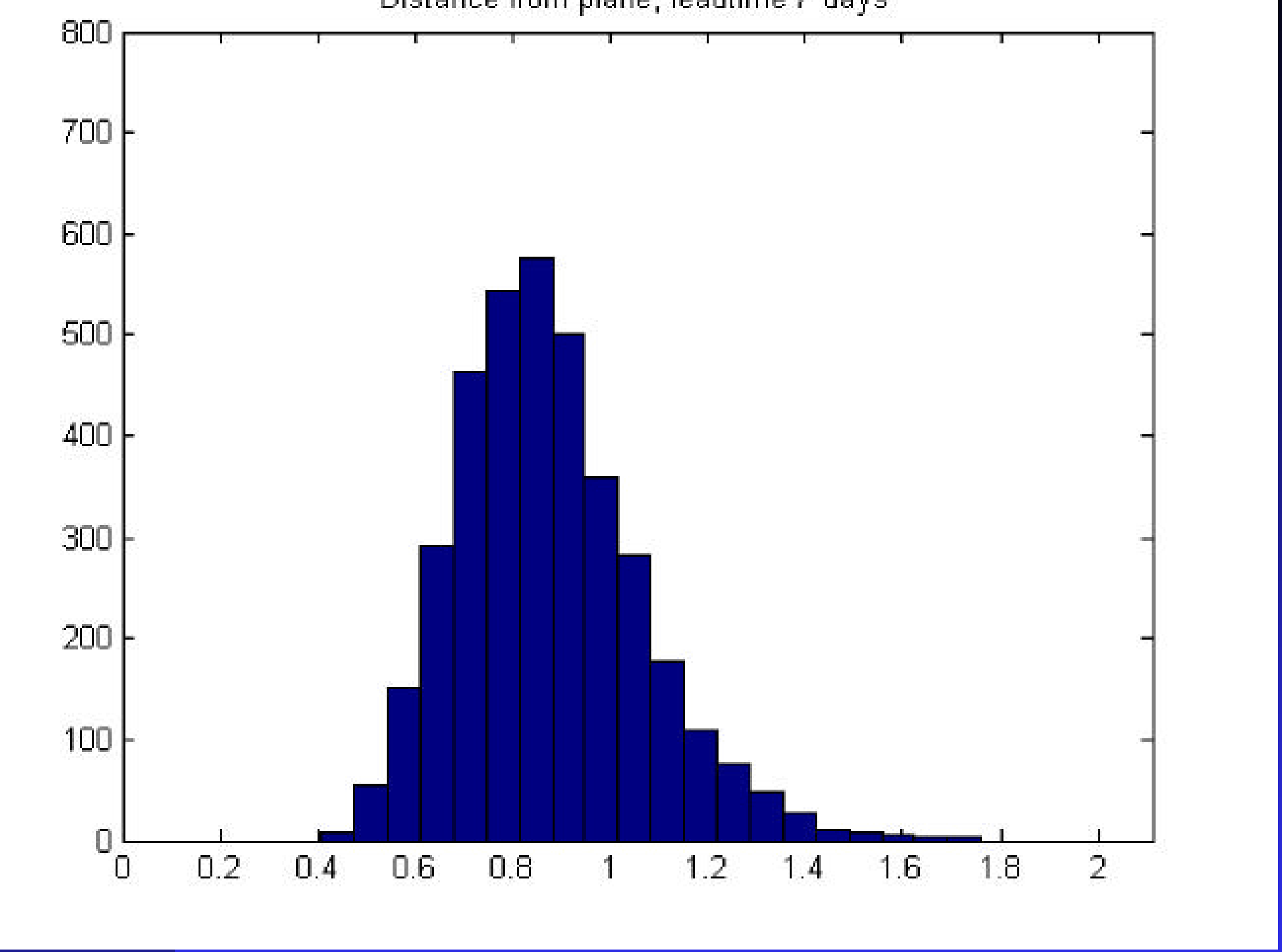

Distance from plane, leadtime 2 days
800
700
600
500
400
300
200
100
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Distance from plane, leadtime 3 days
800
700
600
500
400
300
200
100
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

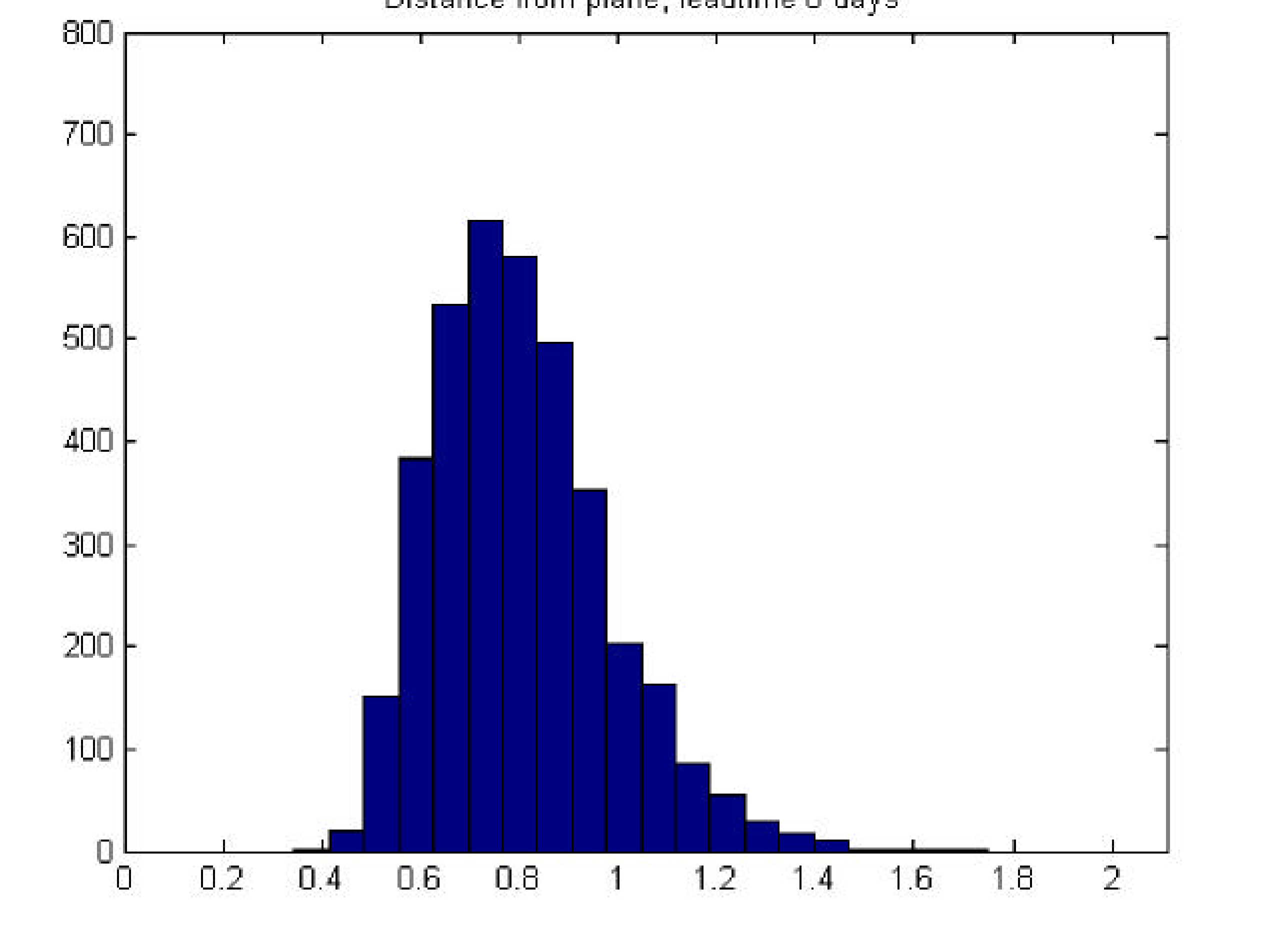

Distance from plane, leadtime 4 days
800
700
600
500
400
300
200
100
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

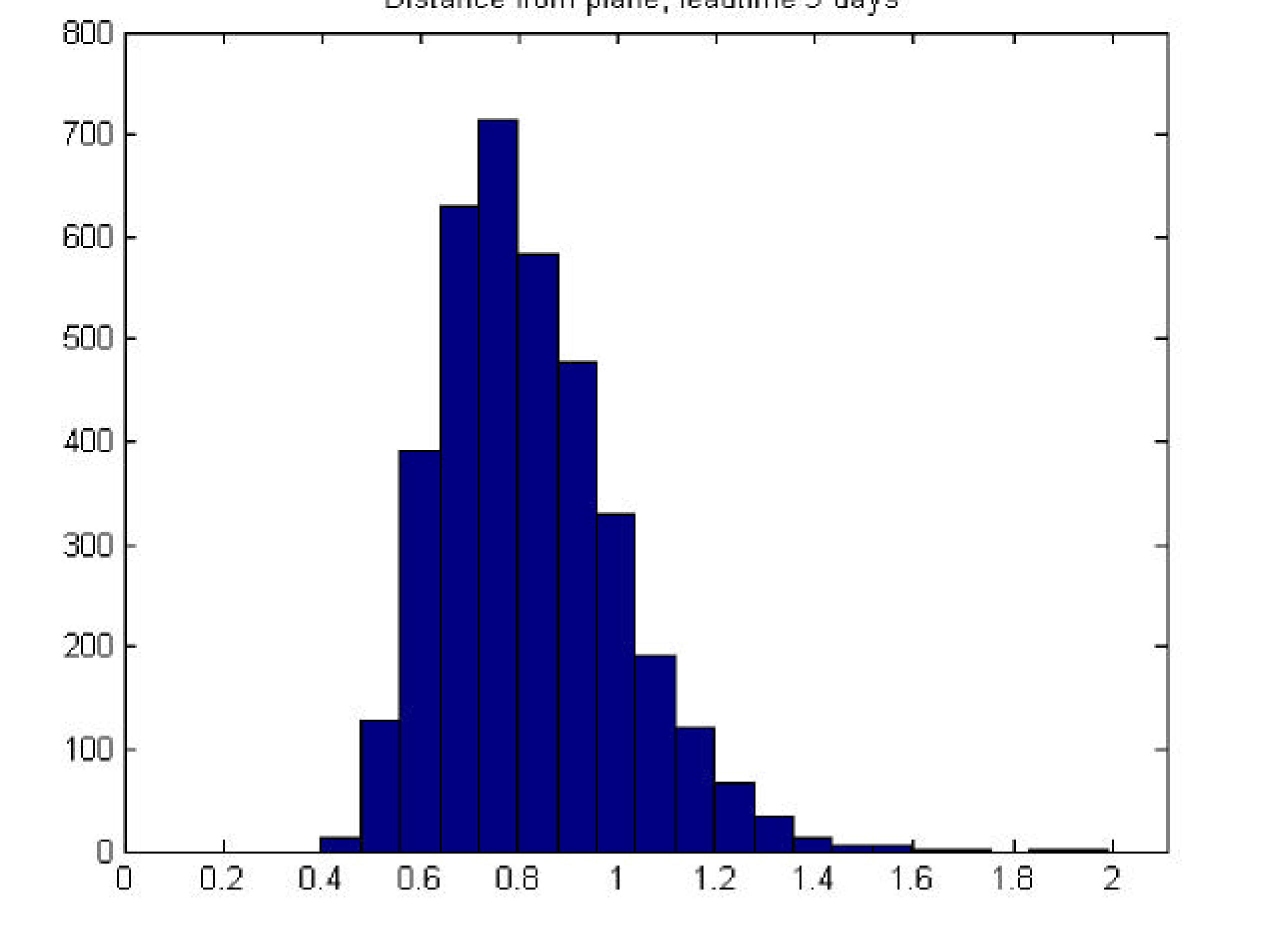

800
700
600
500
400
300
200
100
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

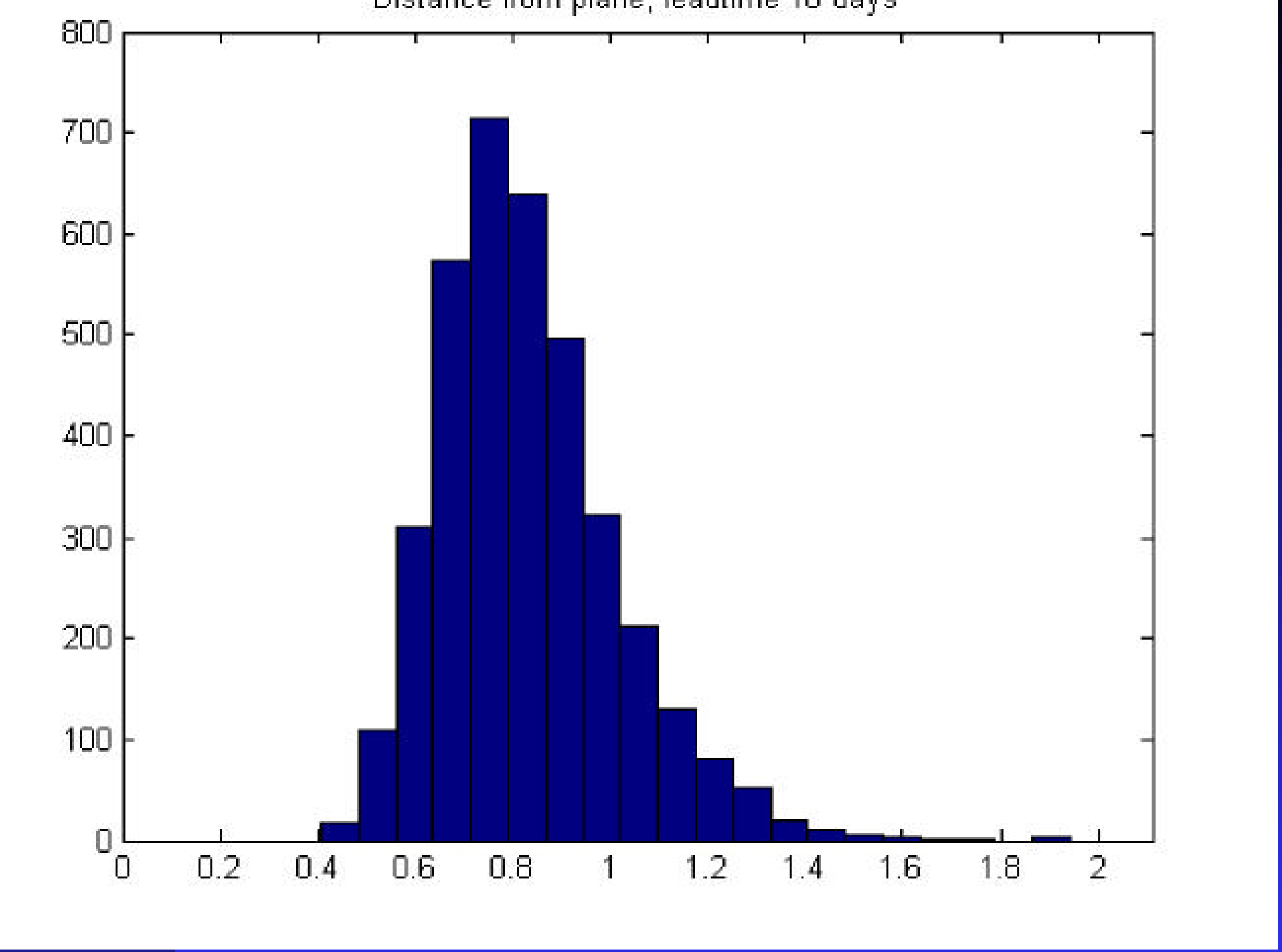

800
700
600
500
400
300
200
100
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

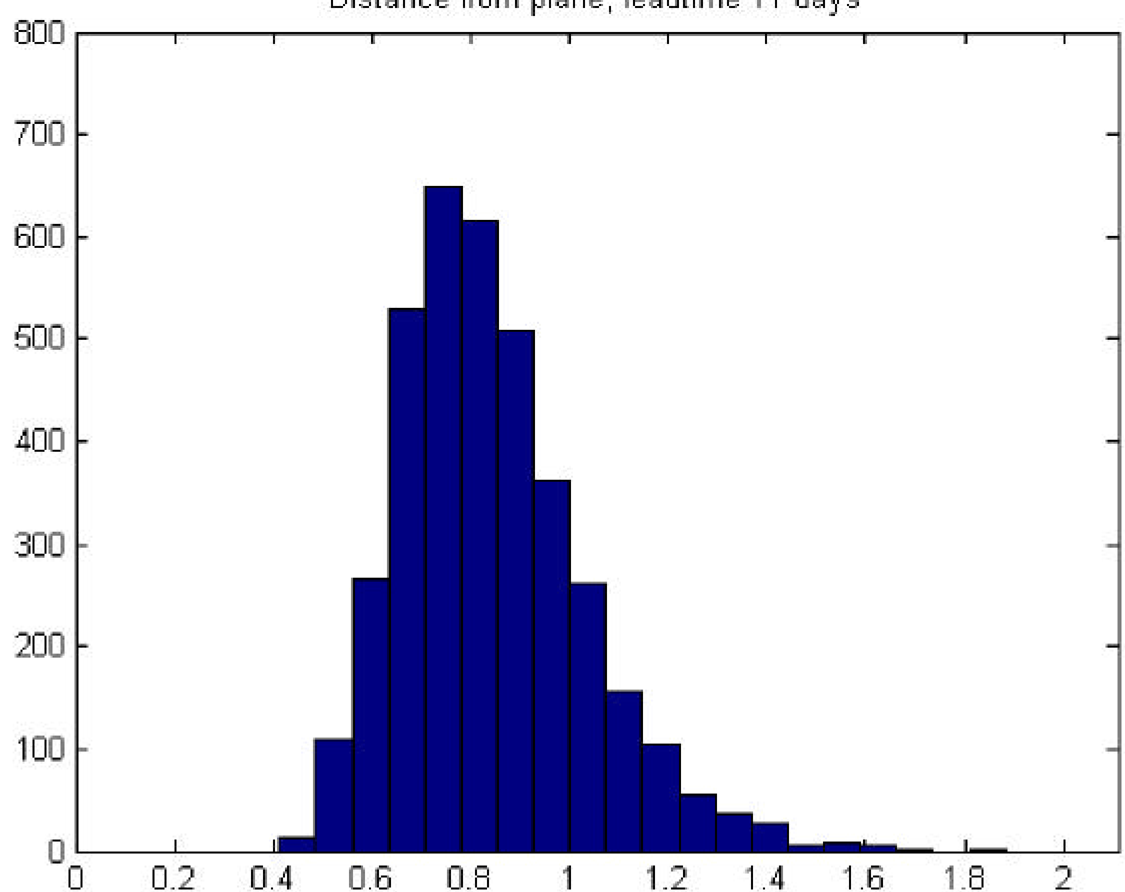


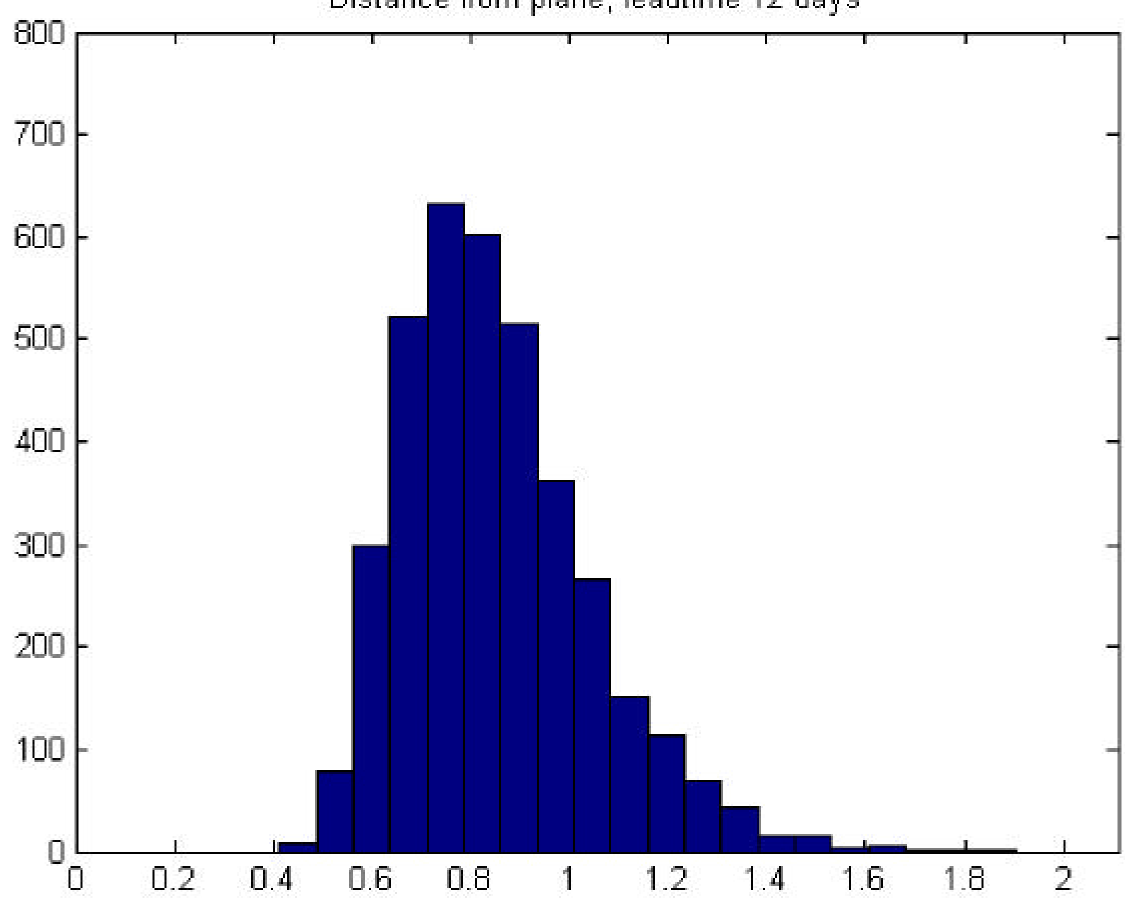
Distance from plane, leadtime 7 days
0
100
200
300
400
500
600
700
800
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

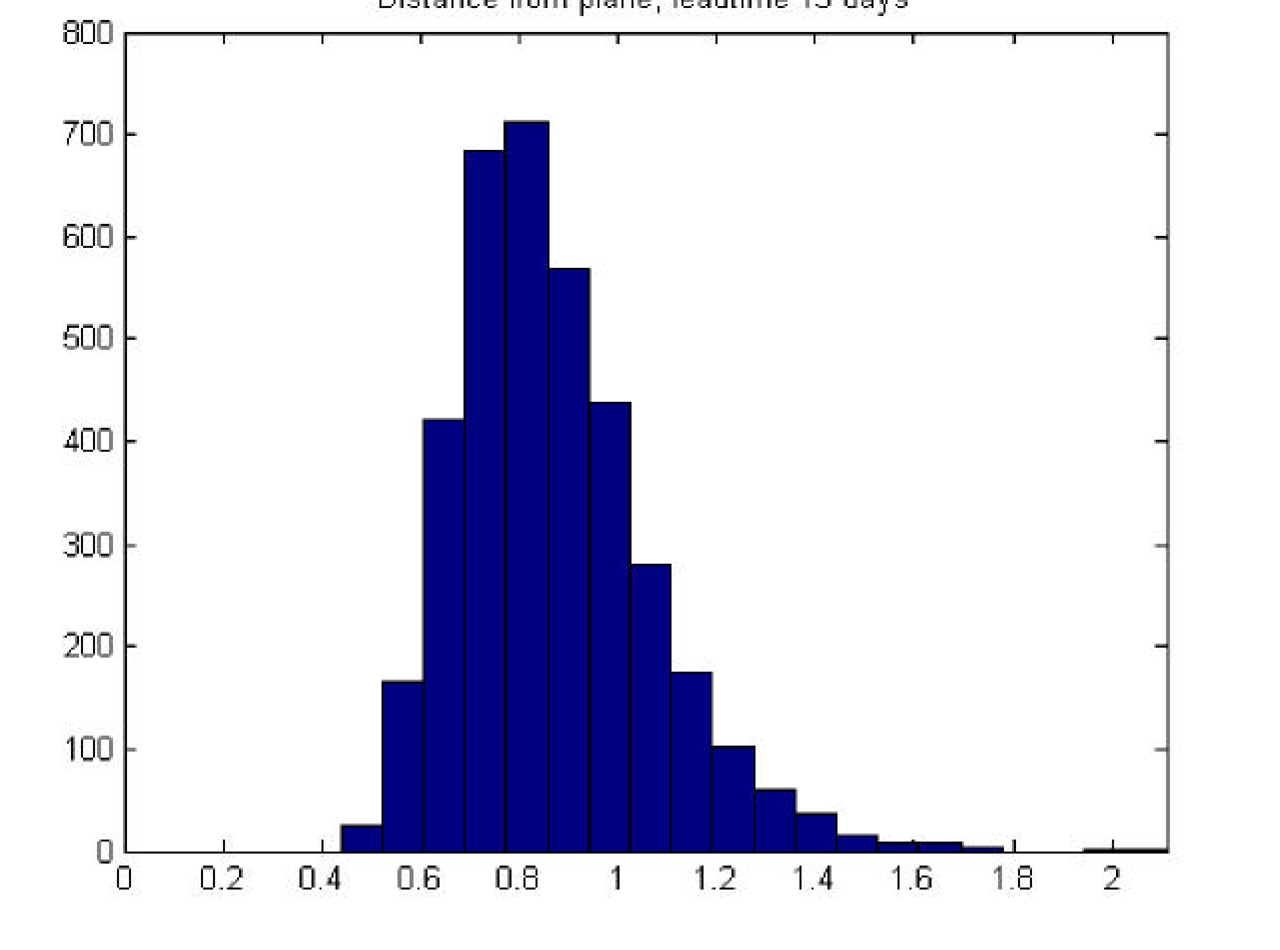

800
700
600
500
400
300
200
100
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Distance from plane, leadtime 5 days
800
700
600
500
400
300
200
100
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

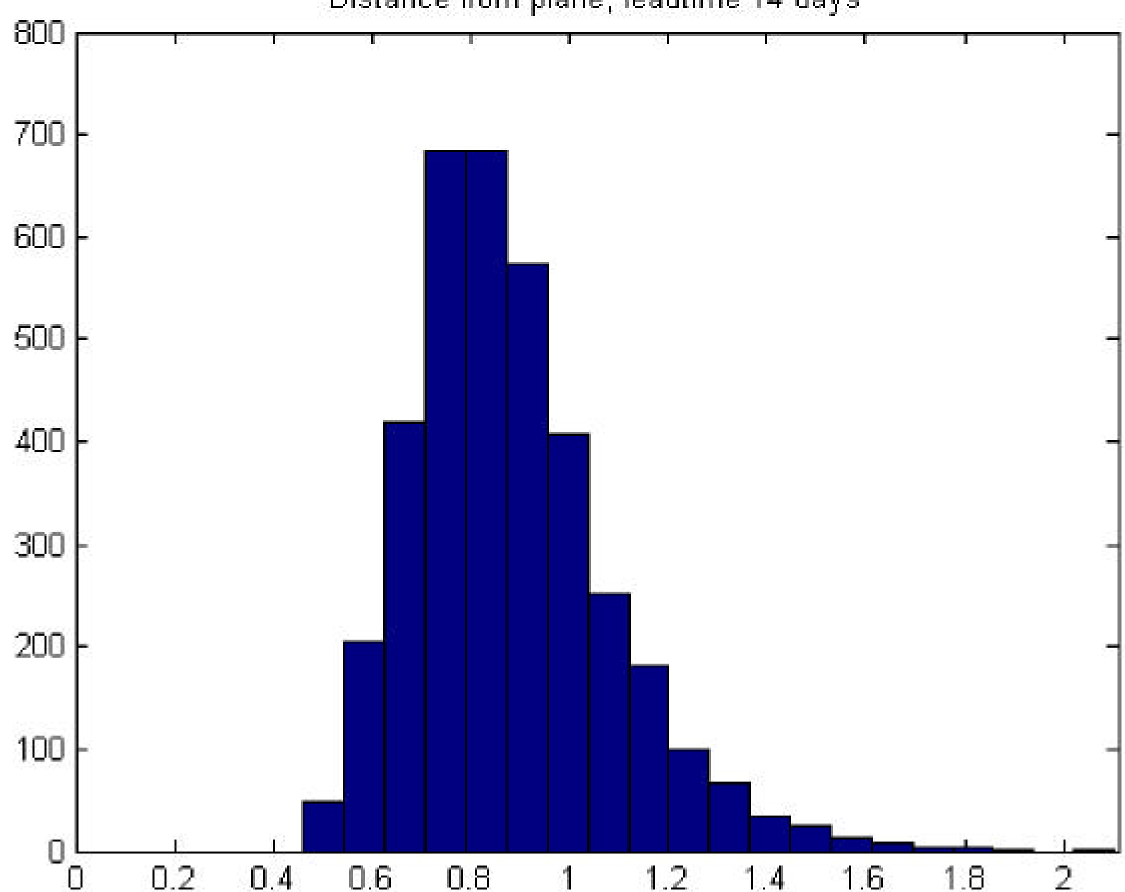

Distance from plane, leadtime 10 days
800
700
600
500
400
300
200
100
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

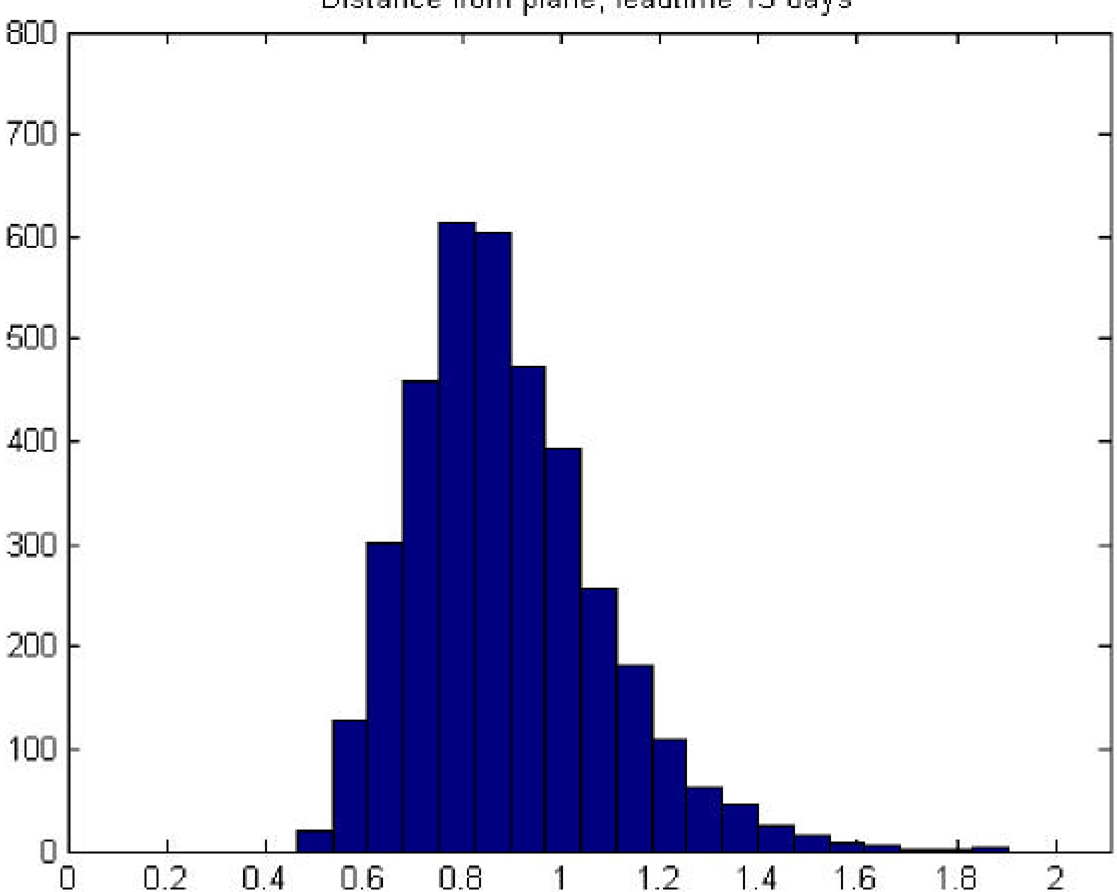

0
100
200
300
400
500
600
700
800
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

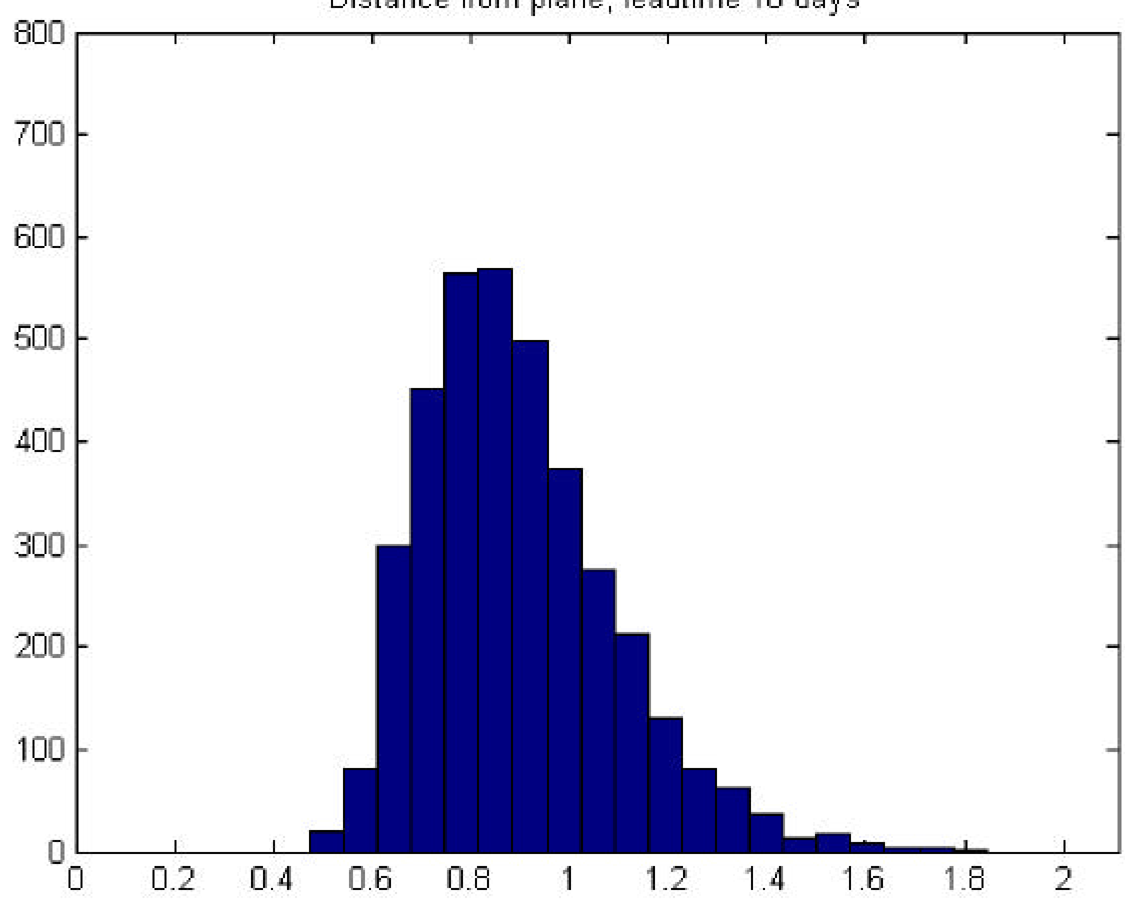

800
700
600
500
400
300
200
100
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2



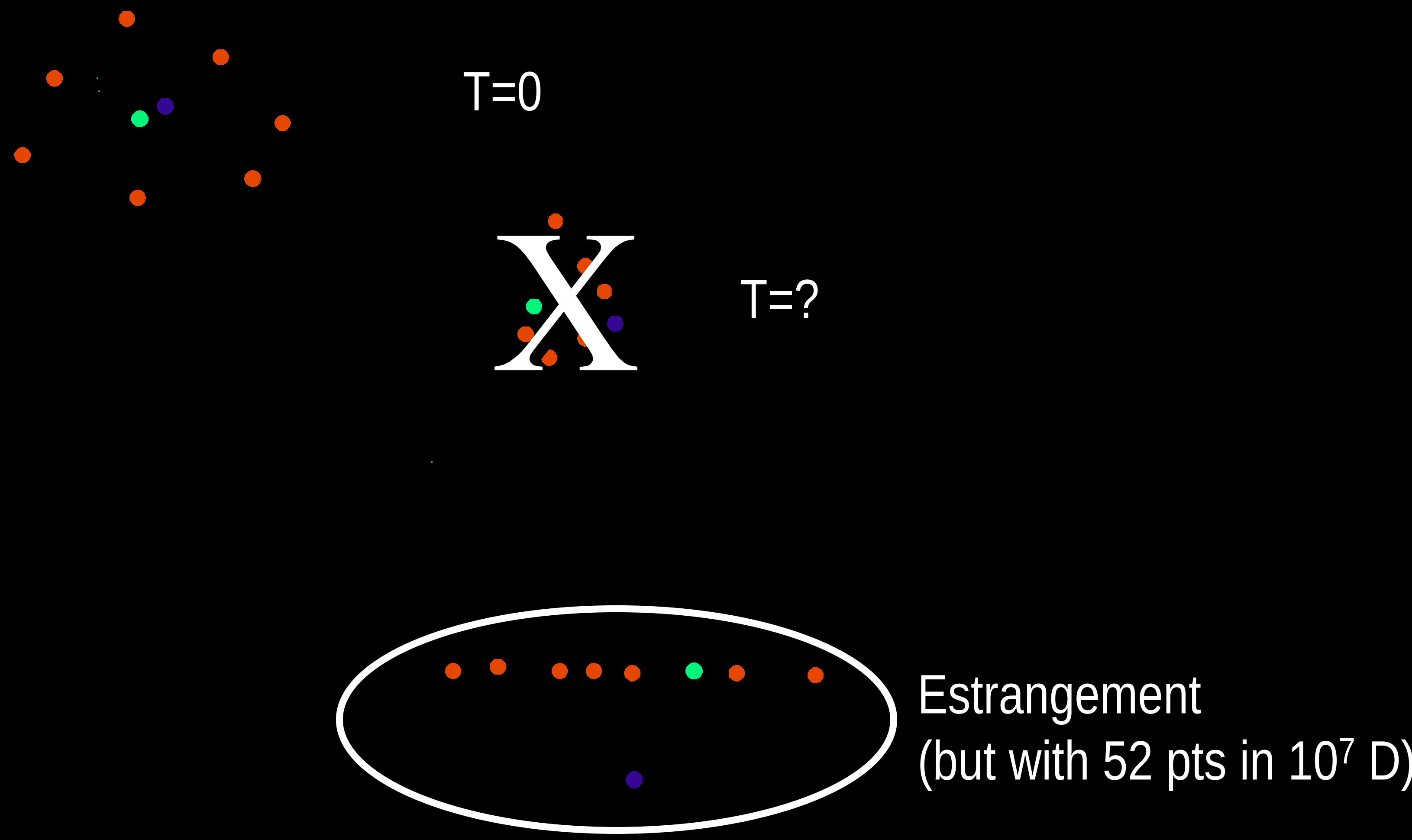
Distance from plane, leadtime 15 days
800
700
600
500
400
300
200
100
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Distance from plane, leadtime 14 days
0
100
200
300
400
500
600
700
800
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Distance from plane, leadtime 15 days
800
700
600
500
400
300
200
100
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


800
700
600
500
400
300
200
100
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Detecting Ensemble Estrangement in practice



"Inflating" the variance will not "capture" the verification.

8

Deployable *ad hoc* tests are valuable

You can make up lots of these type of consistency tests:

To verify relevance of claims of "optimality":

I Gilmour, LA Smith & R Buizza (2001) Linear Regime Duration: Is 24 Hours a Long time in Weather Forecasting? JAS **58**:3525-3539.

Or relevance of various approximations in adaptive obs:

J Hansen & LA Smith (2000) The role of operational constraints in selecting supplementary observations. JAS **57** (17): 2859-2871.

Or indications of drift and systematic state-dependent model error:

D Orrell, LA Smith, J Barkmeijer and TN Palmer (2001) Model error in weather forecasting. Nonlin Processes in Geophysics **8**:357-371.

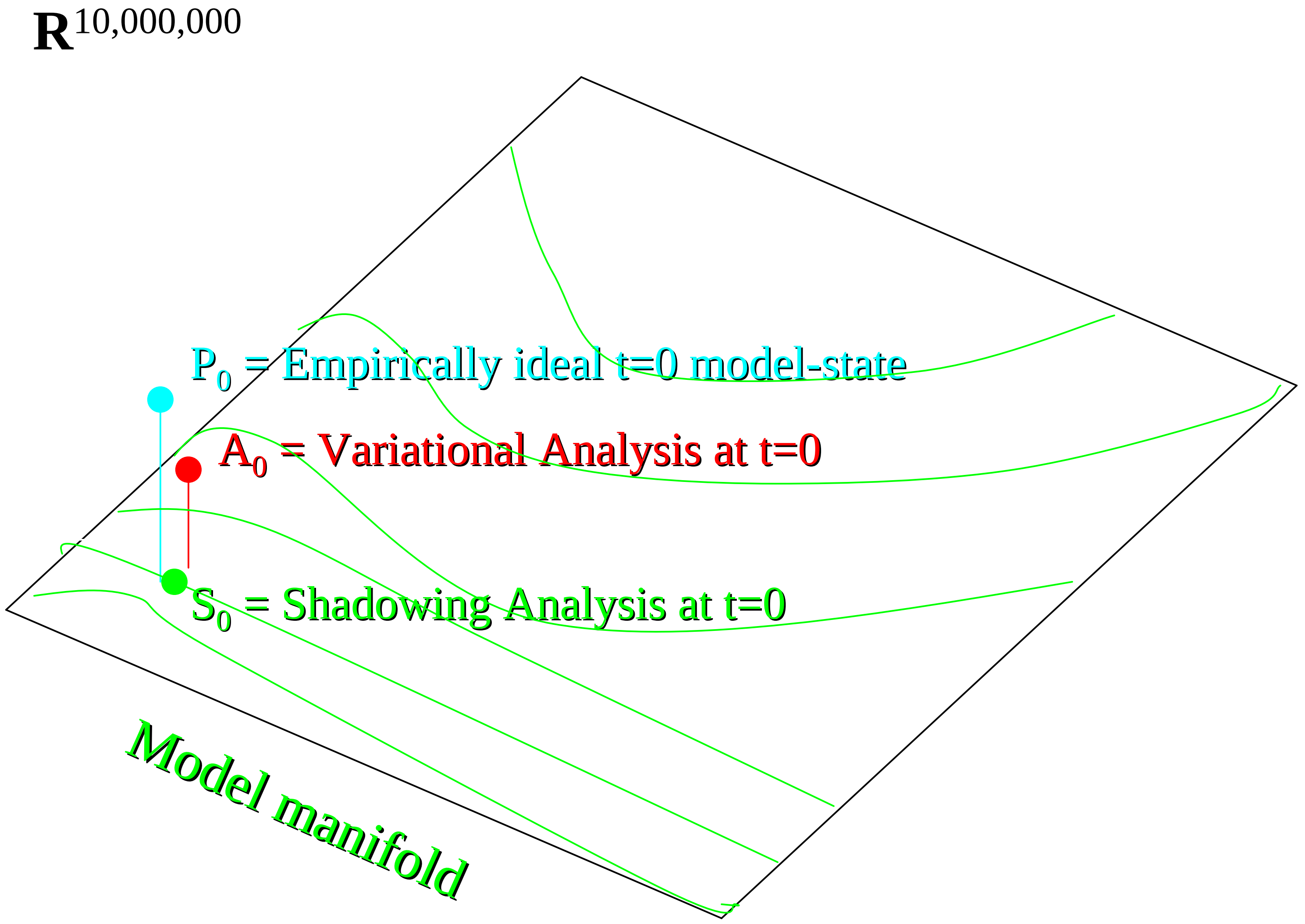
Can we linearly interpolate climate sensitivity in HADAM3 parameters

D Stainforth et al, Nature (2005) (no!)

Rank Histograms in higher dimensions

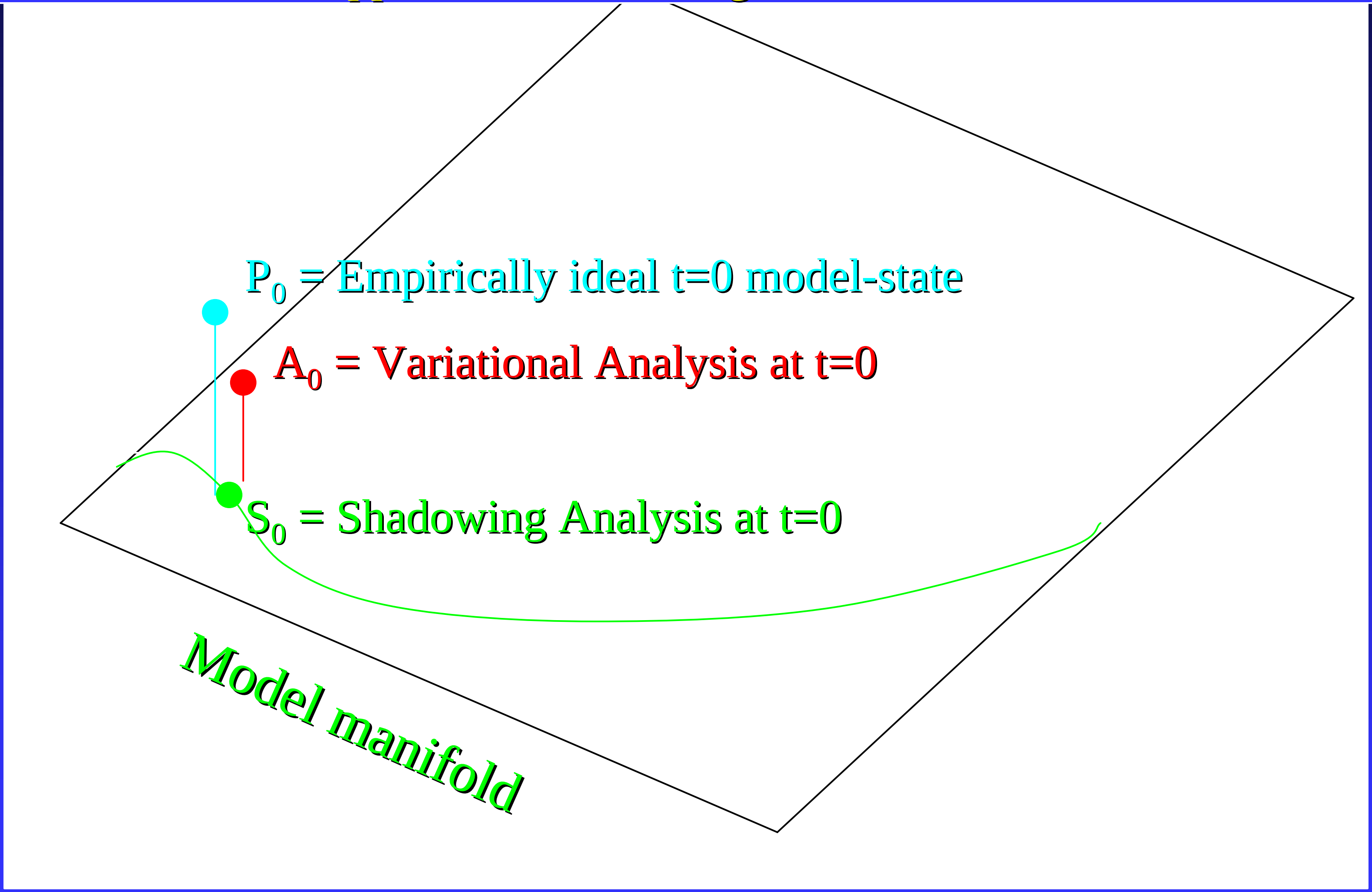
Smith and Hansen (2004) MWR

I'll skip these and look at model error in the ***big*** picture...


R^10,000,000
P0 = Empirically ideal t=0 model-state
A0 = Variational Analysis at t=0
S0 = Shadowing Analysis at t=0
Model manifold

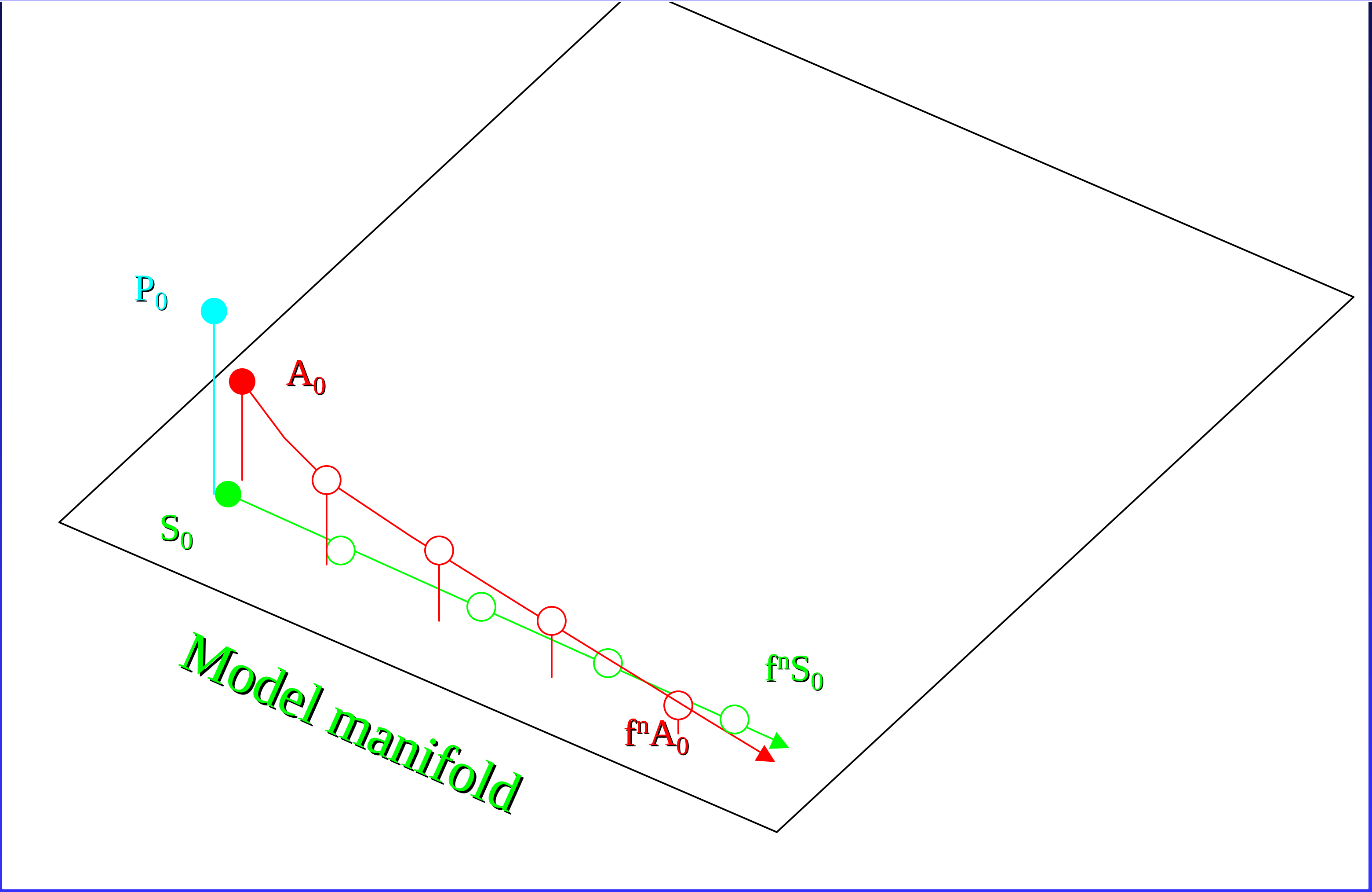
We might keep P_n as a target/verification,
but P_0 is unlikely to provide model-initial condition(s).

Variational Assimilation pulls the initial conditions away from the manifold.
What happens when we "let go" and forecast…

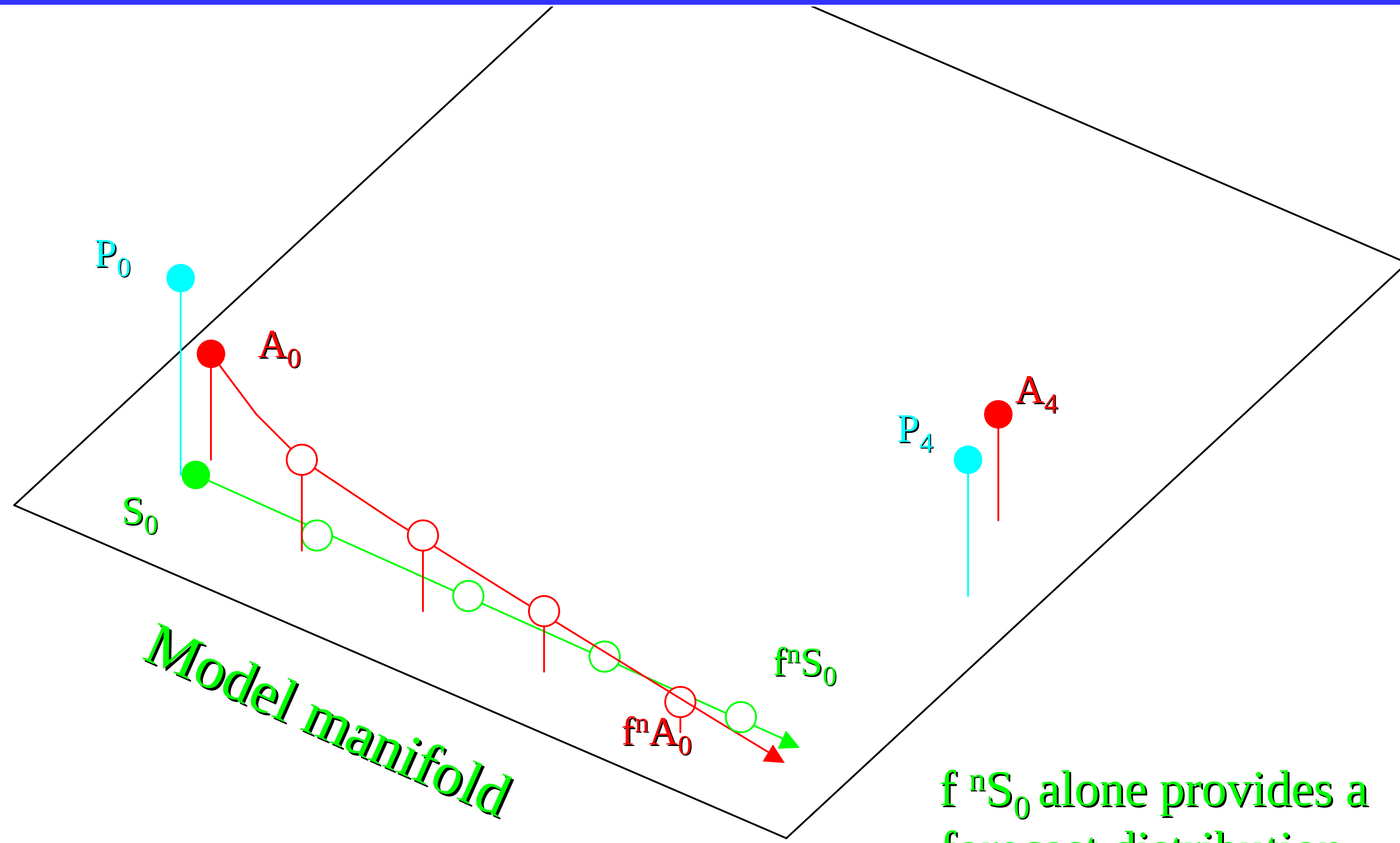


What happens when we "let go" and forecast…

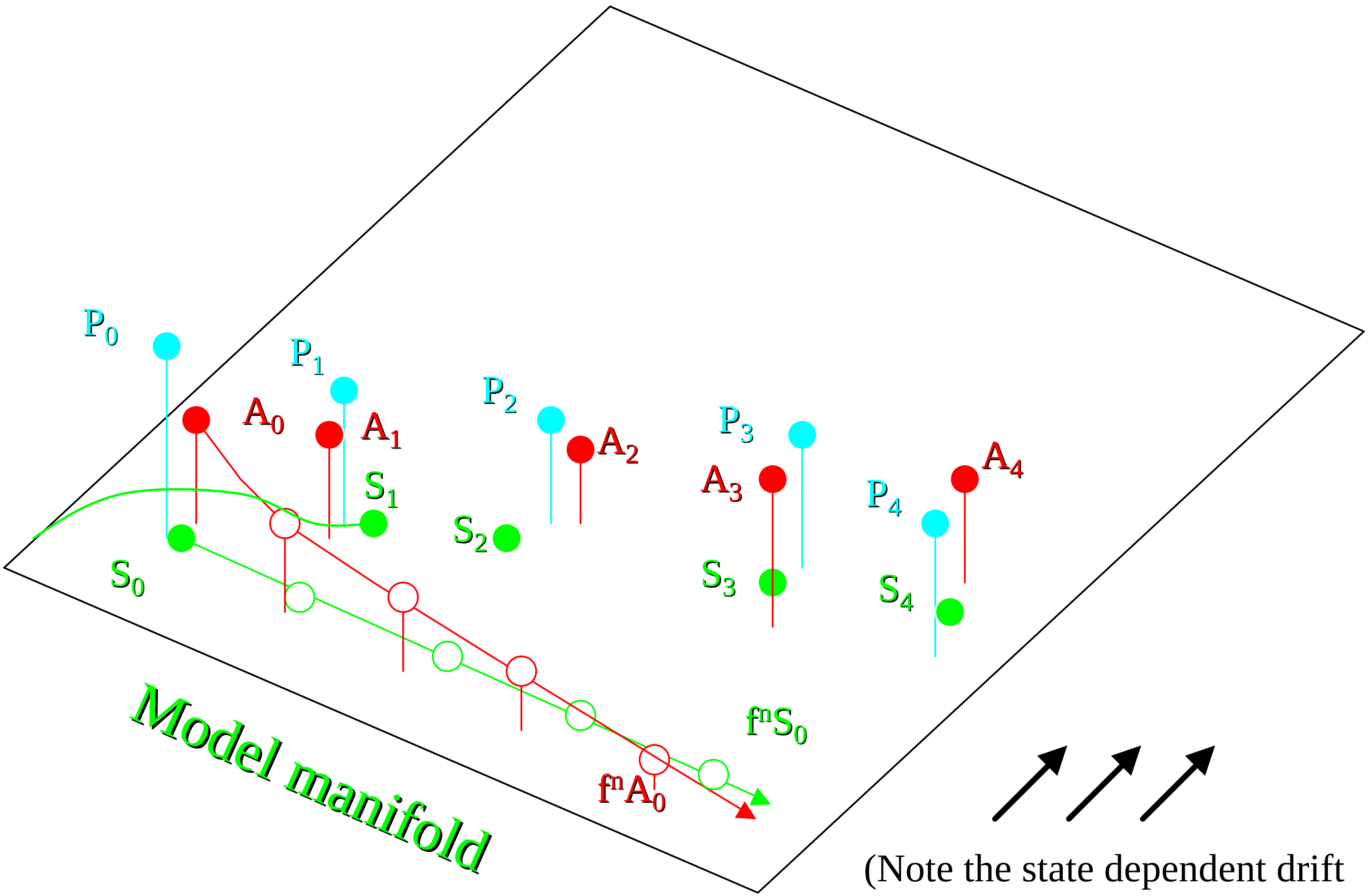
A_0 immediately falls toward *somewhere* on the manifold.



We are allowed a projection operator to map $f^{\,n}S_0$ into a distribution; we take this freedom even if we verify against P!

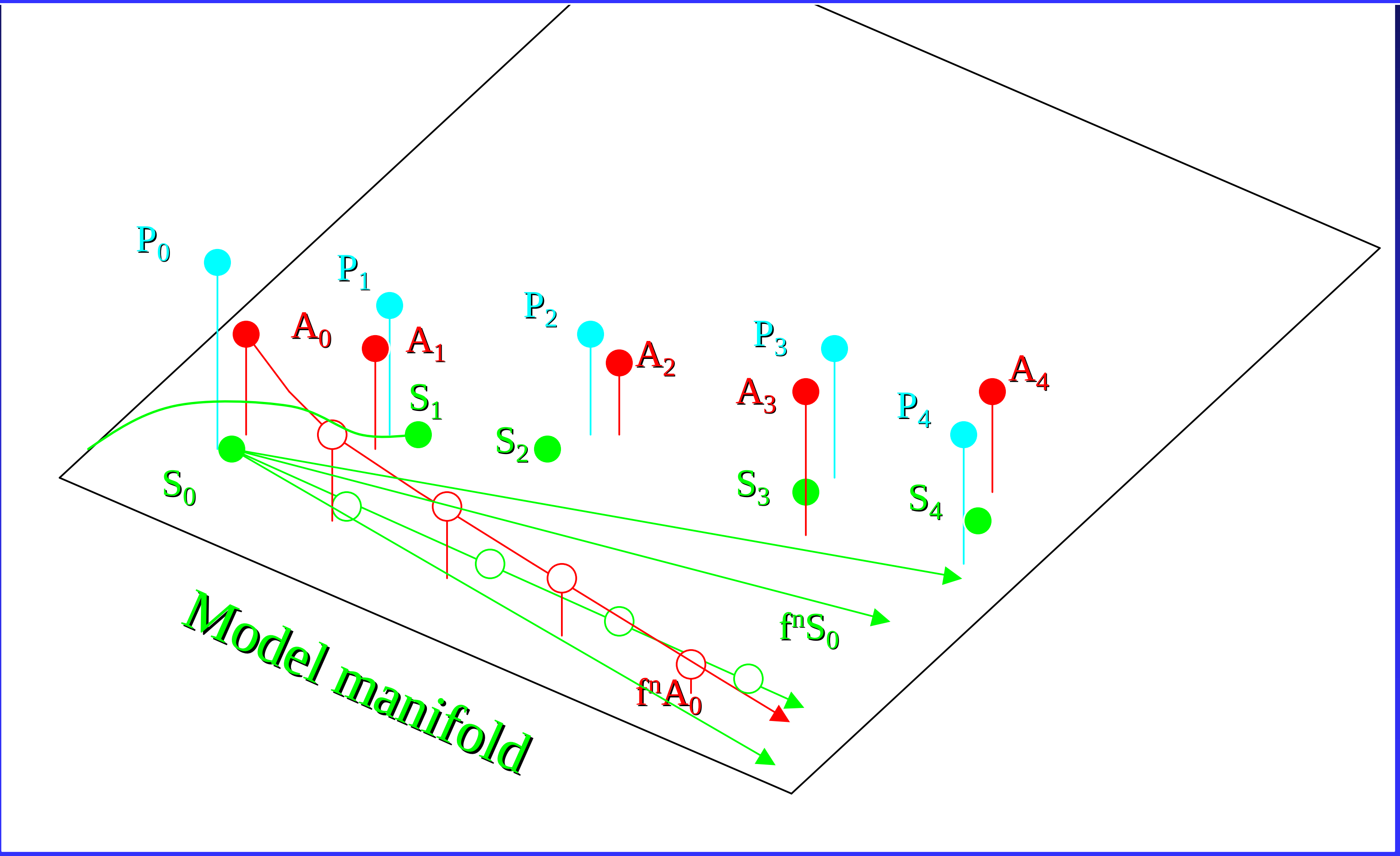


$f^{\,n}S_0$ alone provides a forecast distribution

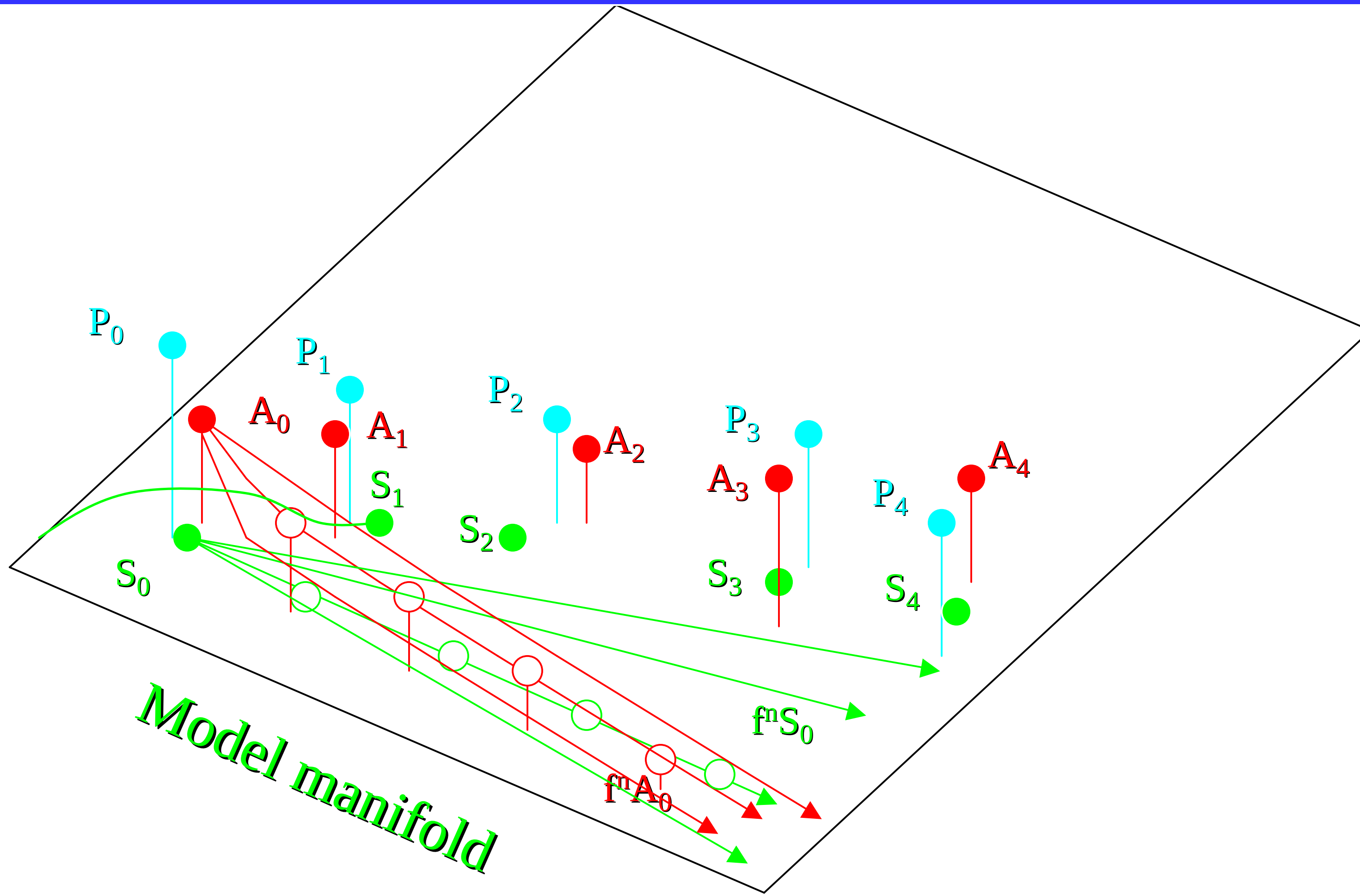


(Note the state dependent drift
due to model inadequacy…)

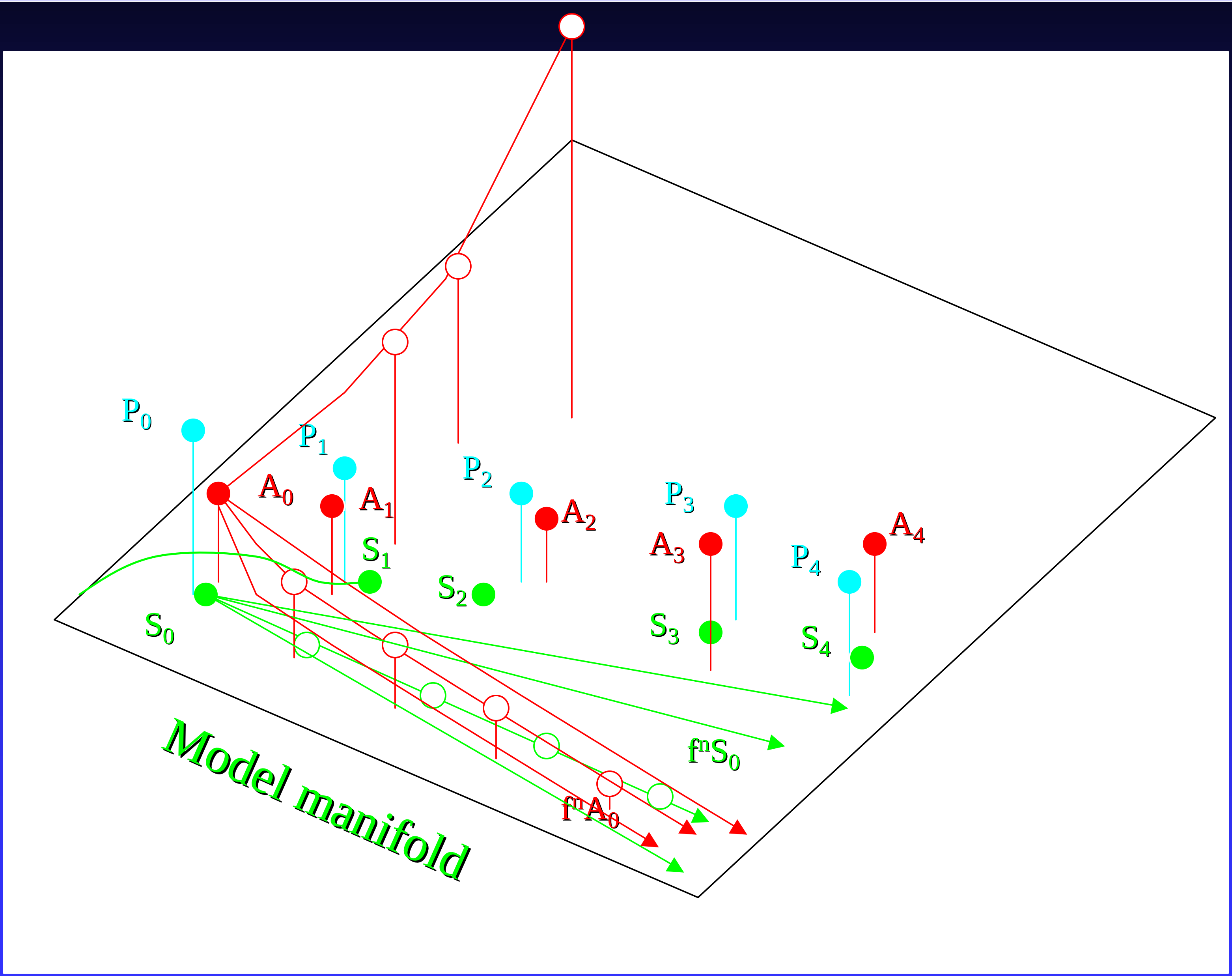
But if we have taken ensembles seriously then we have an ensemble of simulations from near S.



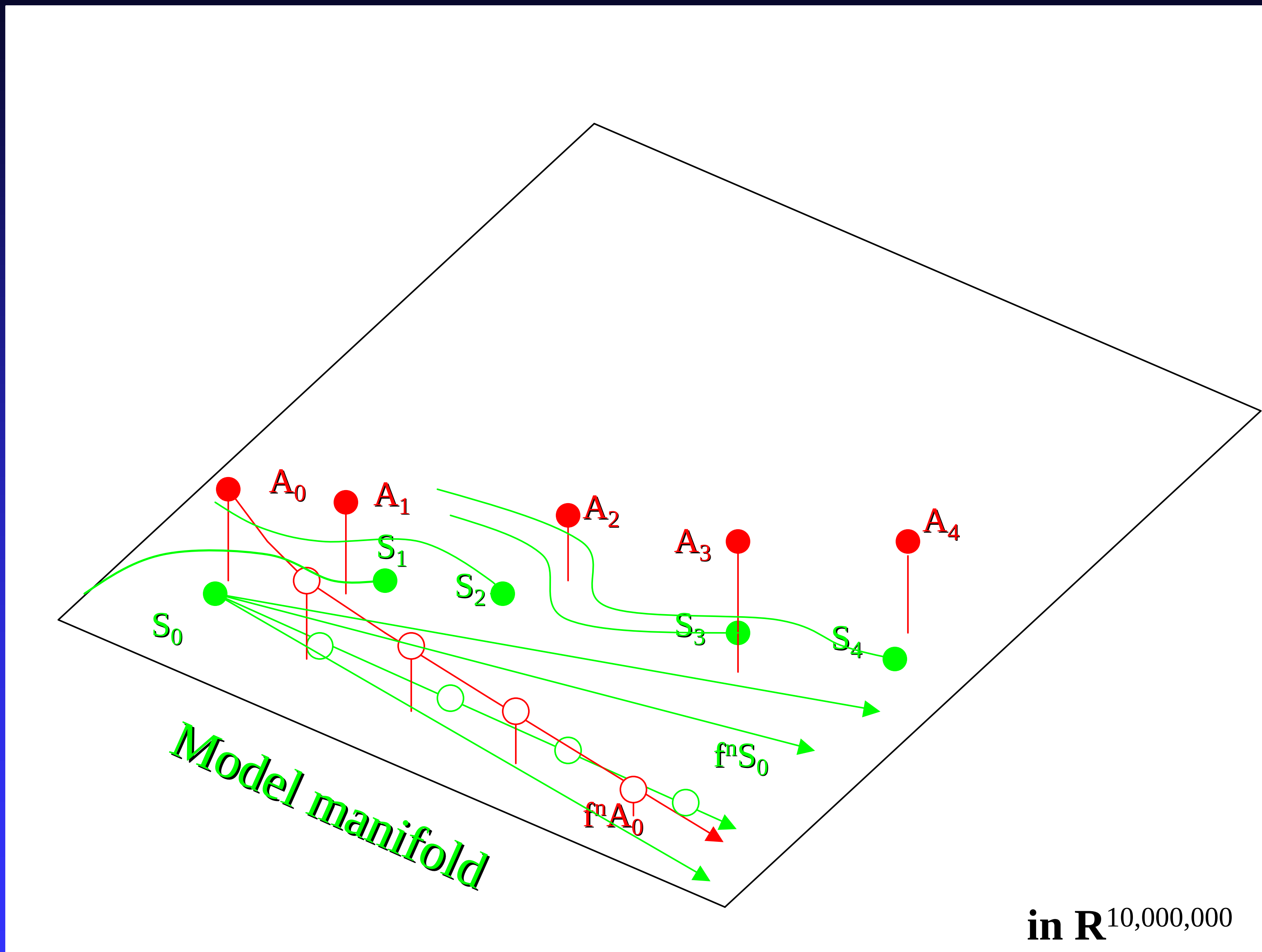
And an ensemble of model simulations from near A.



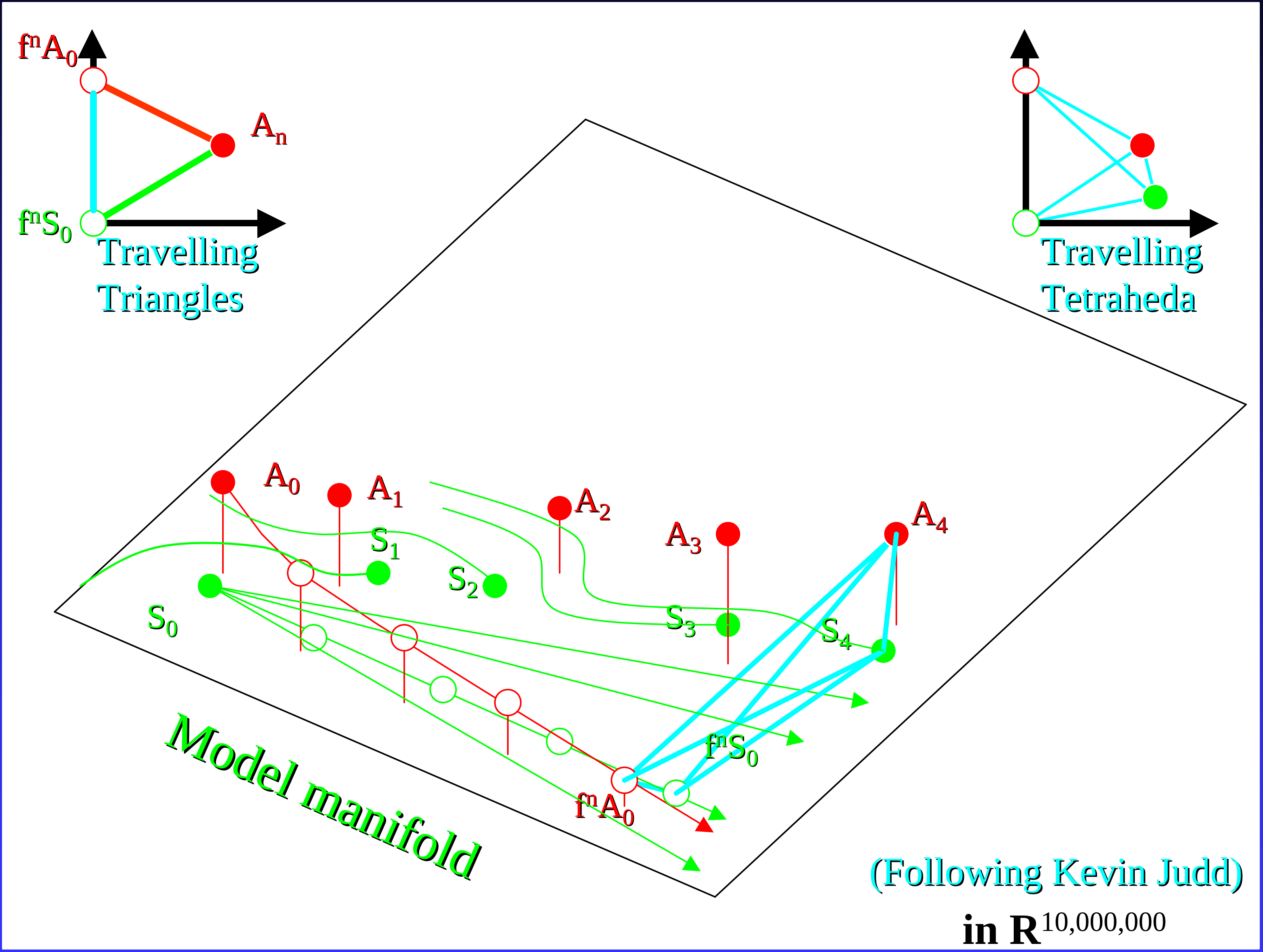
Of course, points near A can fall onto other bits of the manifold.



What can we know operationally?

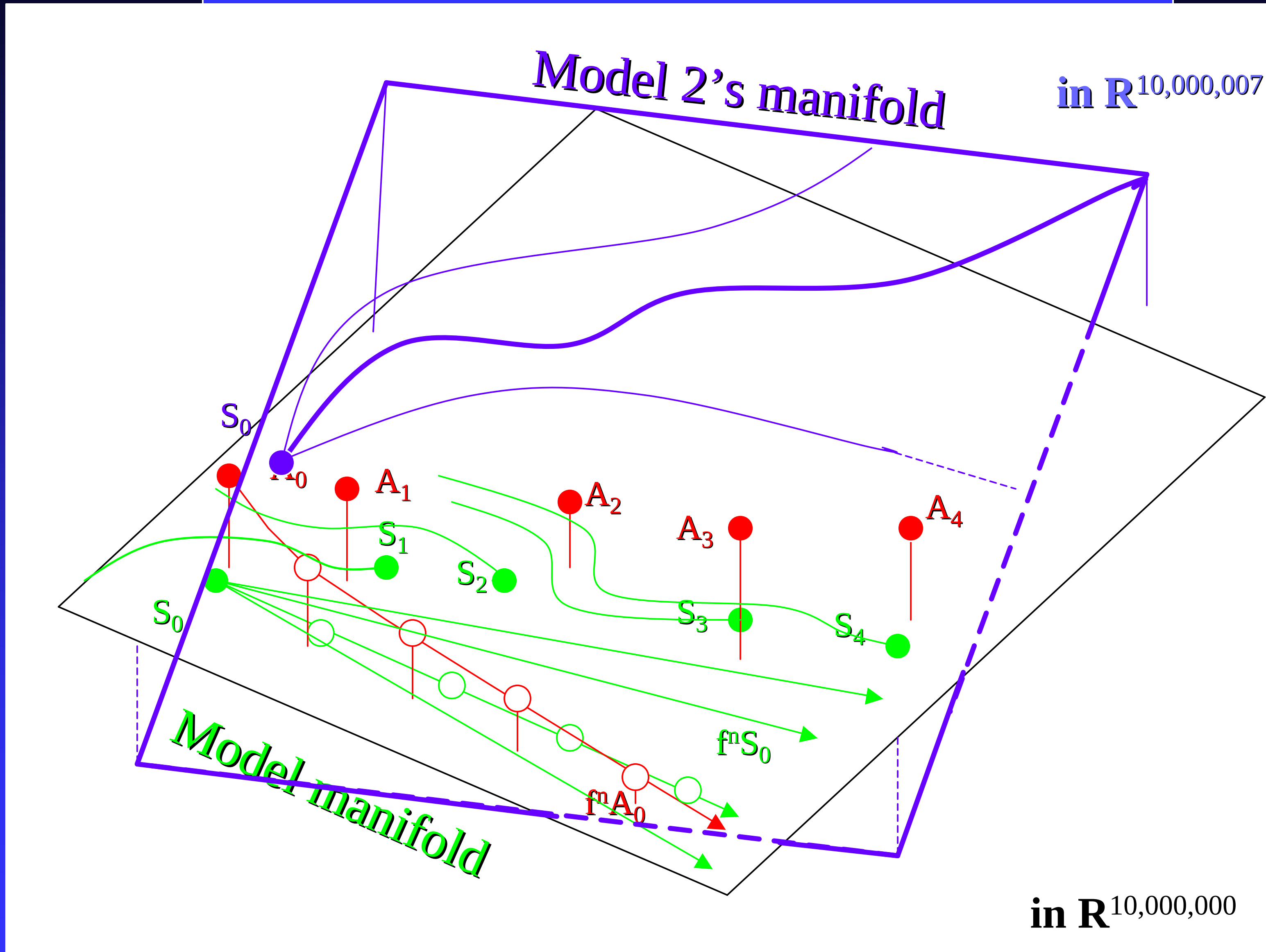


But could we ever interpret such diagrams operationally?



What is the aim of DA given two models?

$P(x(t > t_0) \mid s_i, F_a(x), a, n_a\ G_b(x'), b, n_b)$



9

Data Assimilation with a human face: Better balance between observations and model

Noise model: Gaussian and informative

P(X|*I*): extremely inhomogeneous

Perfect Bayes: ideal result (assuming perfect model class)

$$\mathrm{prob}(X \mid Y, I) = \frac{\mathrm{prob}(Y \mid X, I) \times \mathrm{prob}(X \mid I)}{\mathrm{prob}(Y \mid I)}$$

(you can leave now: the problem if finished)

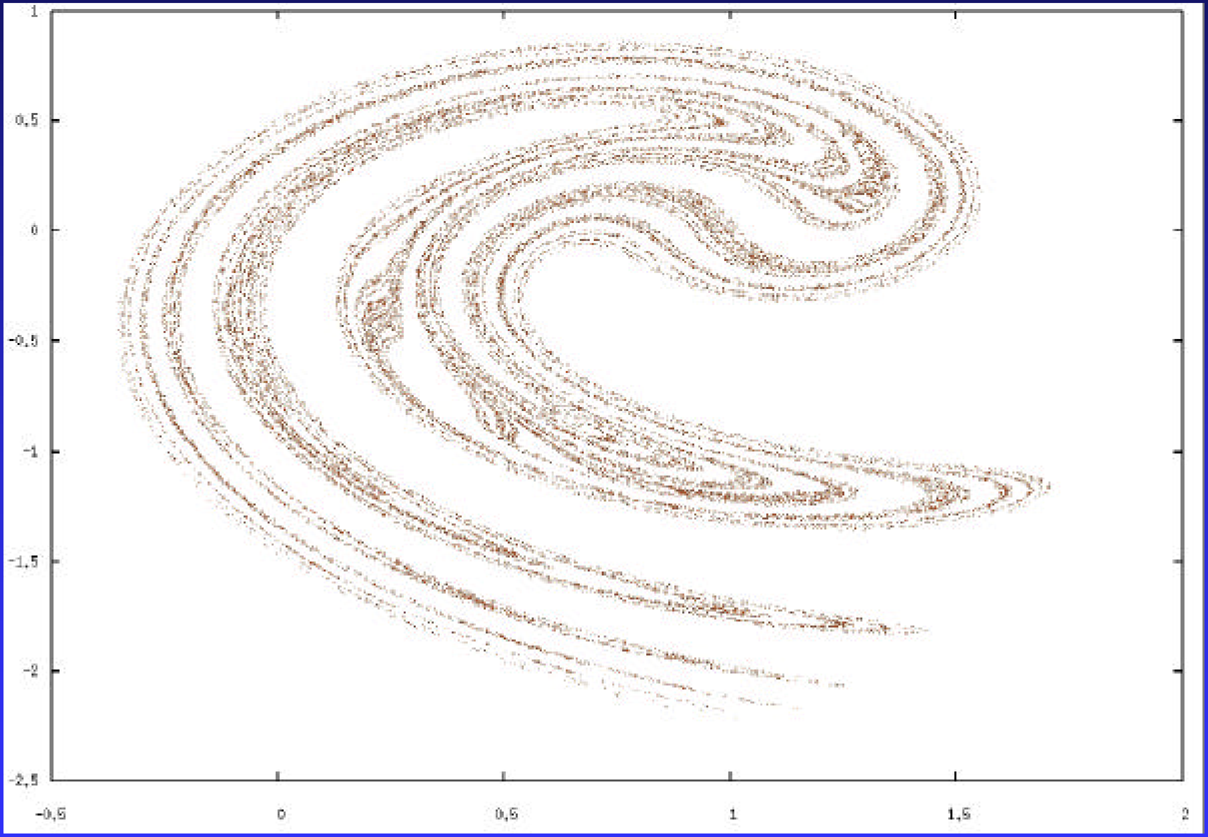
If you stay, prepare to go *ad hoc* (and sample somehow)

- (Real World) Accountable Bayes (ABIS)
- En+1 KF Ensembles
- ISIS Ensembles

Use some large finite computational resource to level the playing field…

Non-Bayesian By Choice (even within PMS)

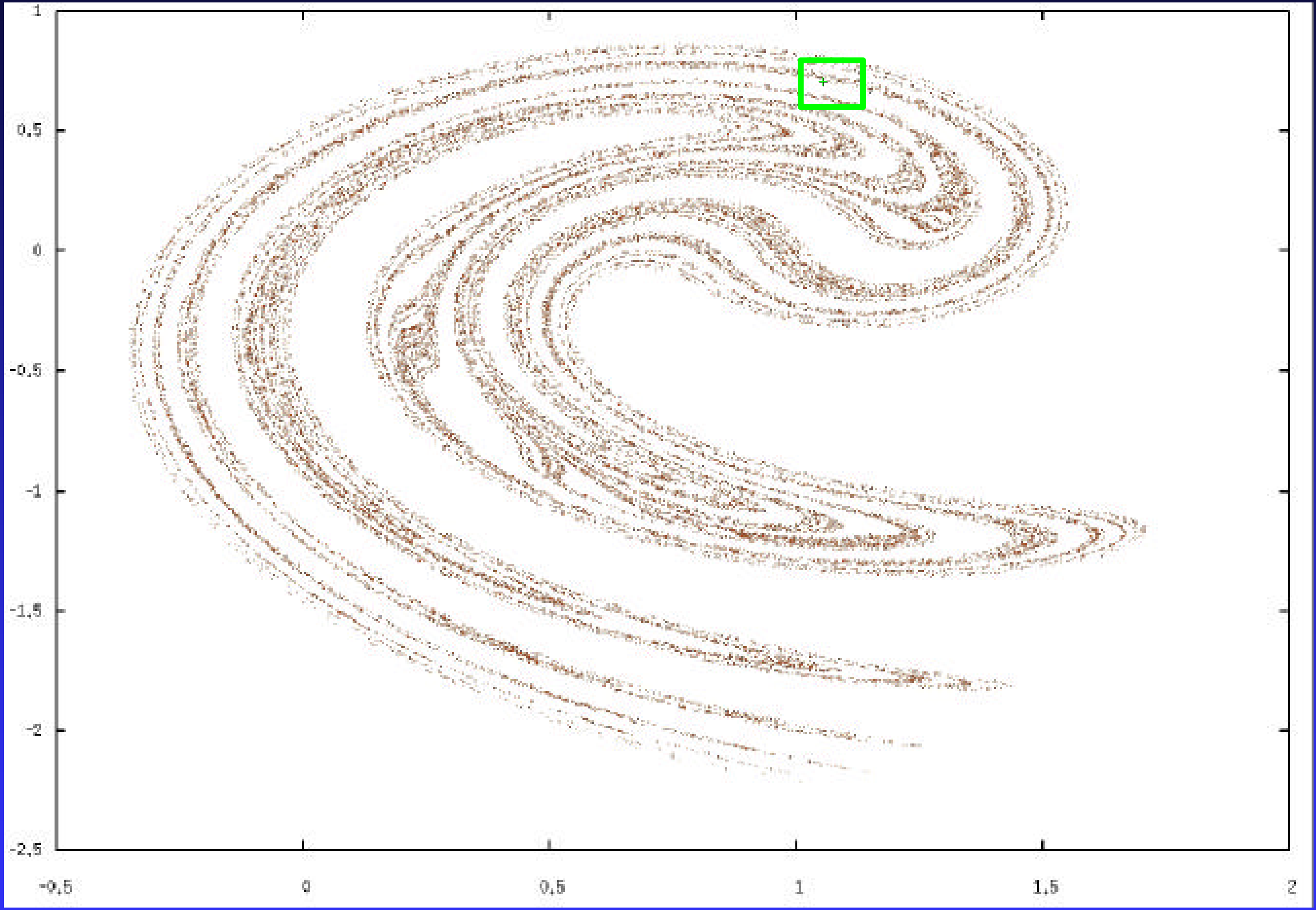
Starting with no knowledge of the current state of the system, my prior is the invariant measure:



(This picture, of course is only a sample from the prior…)

Non-Bayesian By Choice (even within PMS)

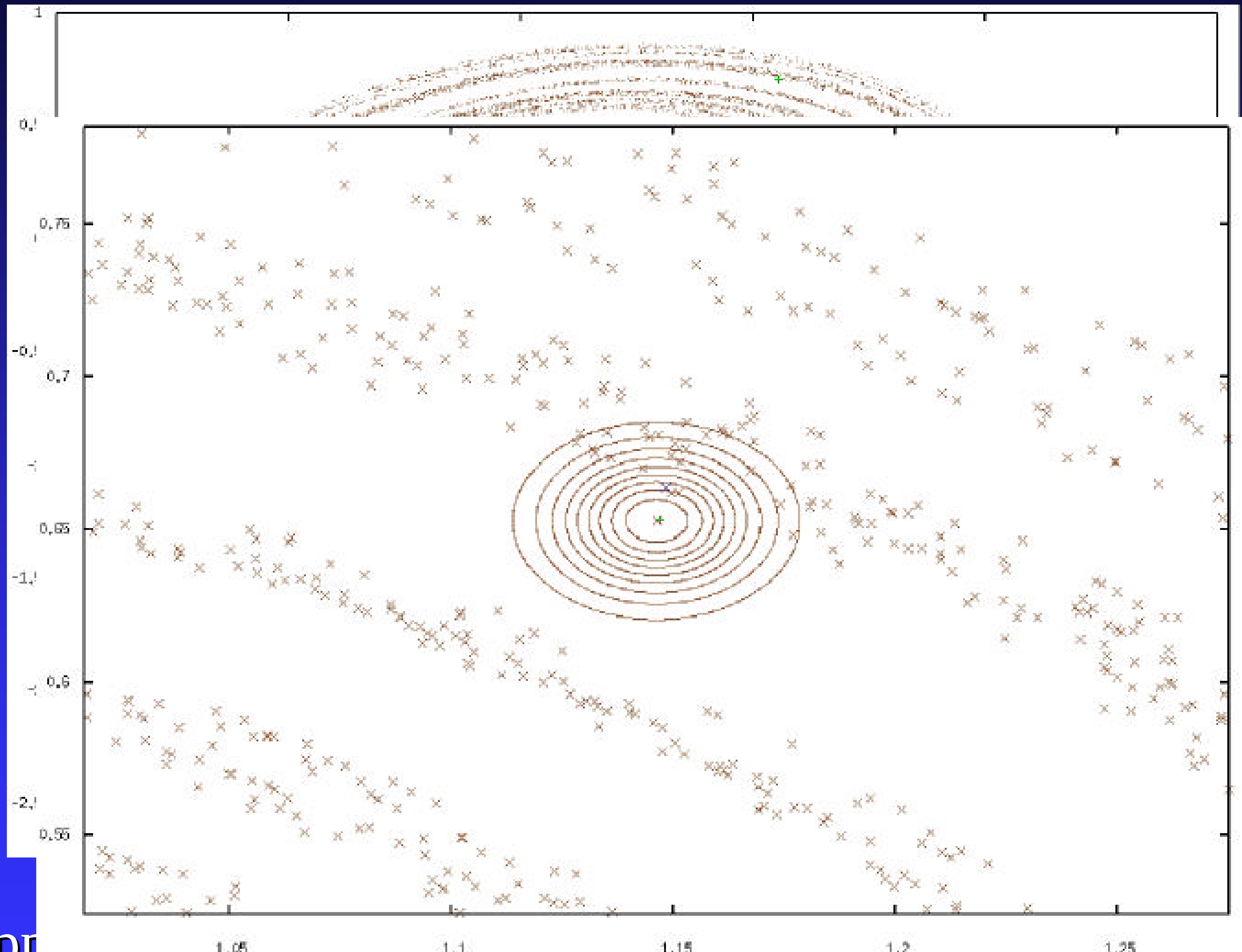
Next I get an observation, knowing my obs noise is Normal:



Applying Bayes' Theorem, I get a posterior: this is the correct answer any algorithm is ultimately judged against…

Non-Bayesian By Choice (even within PMS)

Next I get an observation, knowing my obs noise is Normal:



App… correct
answer any algorithm is ultimately judged against…

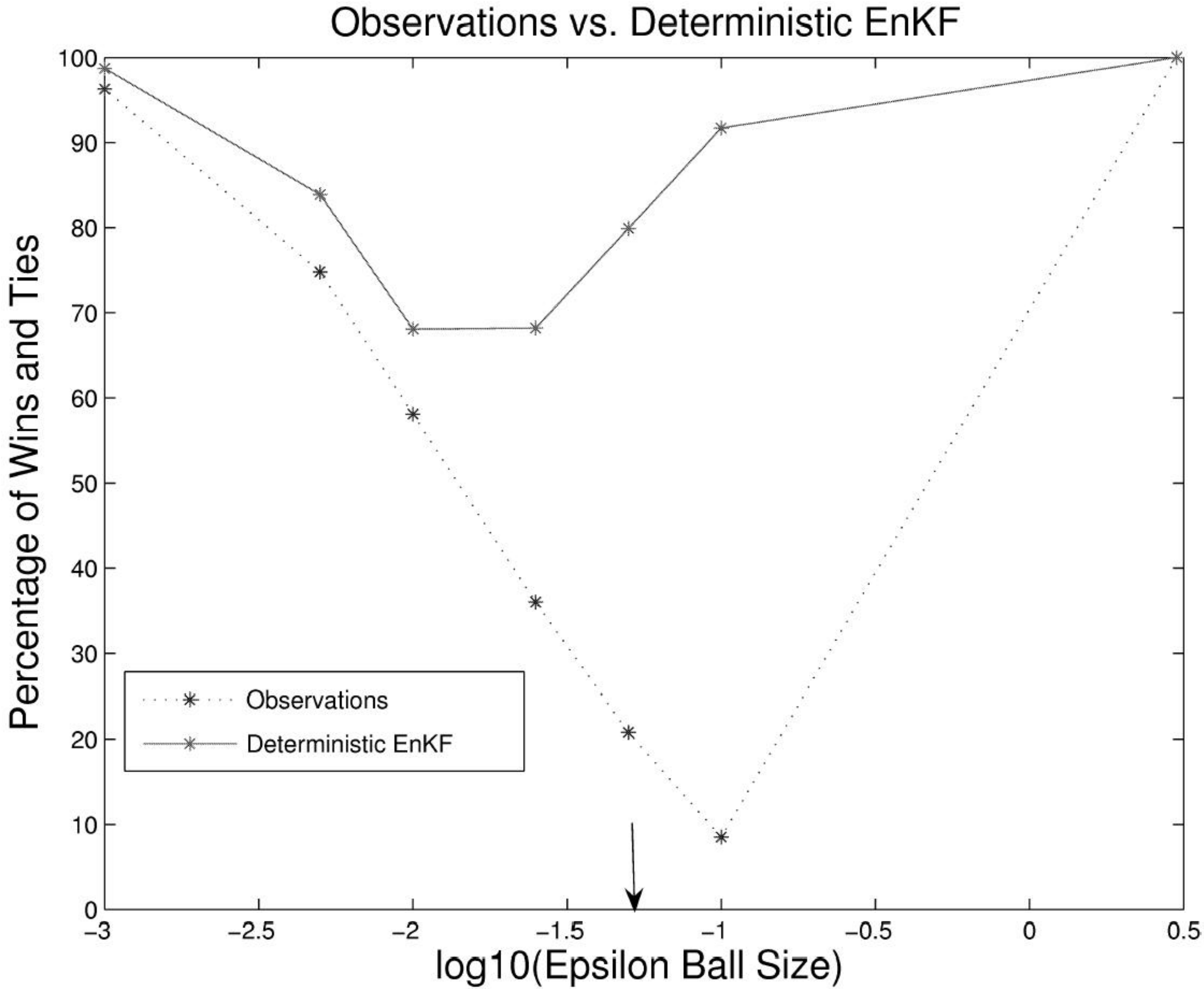
Shree's slides

I want to compare a variety of DA methods that give me a sample of points (each with weights) when estimating the location of "truth" in PMS given noisy obs.

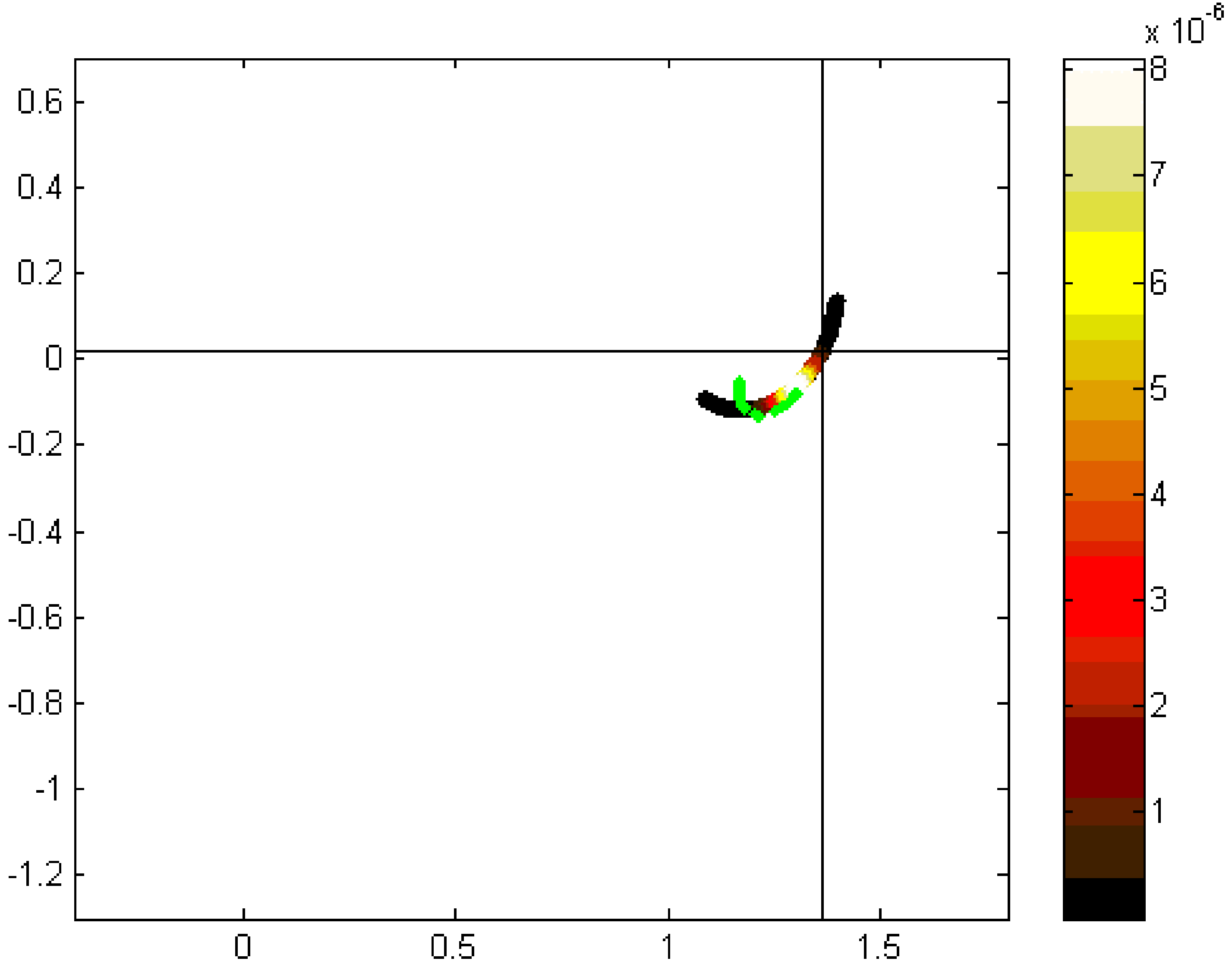
EnKF ISIS Obs Noise

To compare these (without dressing, &c) we will place a series of epsilon balls about truth, compute the total weight each method assumes to a ball of a given radius, then see which method wins: ties count for both methods.

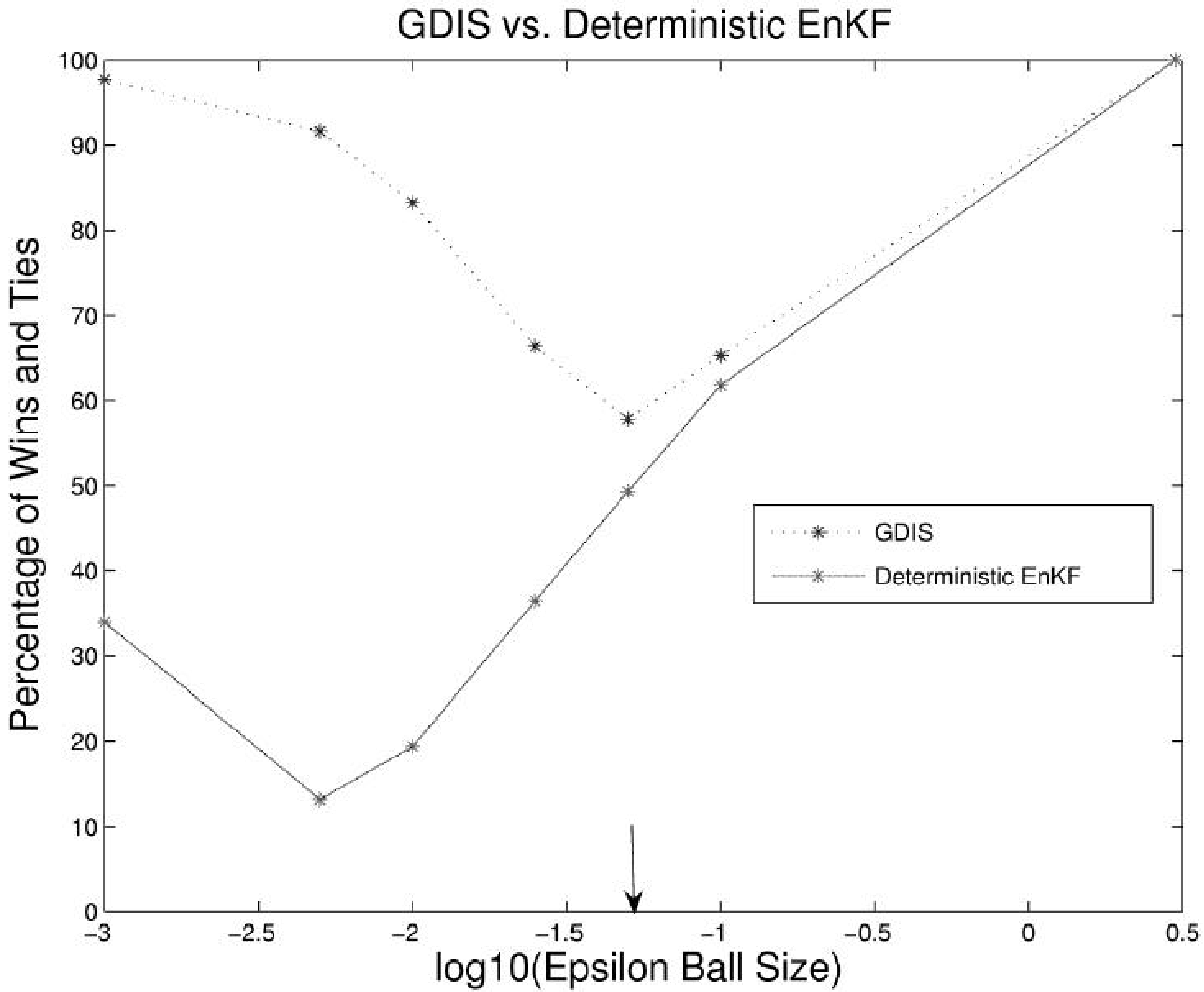
All sampling methods are somewhat *ad hoc*, I aim to level playing field in terms of computational resource; (this is done only roughly in the following graphs)

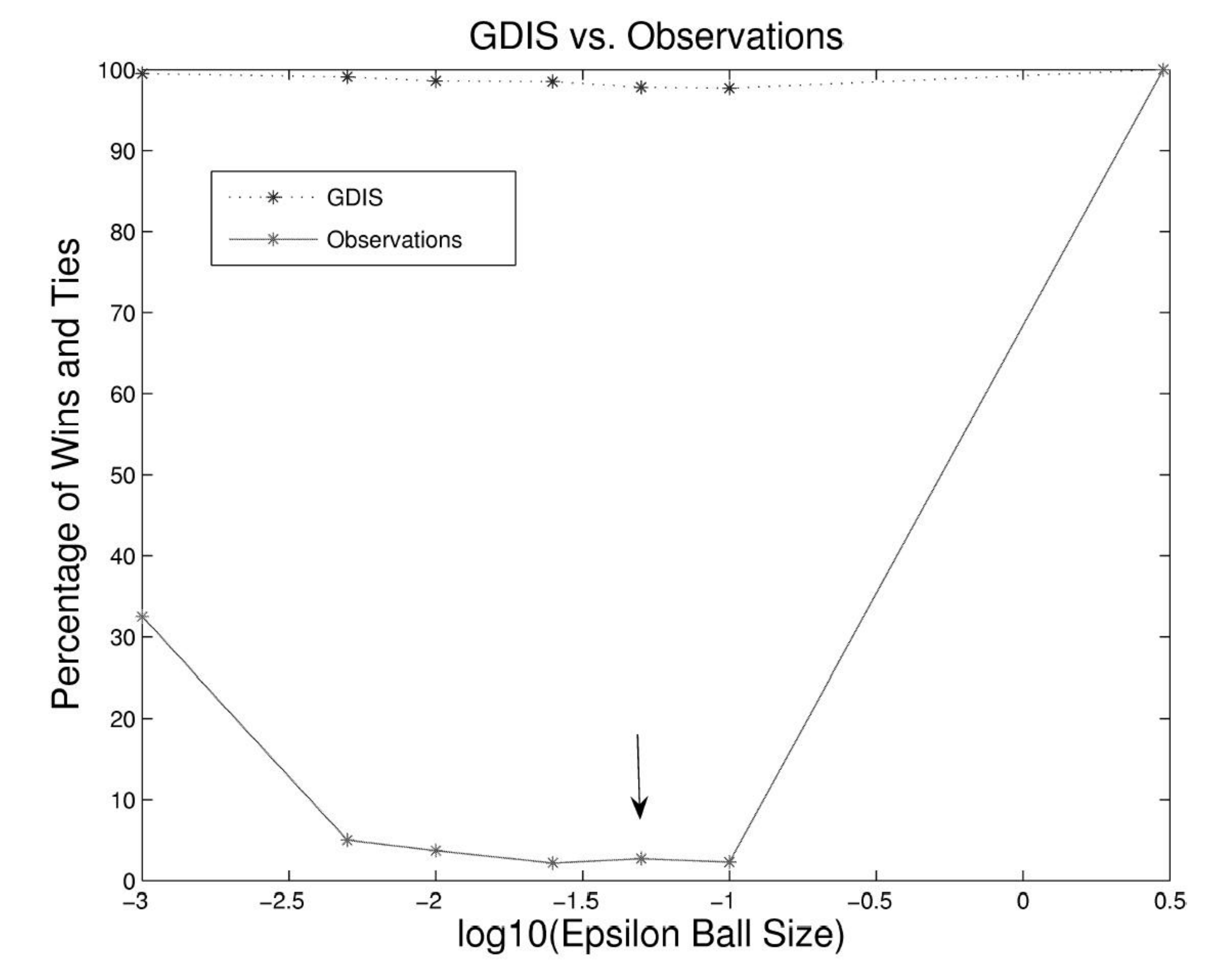


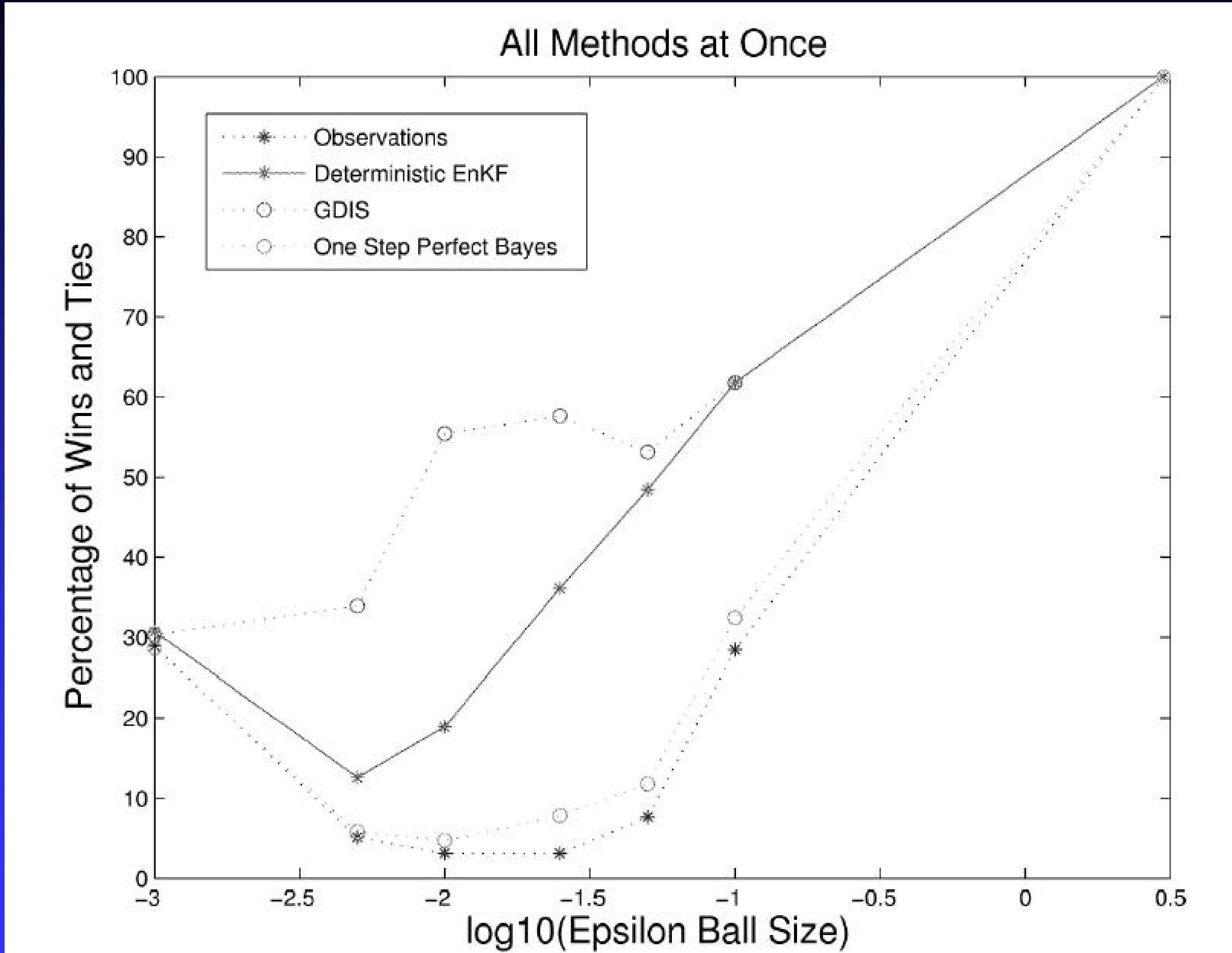
Observations vs. Deterministic EnKF
Percentage of Wins and Ties
log10(Epsilon Ball Size)
Observations
Deterministic EnKF
0
10
20
30
40
50
60
70
80
90
100
−3
−2.5
−2
−1.5
−1
−0.5
0
0.5

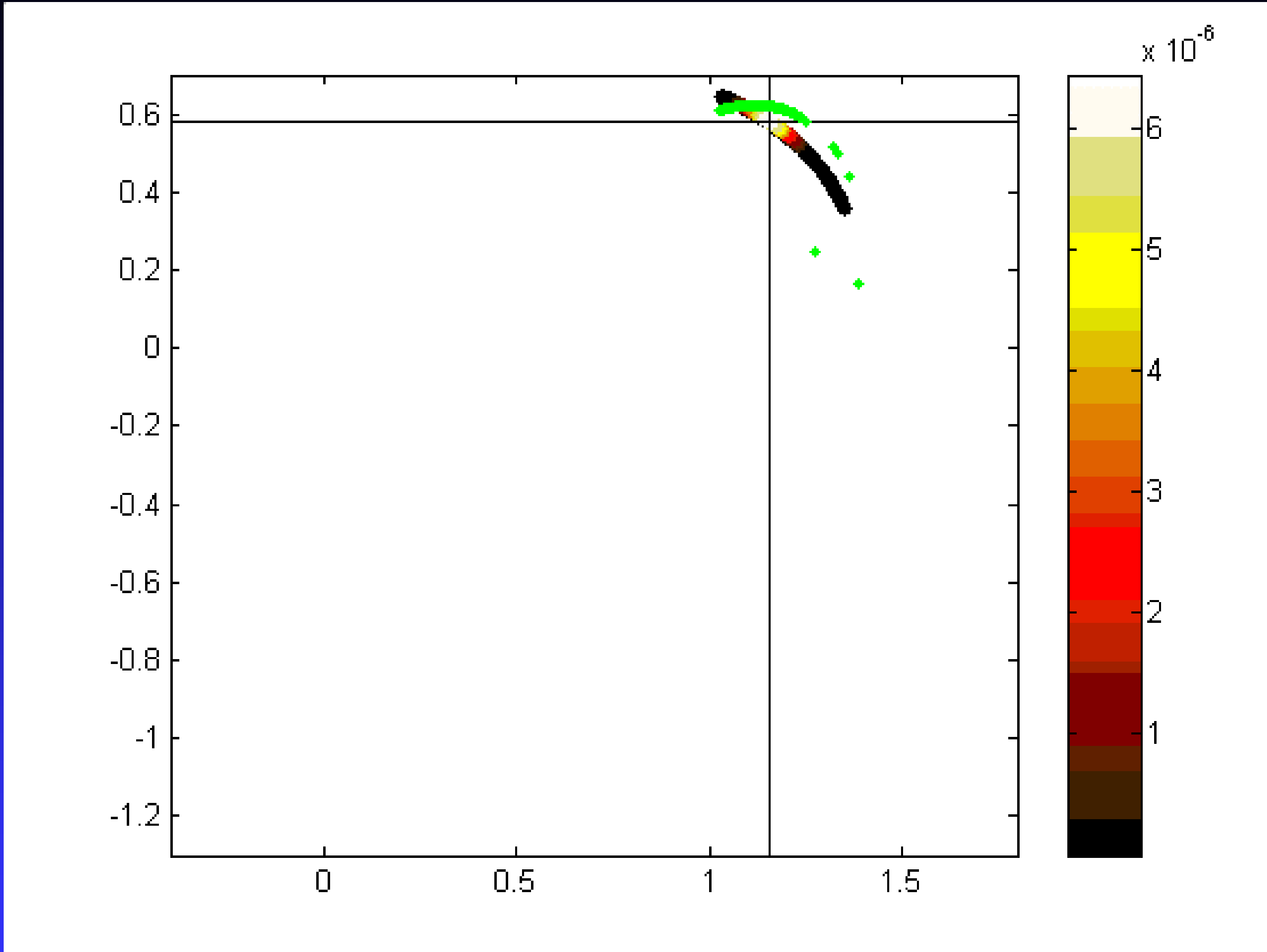


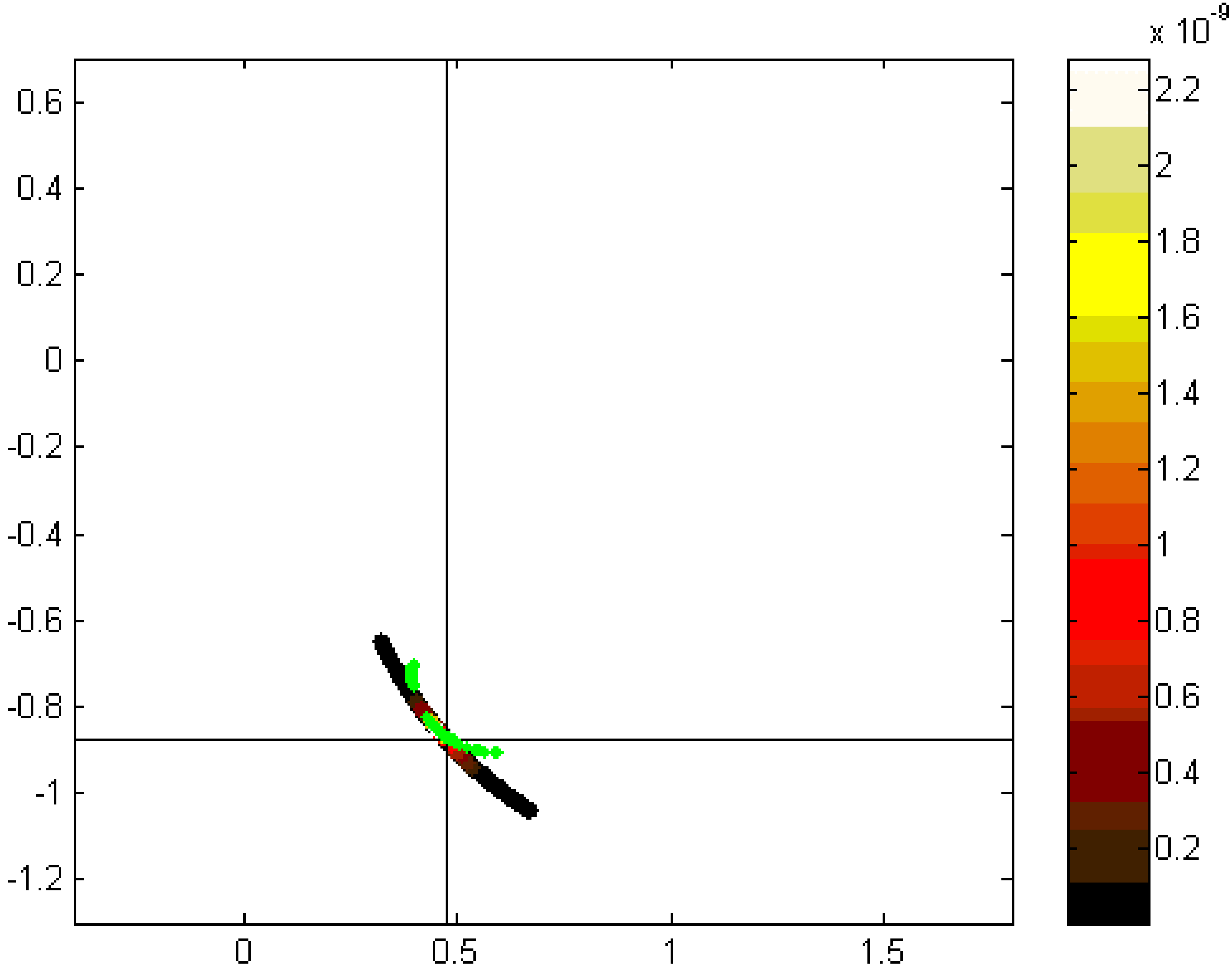
x 10^-6
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0
0.5
1
1.5
8
7
6
5
4
3
2
1


GDIS vs. Deterministic EnKF
Percentage of Wins and Ties
log10(Epsilon Ball Size)
GDIS
Deterministic EnKF
100
90
80
70
60
50
40
30
20
10
0
−3
−2.5
−2
−1.5
−1
−0.5
0
0.5


GDIS vs. Observations
100
90
80
70
60
50
40
30
20
10
0
Percentage of Wins and Ties
GDIS
Observations
−3
−2.5
−2
−1.5
−1
−0.5
0
0.5
log10(Epsilon Ball Size)


All Methods at Once
Observations
Deterministic EnKF
GDIS
One Step Perfect Bayes
Percentage of Wins and Ties
log10(Epsilon Ball Size)
0
10
20
30
40
50
60
70
80
90
100
−3
−2.5
−2
−1.5
−1
−0.5
0
0.5


x 10^-6
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0
0.5
1
1.5
6
5
4
3
2
1


0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0
0.5
1
1.5
x 10-9
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2

As models improve, coping with model inadequacy
Will become more important, not less.

(we'll have more to gain!)

And while in a sense the problem will be come more tractable
it will *never* go away (if we are looking for things like PDFs).

There is no stochastic 'fix'.

10

THE NATION'S NEWSPAPER

USA TODAY

NO. 1 IN THE USA

The debate's over: Globe *is* warming

Politicians, corporations and religious groups differ mainly on how to fix the problem

By Dan Vergano
USA TODAY

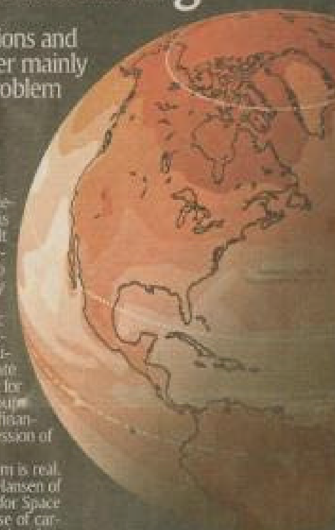
Don't look now, but the ground has shifted on global warming. After decades of debate over whether the planet is heating and, if so, whose fault it is, divergent groups are joining hands with little fanfare to deal with a problem they say people can no longer avoid.

Cover story

General Electric is the latest big corporate convert; politicians at the state and national level are looking for solutions; and religious groups are taking philosophical and financial stands to slow the progression of climate change.

They agree that the problem is real. A recent study led by James Hansen of the NASA Goddard Institute for Space Studies confirms that, because of carbon dioxide emissions and other greenhouse gases, Earth is trapping more energy from the sun than it is re-

Please see COVER STORY next page ►

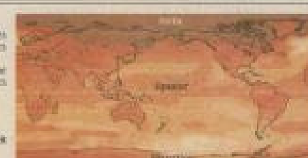


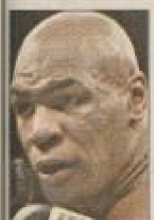
A warming world

This simulation compares air temperatures near Earth's surface during the last 20 years of the 20th century with projections of temperatures during the last 20 years of the 21st century. The greatest warming occurs in the Arctic and Antarctica.

Projected temperature increase from 2000 to 2100

0 1.8 3.6 5.4 7.2 Fahrenheit





Tyson: 'I'm just happy it's over'

Mike Tyson reflective after weekend defeat by Kevin McBride: 'I just don't have it in me anymore'
■ 8C

Whys is the Speed of Light like Climate Sensitivity?

Olaf Roemer (Philosophical Transactions; June 25, 1677) reported a finite speed of light. Generally disbelieved until Bradley gave independent confirmation of the finite speed of light in January 1729.

In ~1953 the speed of light was revised by several standard errors, almost certainly due to "anchoring" (the search and "correction" of systematic experimental errors until the result agrees with previously accepted values, and no further).

If anchoring has this effect on the empirical measurement of *c*, then can we really ignore it in estimates of "climate sensitivity"?

(This is nothing to be embarrassed about)

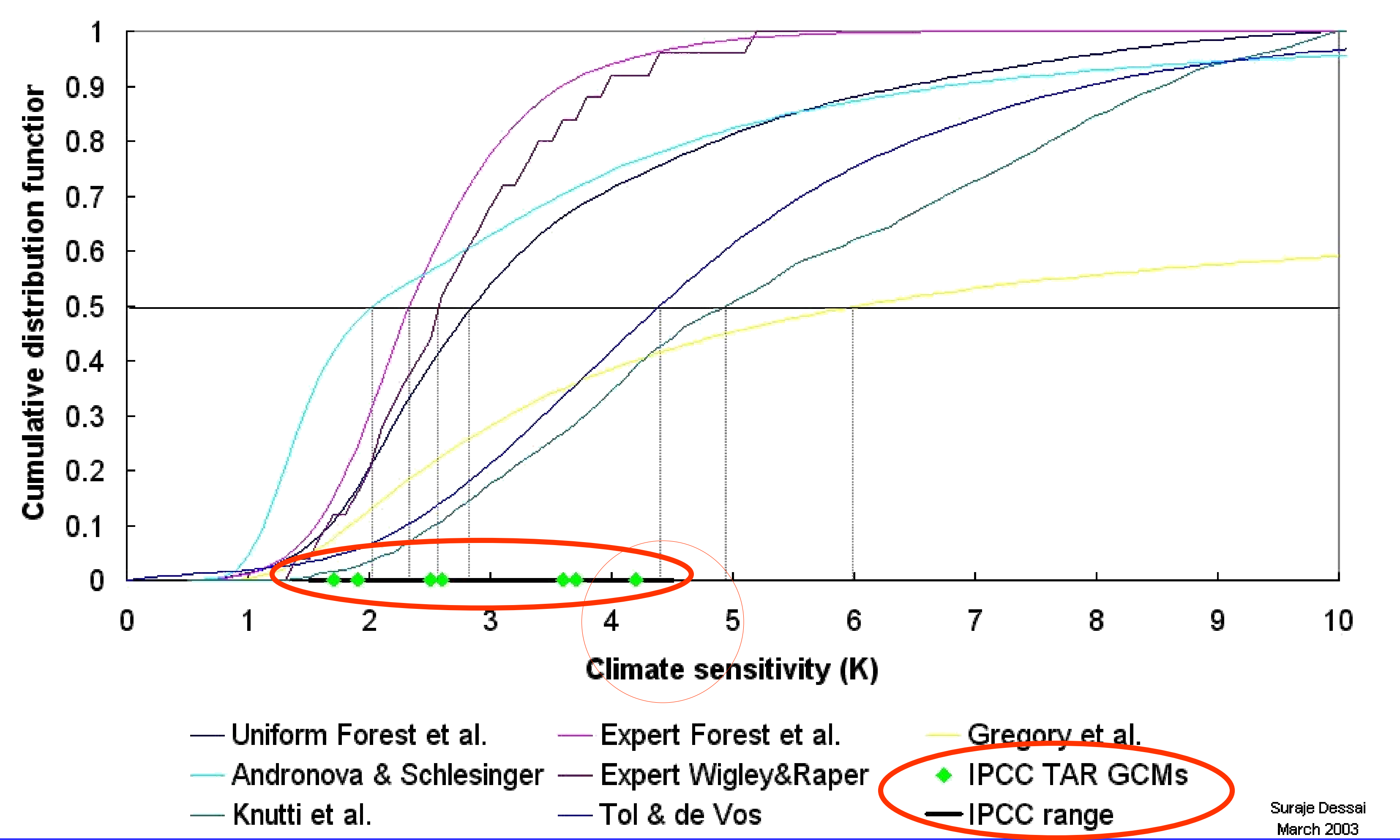
S
A

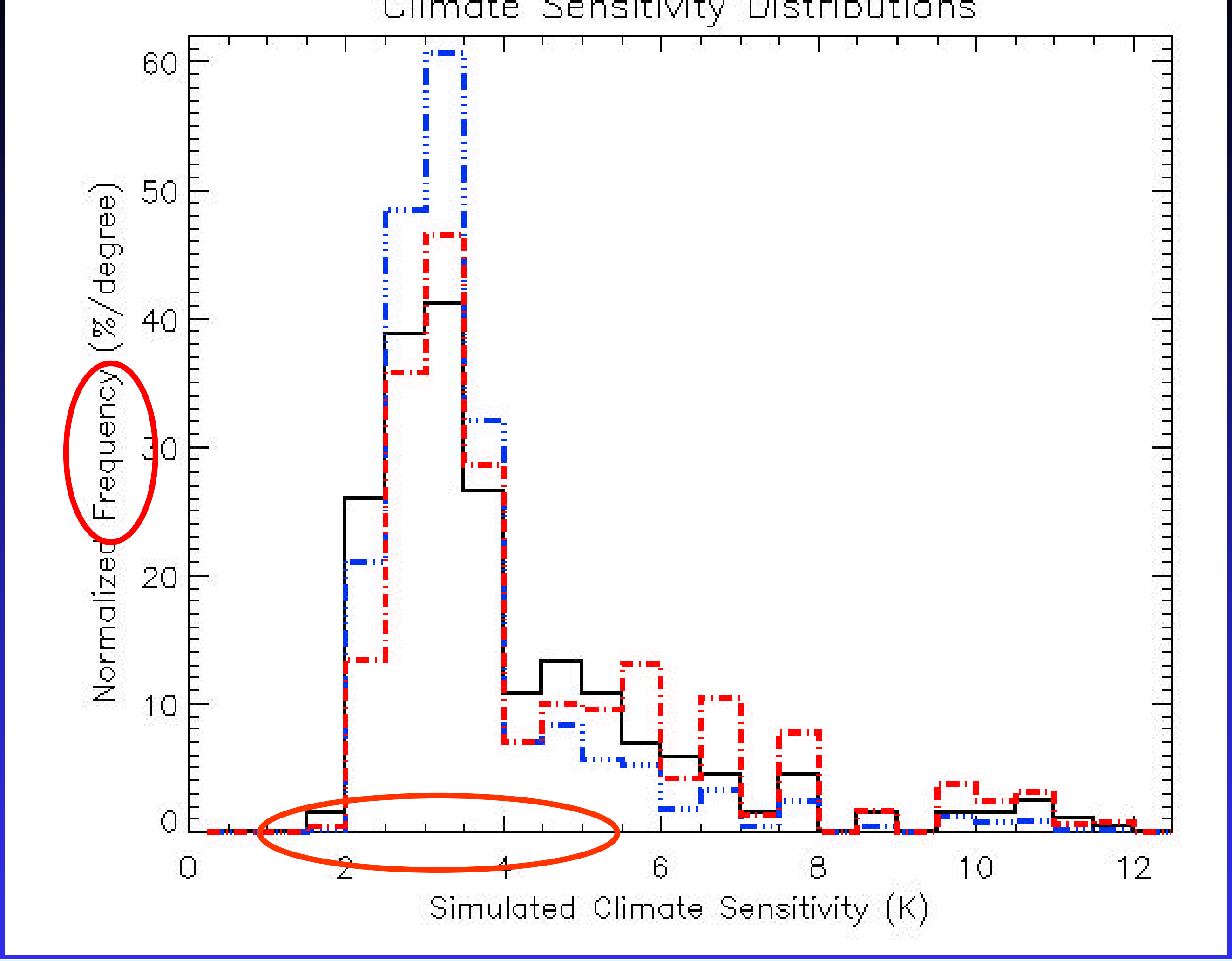
Climate Modelling is in-sample by definition.

This is nothing to be ashamed of (but should not be ignored).

Uncertainty in Climate Sensitivity

Climate sensitivity is defined as the equilibrium global mean surface temperature change for a doubling of CO_2 levels.





Black: As sampled in cp.n
Blue: Entrainment coef constant
Red: Cloud-to-rain conversion threshold constant
Note that sampling uniform in *a* differs from uniform in *1/a*.

Climate is defined as a distribution of weather states; we must sample initial states in order to describe this distribution and to obtain statistically meaningful results on instabilities.

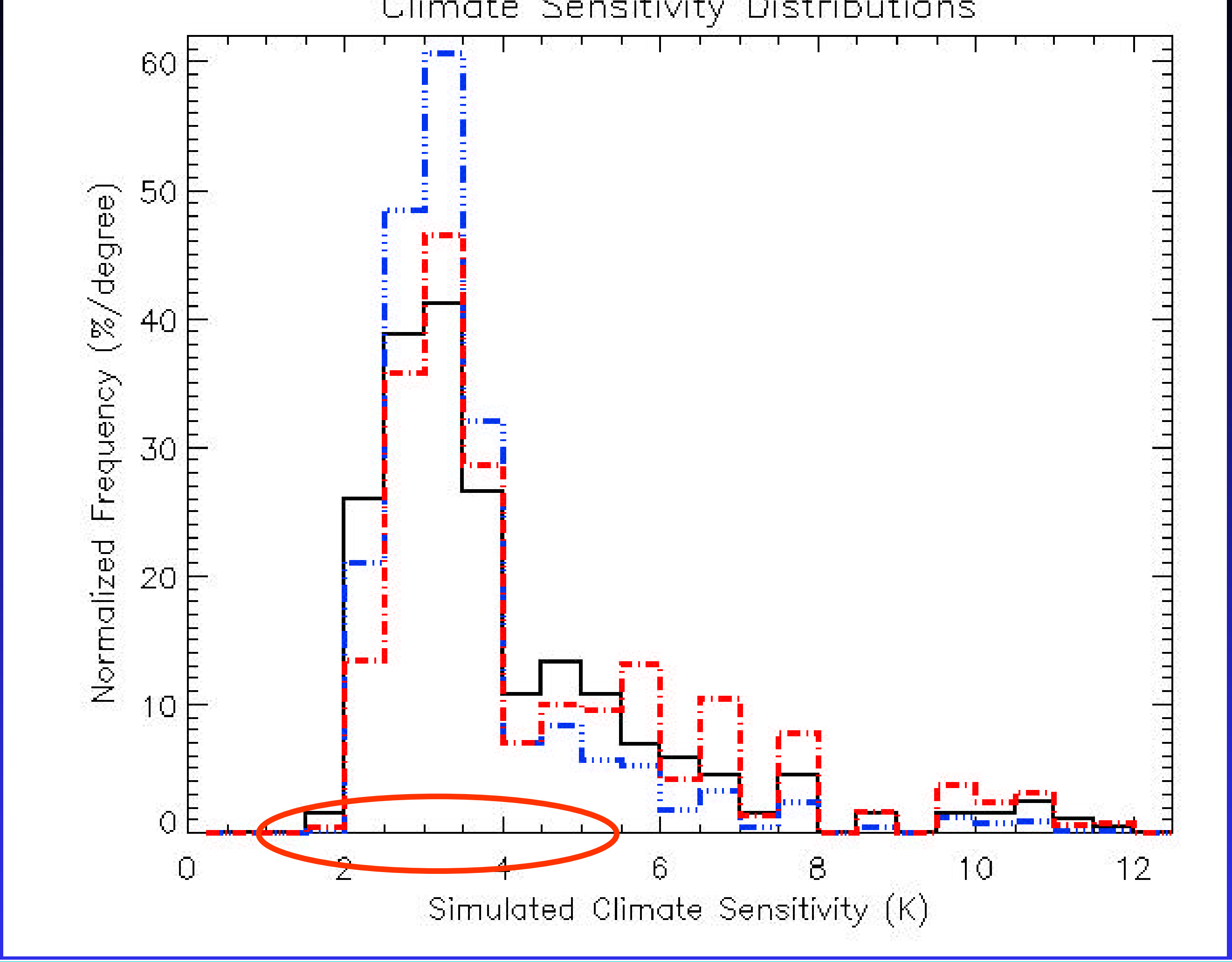
Model dynamics reduces the impact of the particular initial conditions

To sample parameter values, however, the input distribution determines the output distribution:

Are all these parameter (and heat flux) values realistic?

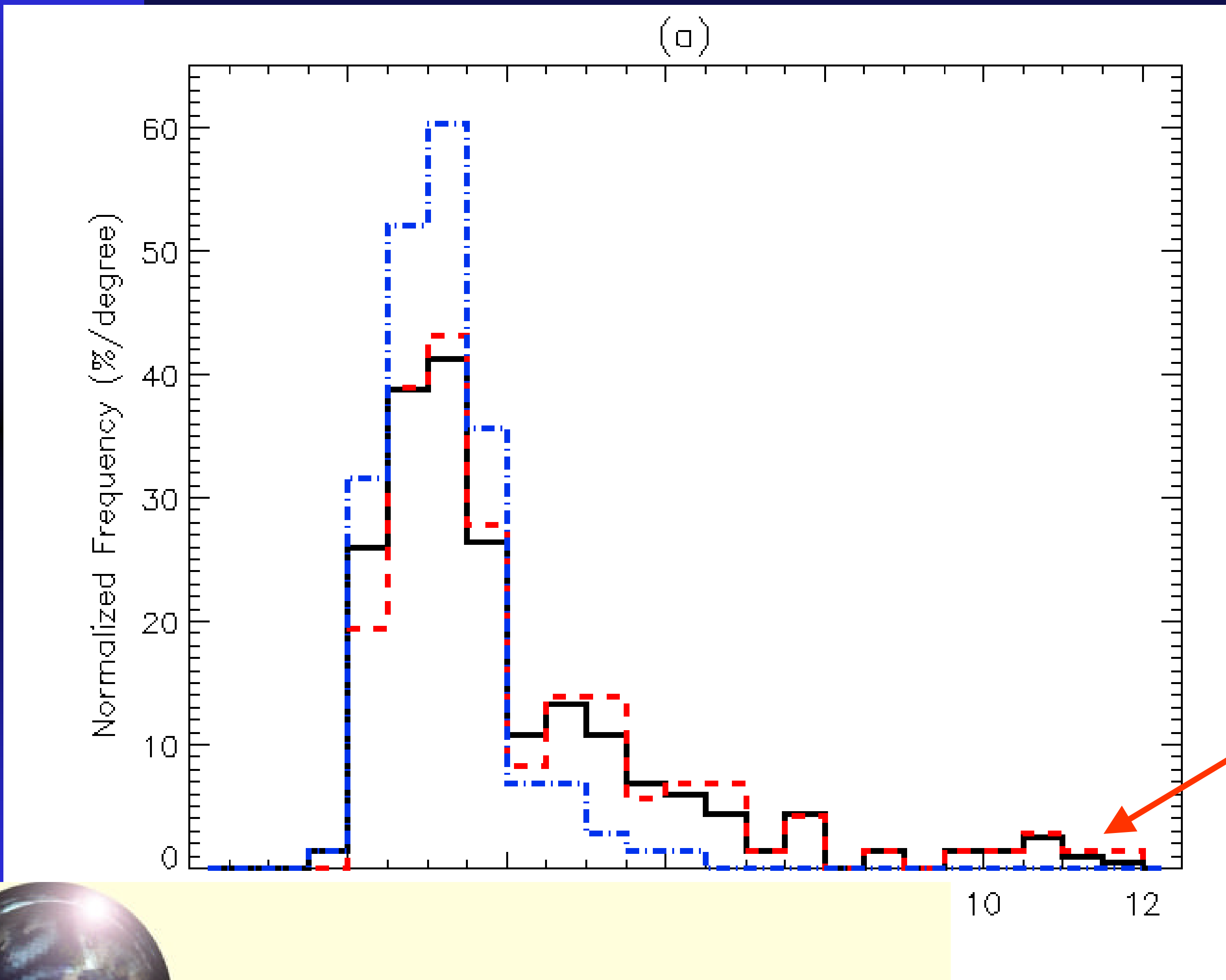
Do they yield “state-of-the-art” climates?

Details of the input distribution determine general shape of the sensitivity distribution!



Black: As sampled in cp.n
Blue: Entrainment coef constant
Red: Cloud-to-rain conversion threshold constant
Note that sampling uniform in *a* differs from uniform in *1/a*.

Data Assimilation and the Press



% > 8°

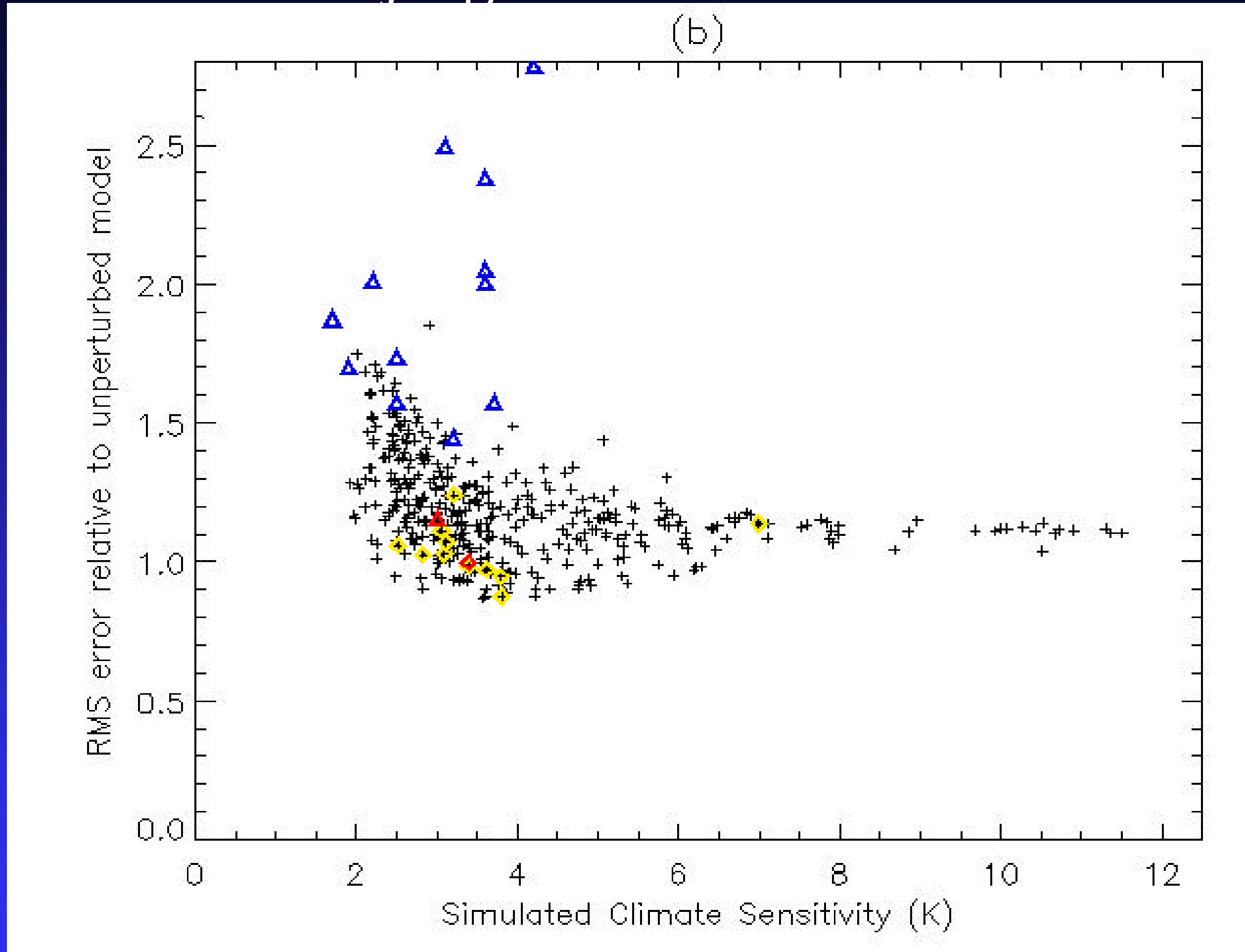
Black:	4.2
Red:	4.9
Blue:	0.0

Data Assimilati
and the Press

climateprediction.net

Is the climate*prediction*.net ensemble "state-of-the-art"?

How should we best judge if a climate model is realistic?



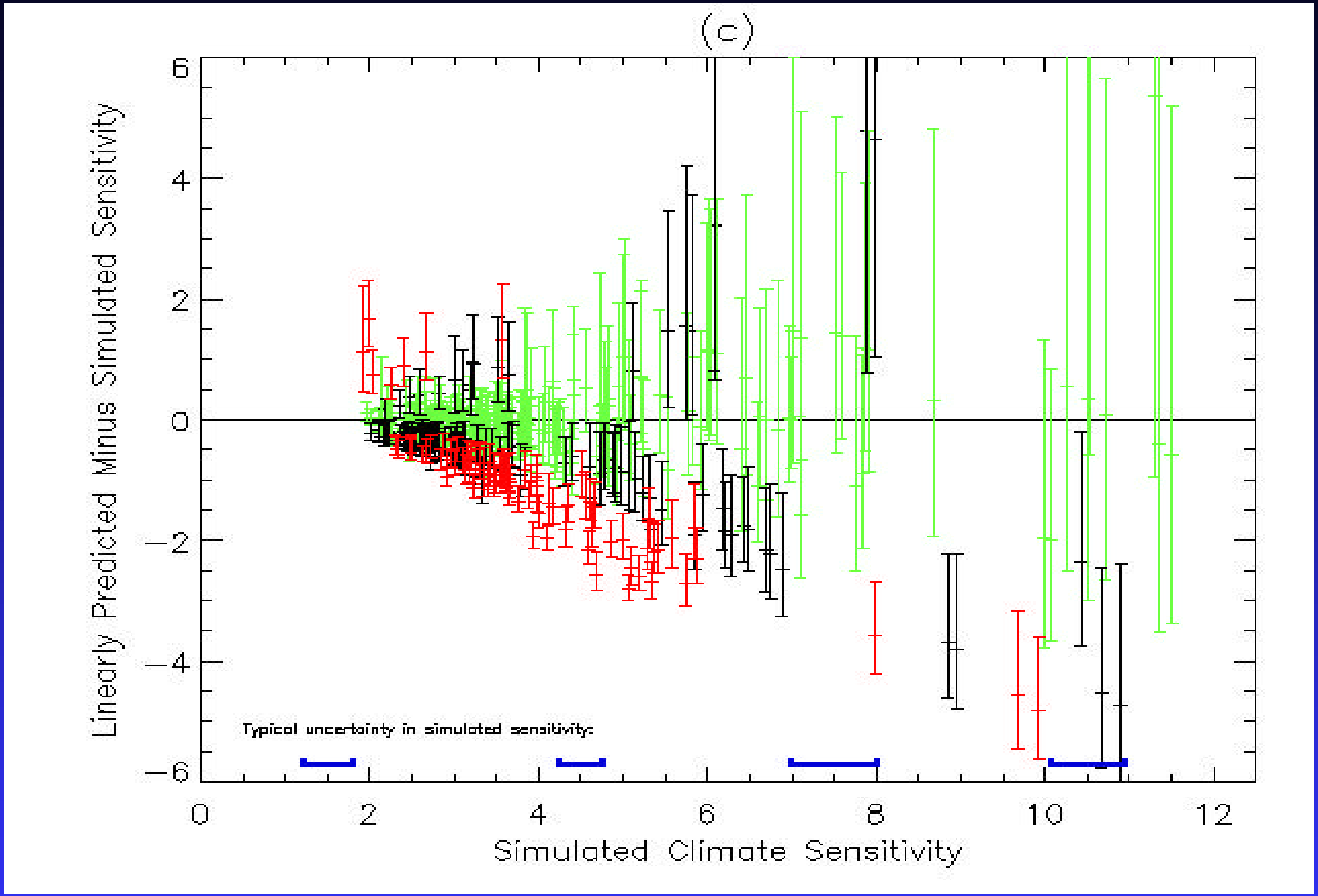
Relative RMS Error relative to unperturbed model.

Yellow Diamond: Single Parameter Perturbation

Black Plus: Multiple Parameter Perturbation

Blue Triangle: CMIP II Model

Red Triangle: HadCM3 (same atmosphere with dynamic ocean



Direct test of linear prediction of climate sensitivity of (Murphy et al 2004); color indicates the error (in standard deviations) Green is less than one, Black up to two, Red greater than two.

The approximation is not reliable.

10’

So where are we?

Aim for Deployable Probabilistic Forecasts with:

Accountability, Resolution, and Relevance

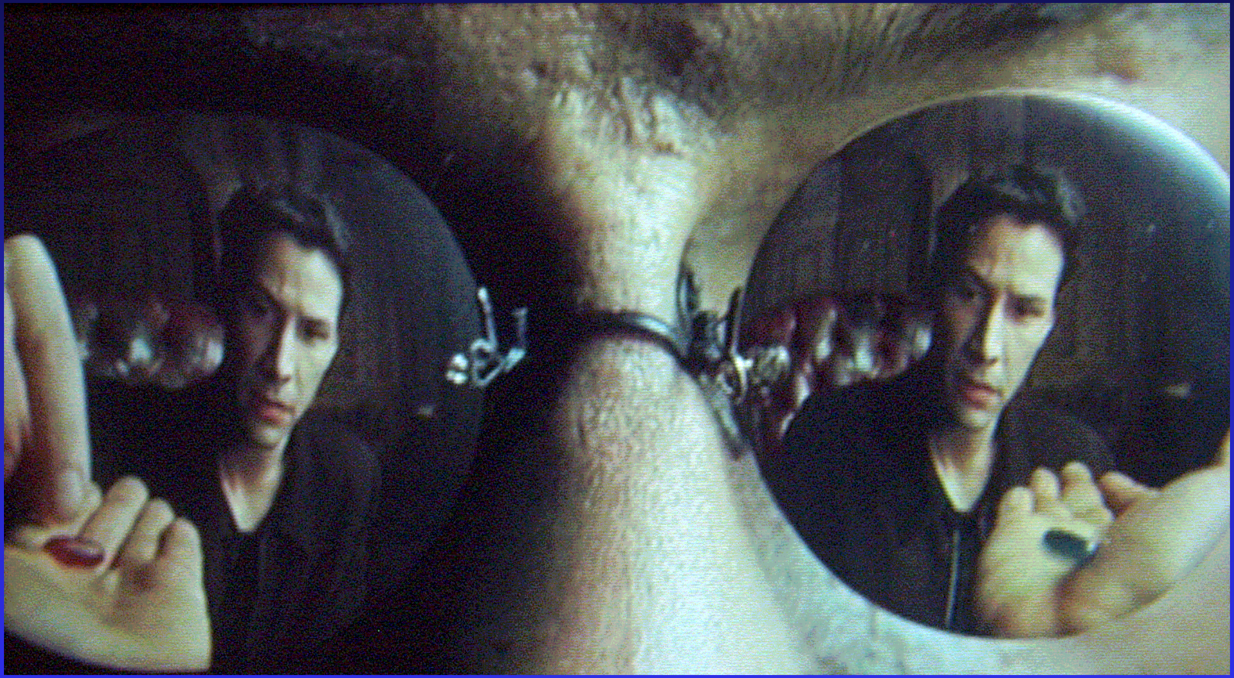
Informative

Assigns non-trivial probabilities (to what happened, no
what is the chance x happene

Suffers only from sampling finite N effects

Towards *better* ignoring *best*

So now you have to make a choice:
You take the blue pill and the lecture ends, you wake-up in your bed and happily do mathematics…



You take the red pill, and try to do physics knowing all models are wrong.

“Remember that all I am offering is the truth. Nothing more”

Morpheus

Nancy Cartwright (1983) *How the Laws of Physics Lie,* OUP

LA Smith (2003) Predictability Past Predictability Present. ECMWF. soon to be in a CUP book (ed. Palmer).

K Judd and LA Smith (2001) *Indistinguishable States I* , Physica D **151**: 125-151 & *(2004) Indistinguishable States II,* **196:** 224-242 .

LA Smith (2000) *Disentangling Uncertainty and Error*, in Nonlinear Dynamic and Statistics (ed A.Mees) Birkhauser.

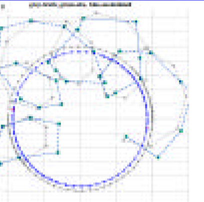
M. Altalo and LA Smith (2004) Environmental Finance **6** (1) 48-49.

Stainforth et al (2005) Uncertainty in predictions of Climate Response Nature **443**,403-406

climateprediction.n

LA Smith (2002) *What might we learn from climate forecasts?*, Proc. National Acad. Sci. **99**: 2487-2492

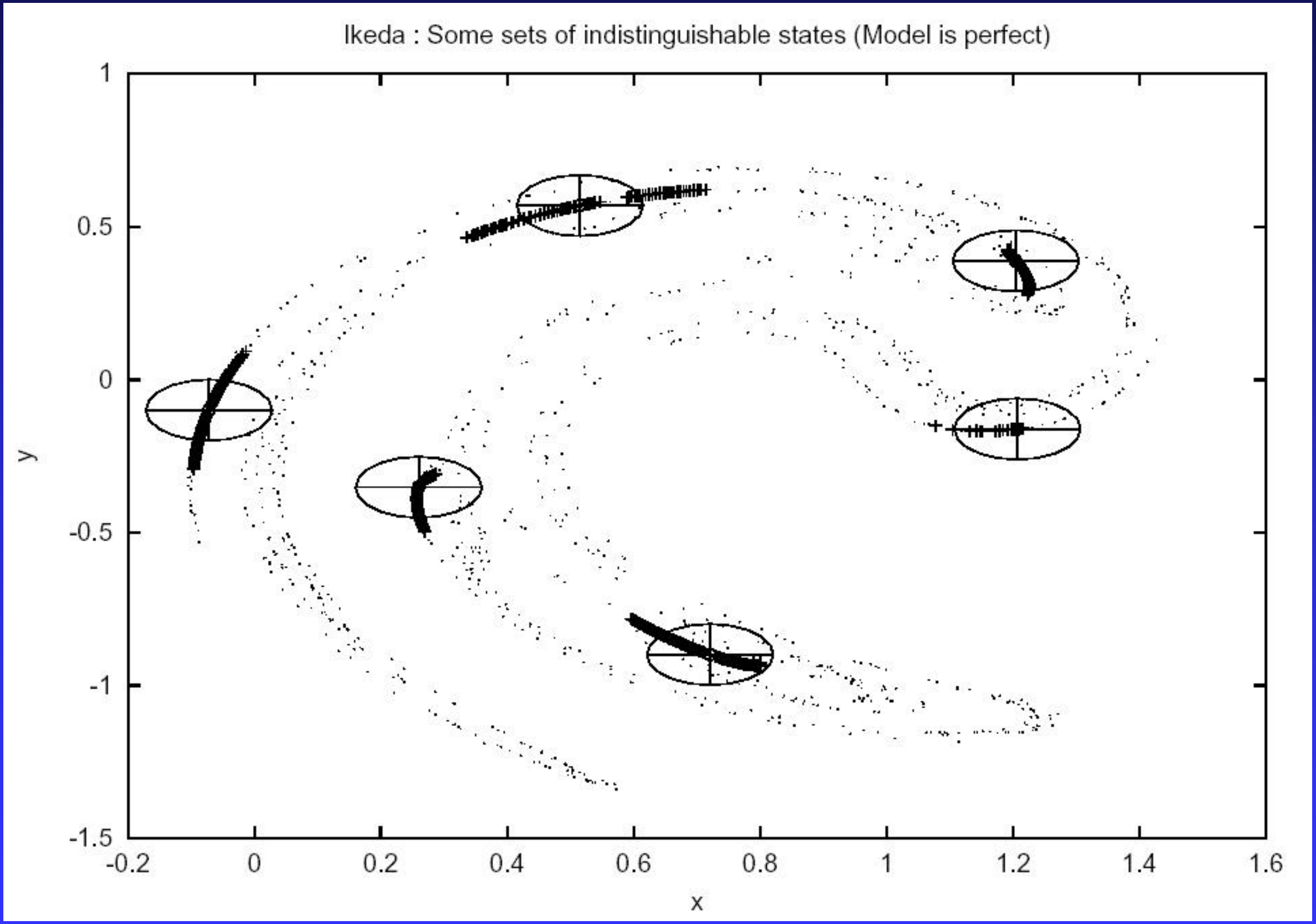
D Orrell, LA Smith, J Barkmeijer and TN Palmer (2001) Model error in weather forecasting. Nonlin Processes in Geophysics **8**:357-371.

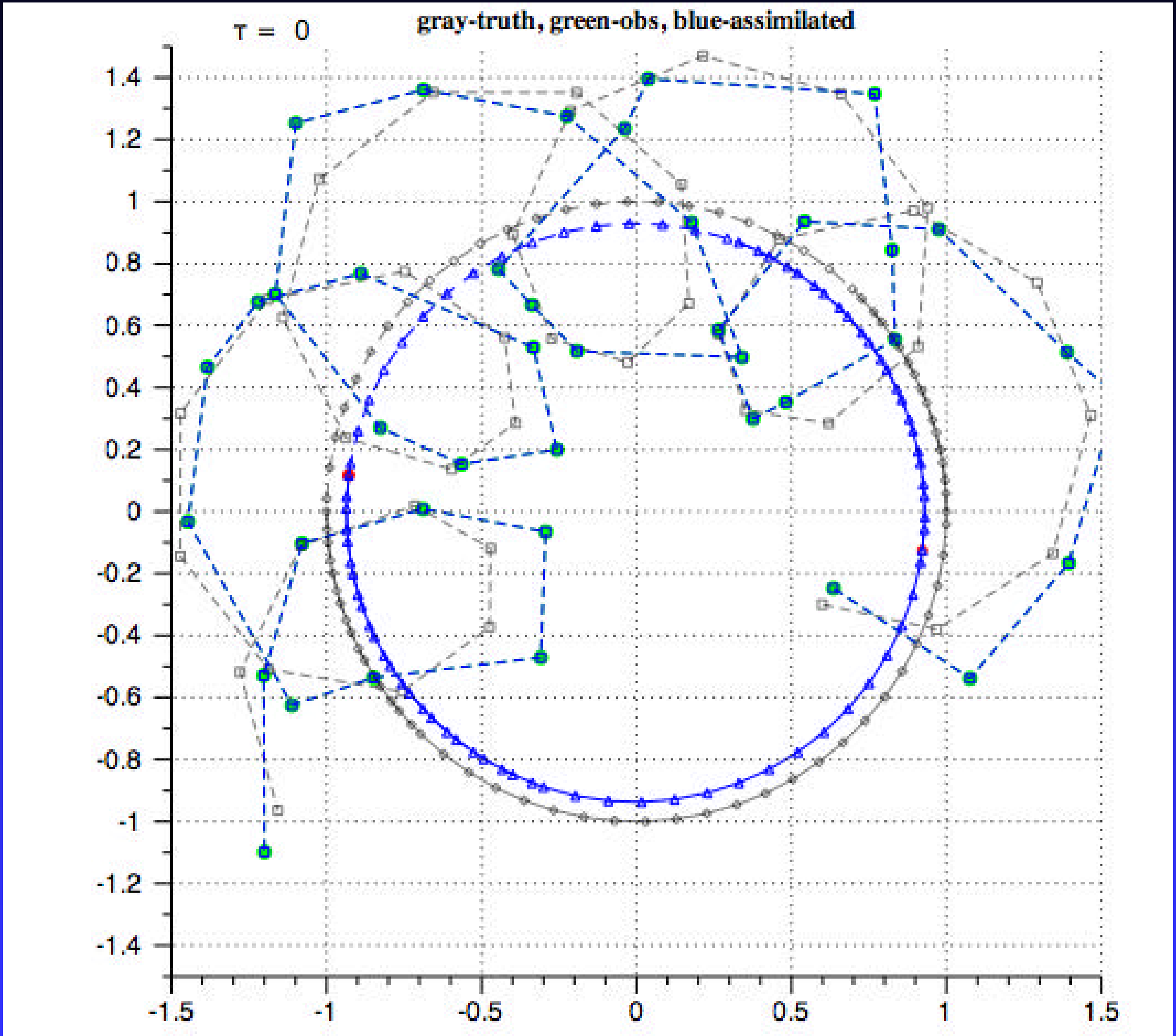


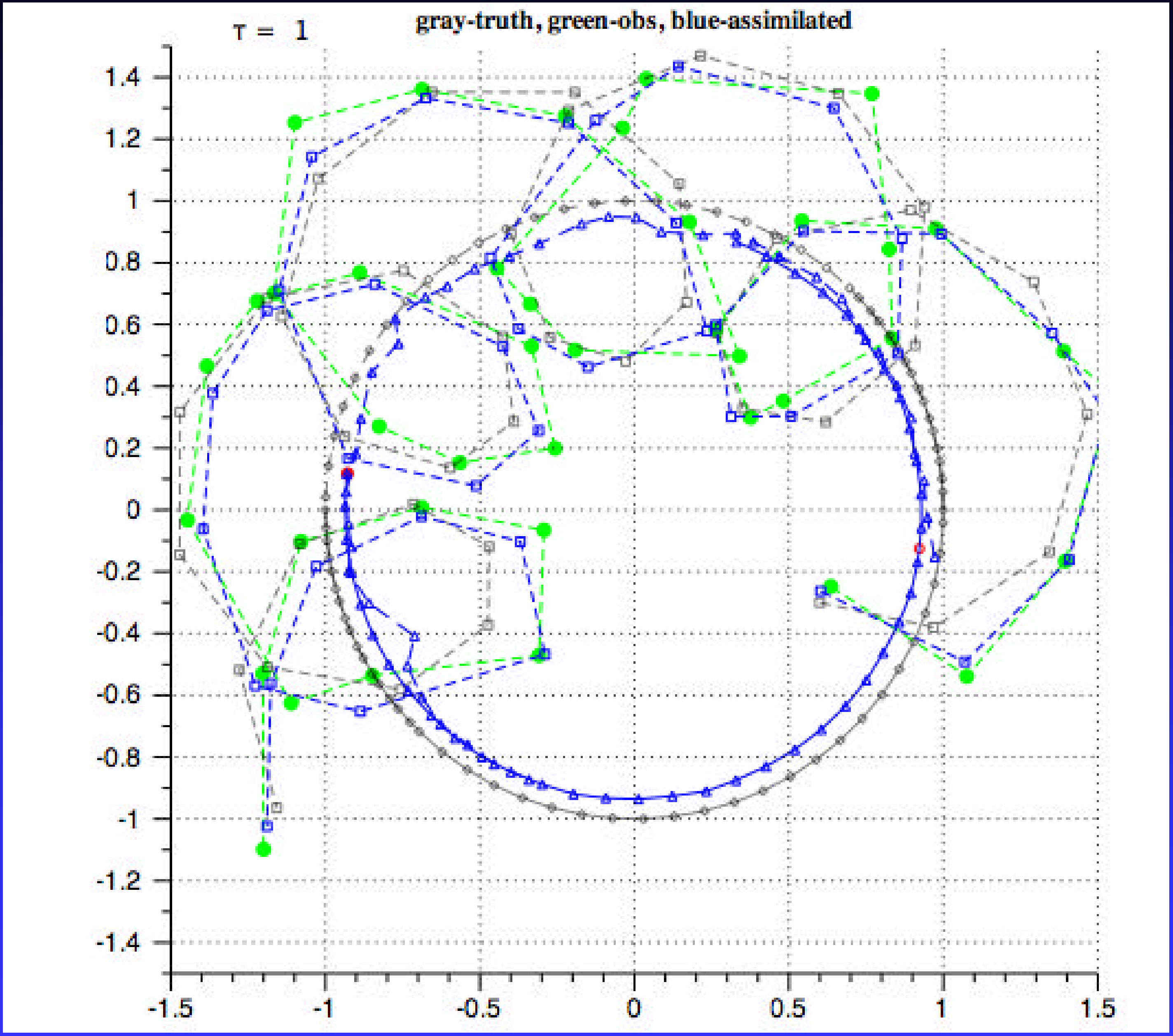
www.lsecats.org lenny@maths.ox.ac.uk

Will being Bayesian buy me better probability forecasts?

Is there an alternative approach, which uses the same resource to find a much higher resolved estimate of (an inferior) PDF?




τ = 0
gray-truth, green-obs, blue-assimilated
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5


τ = 1
gray-truth, green-obs, blue-assimilated
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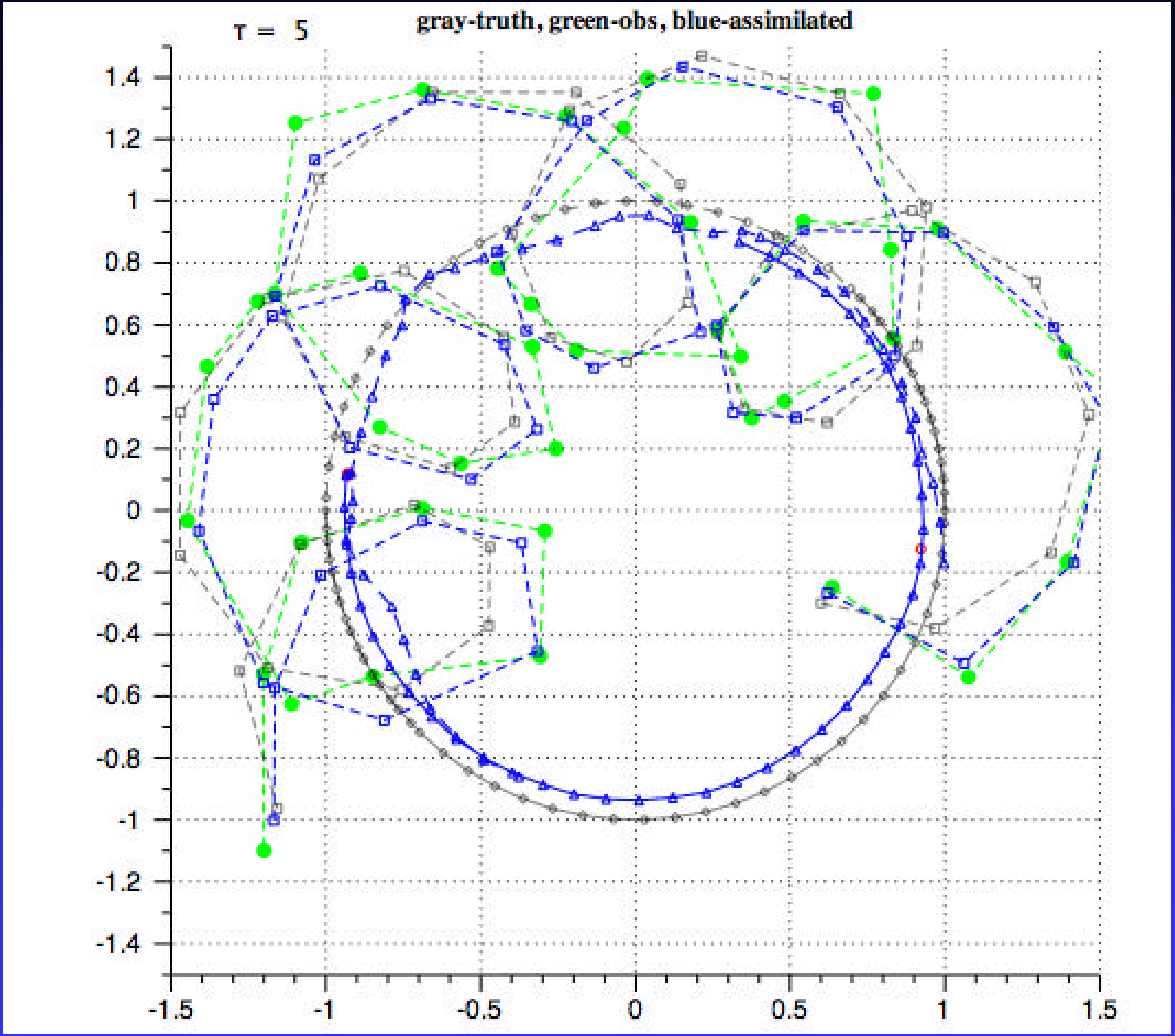
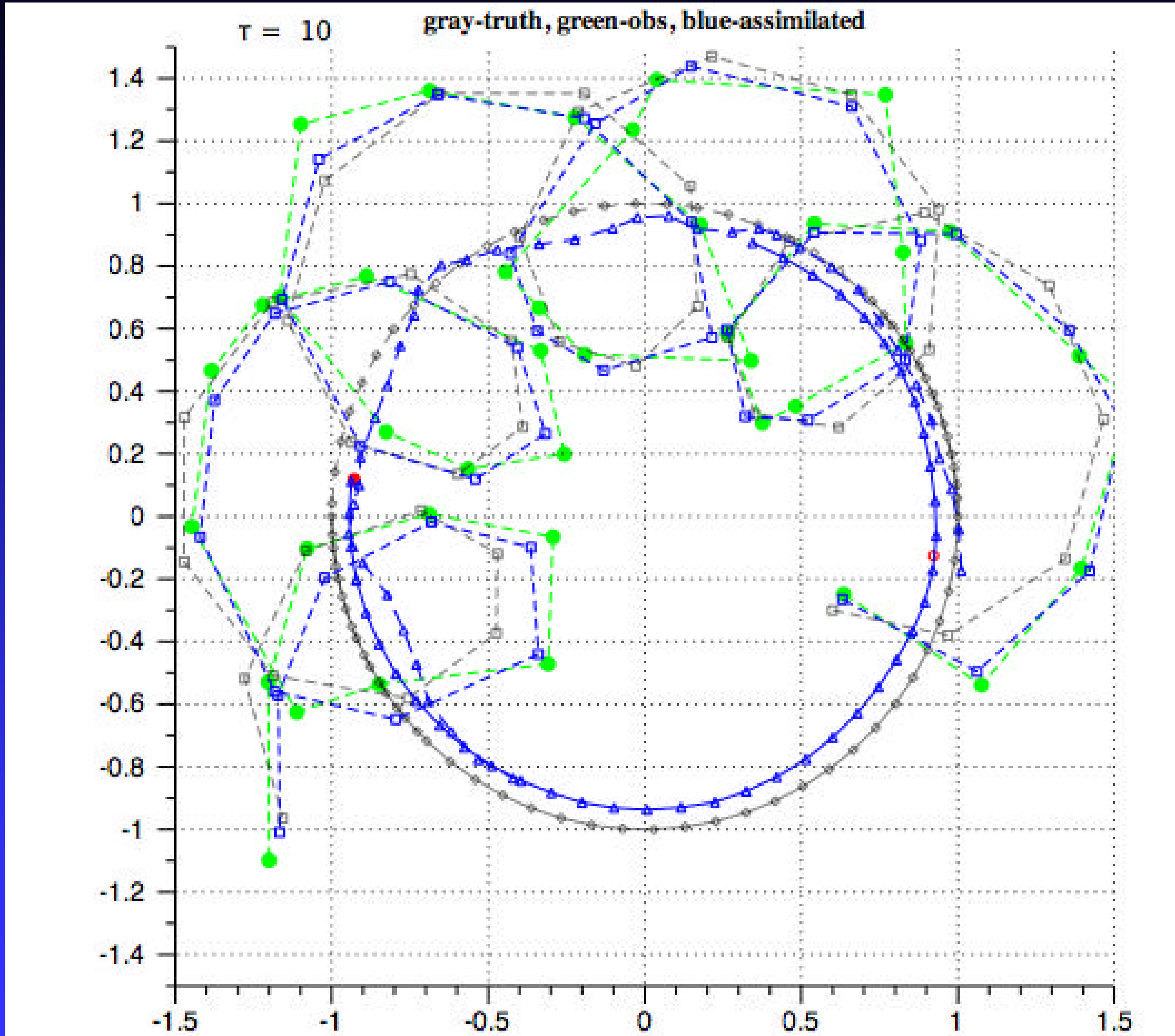
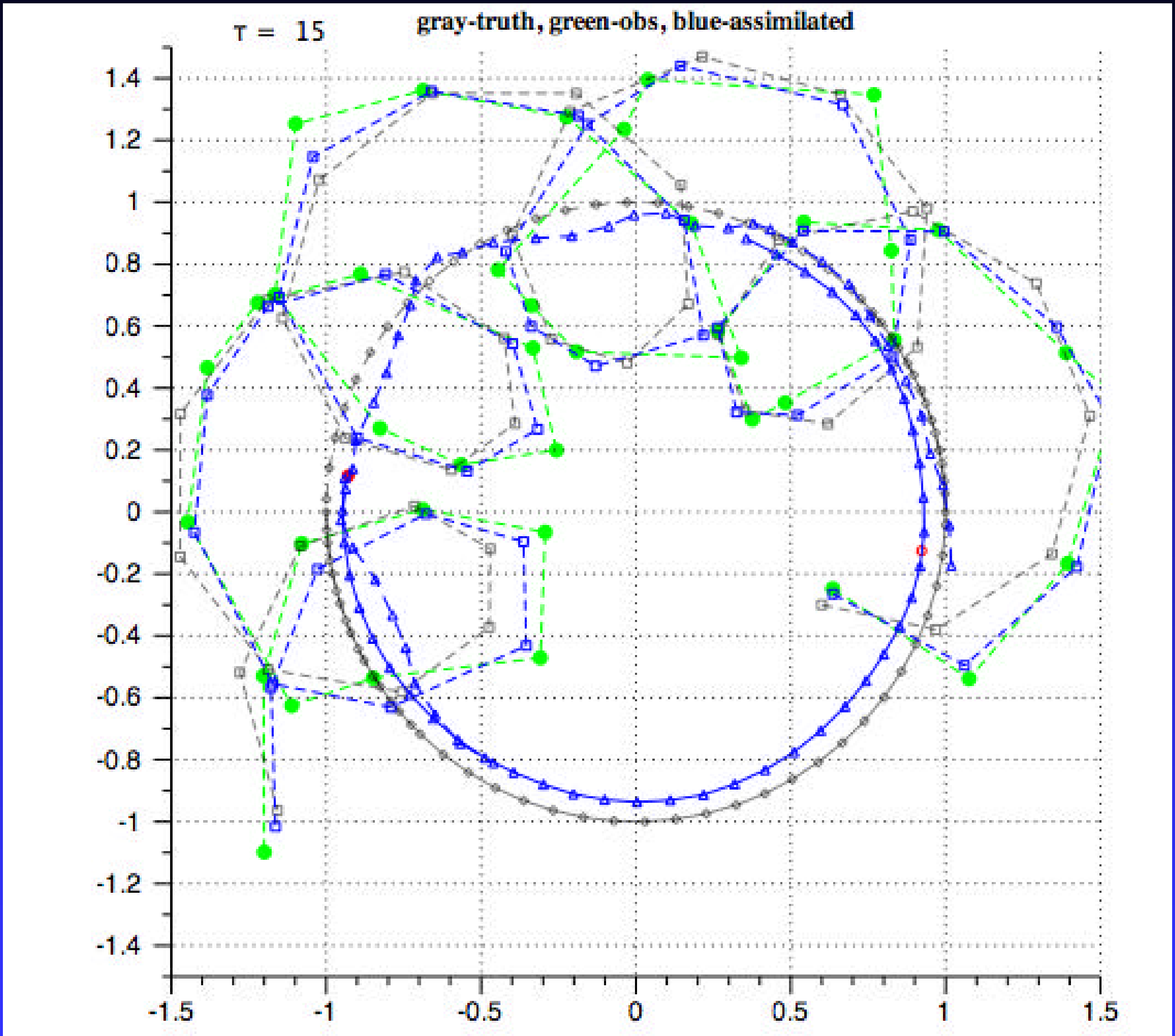
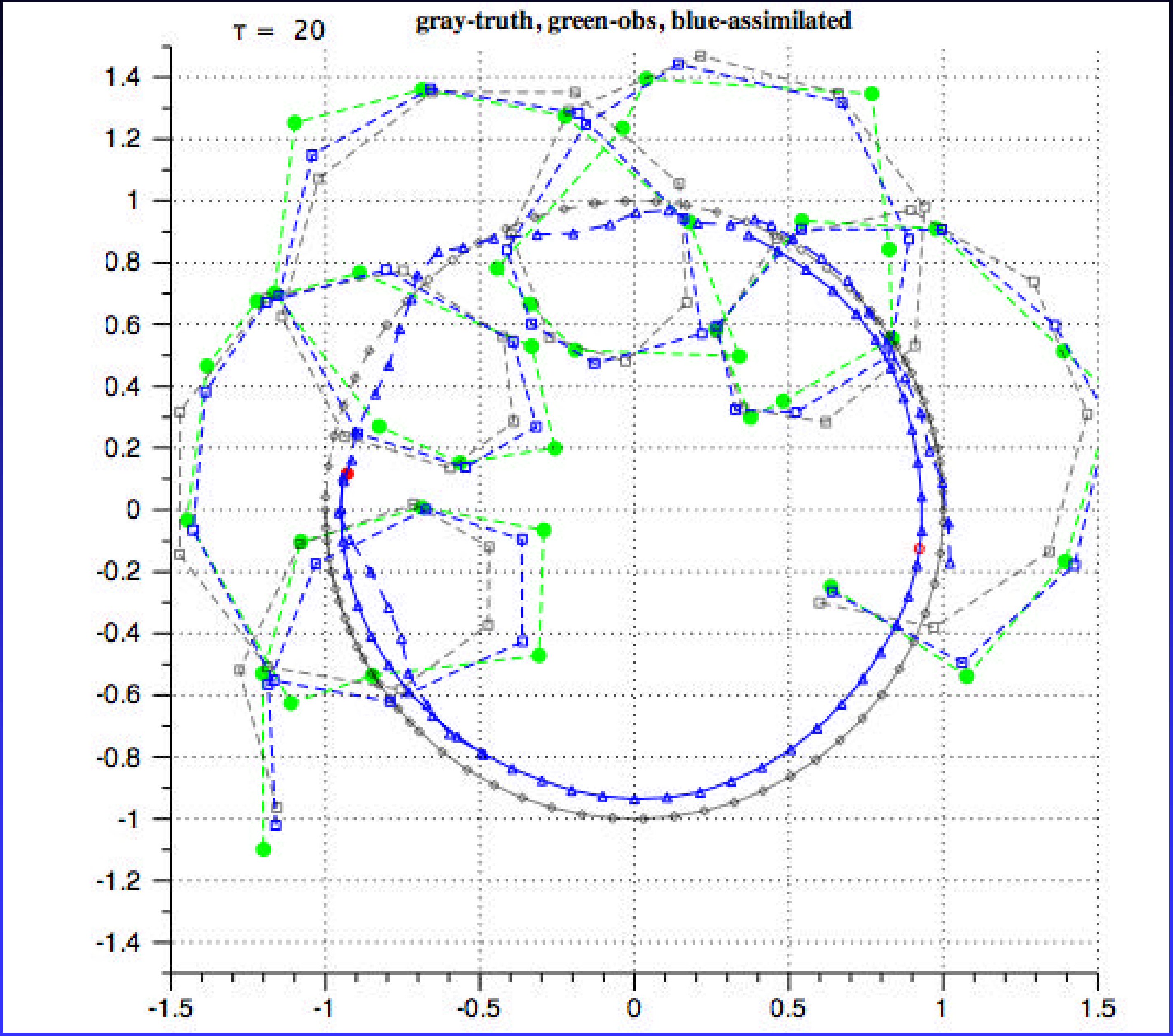
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5


T = 5
gray-truth, green-obs, blue-assimilated
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5


τ = 10
gray-truth, green-obs, blue-assimilated
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5


τ = 15
gray-truth, green-obs, blue-assimilated
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5


τ = 20
gray-truth, green-obs, blue-assimilated
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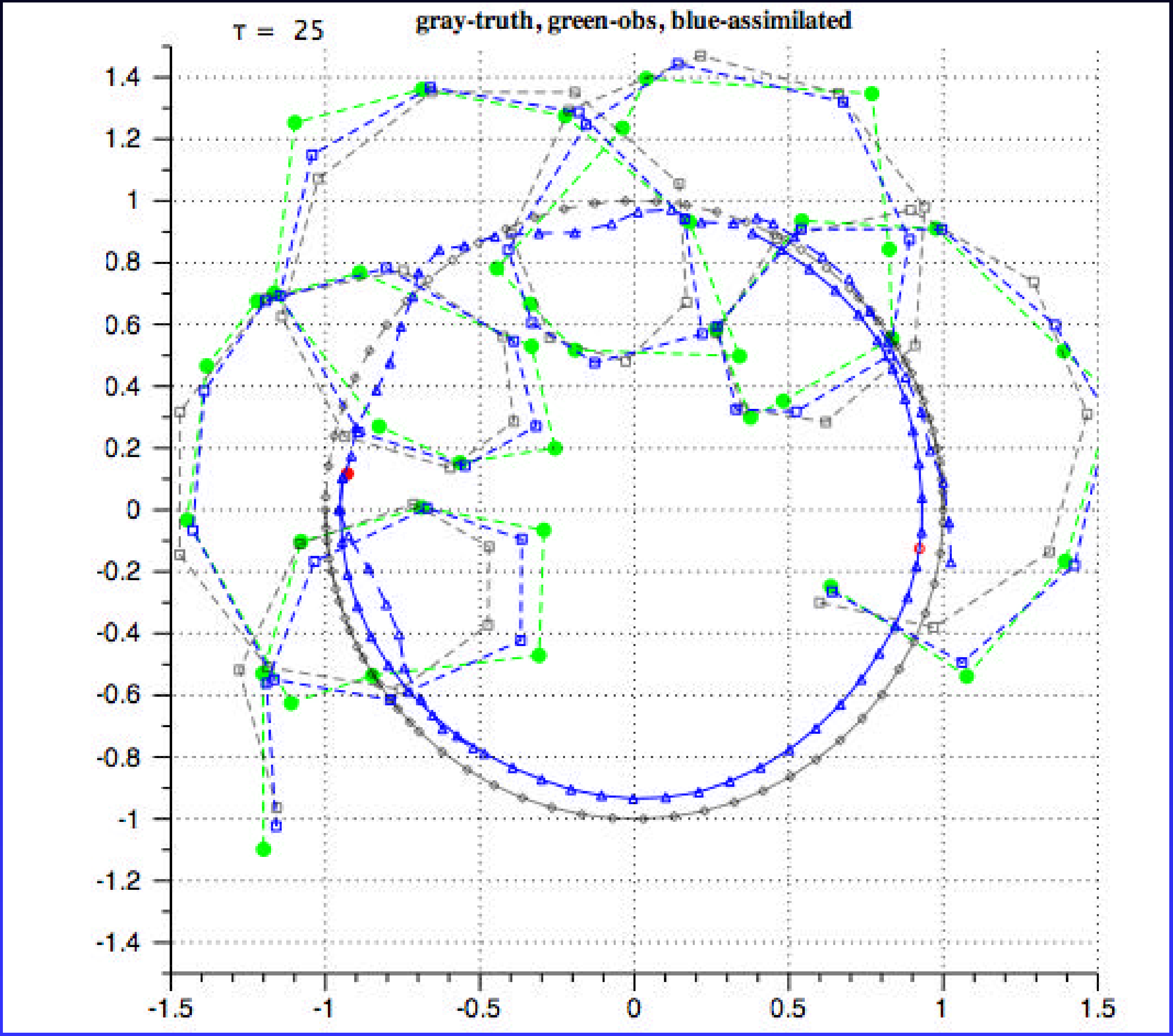
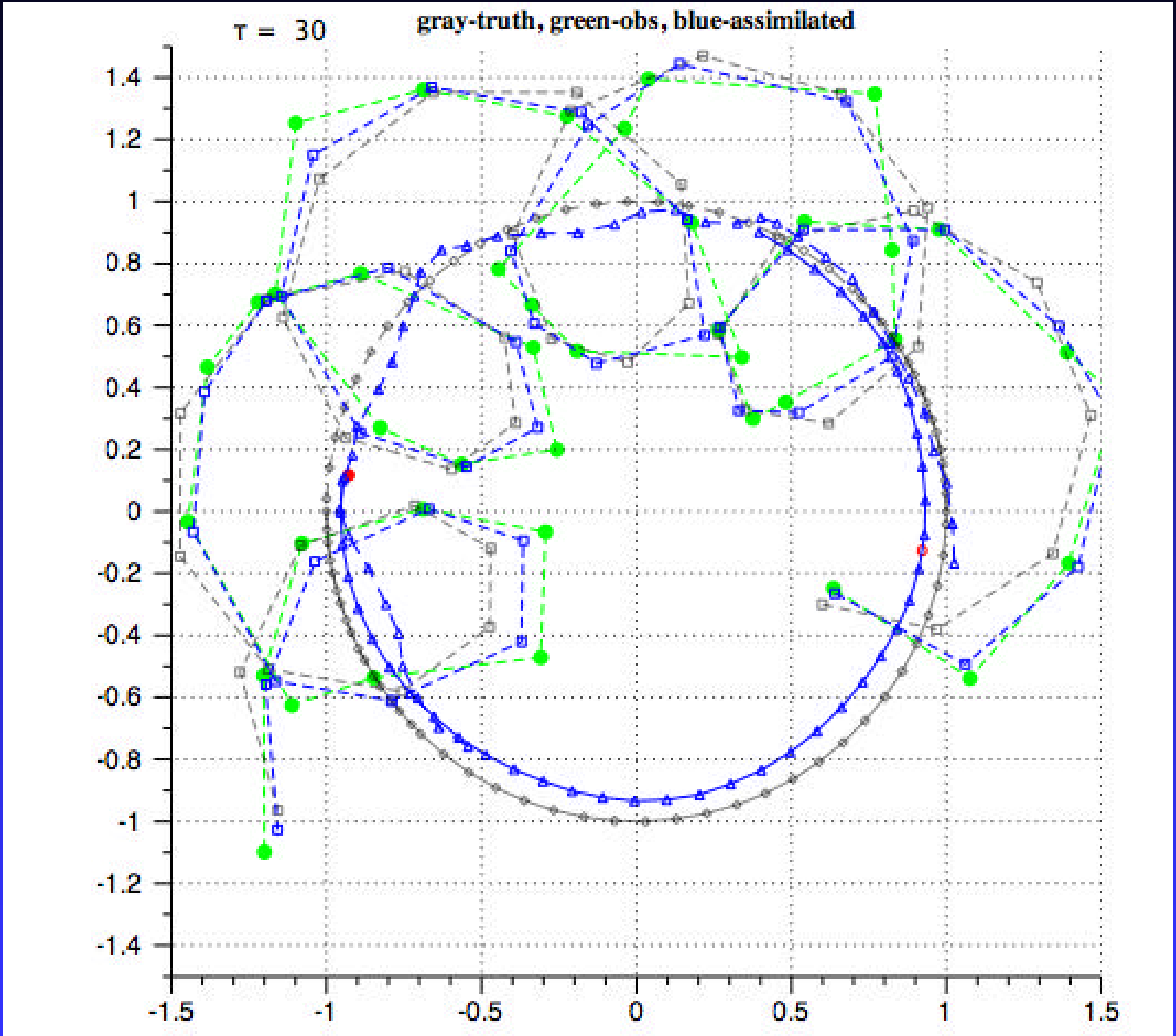
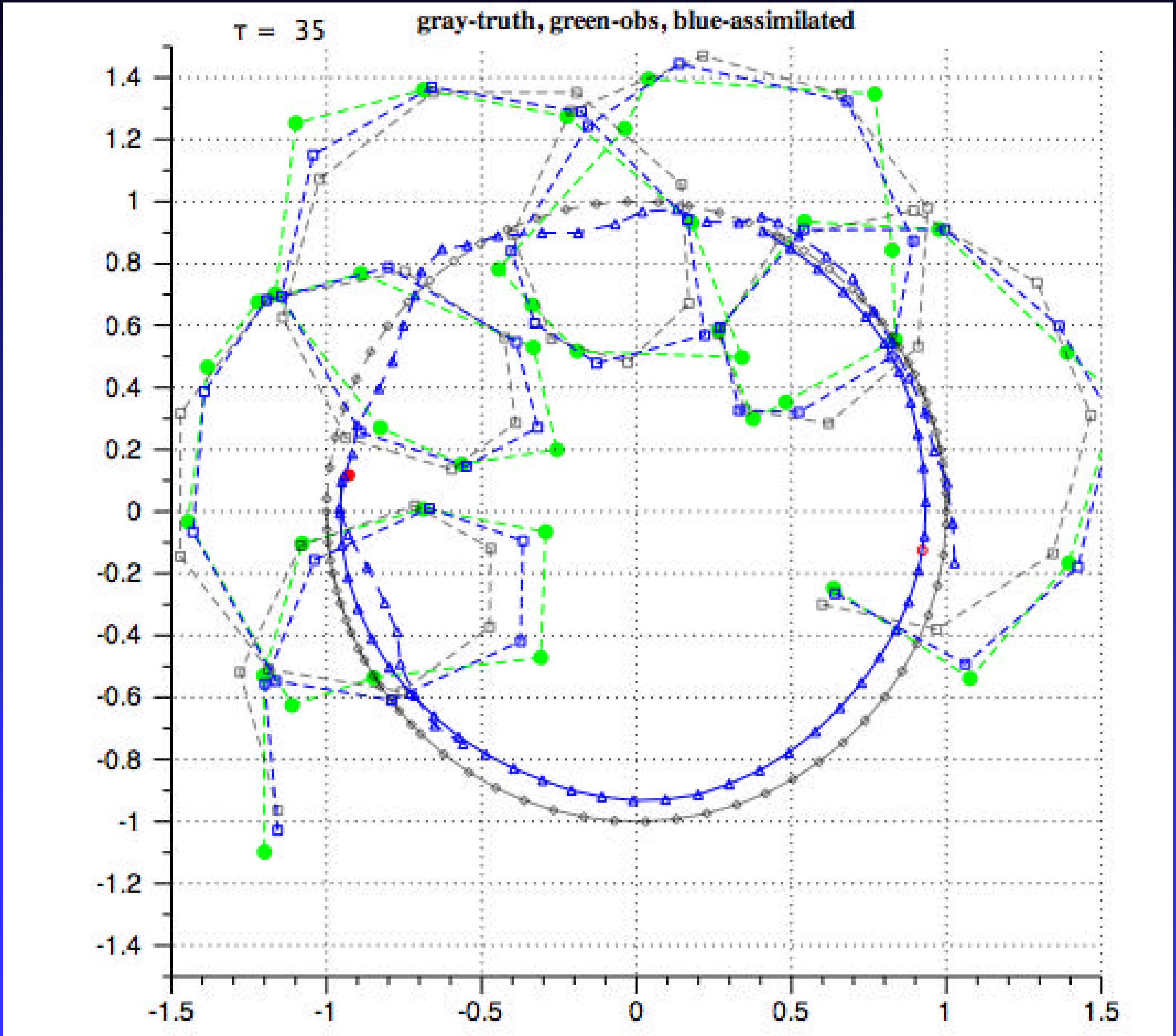
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5

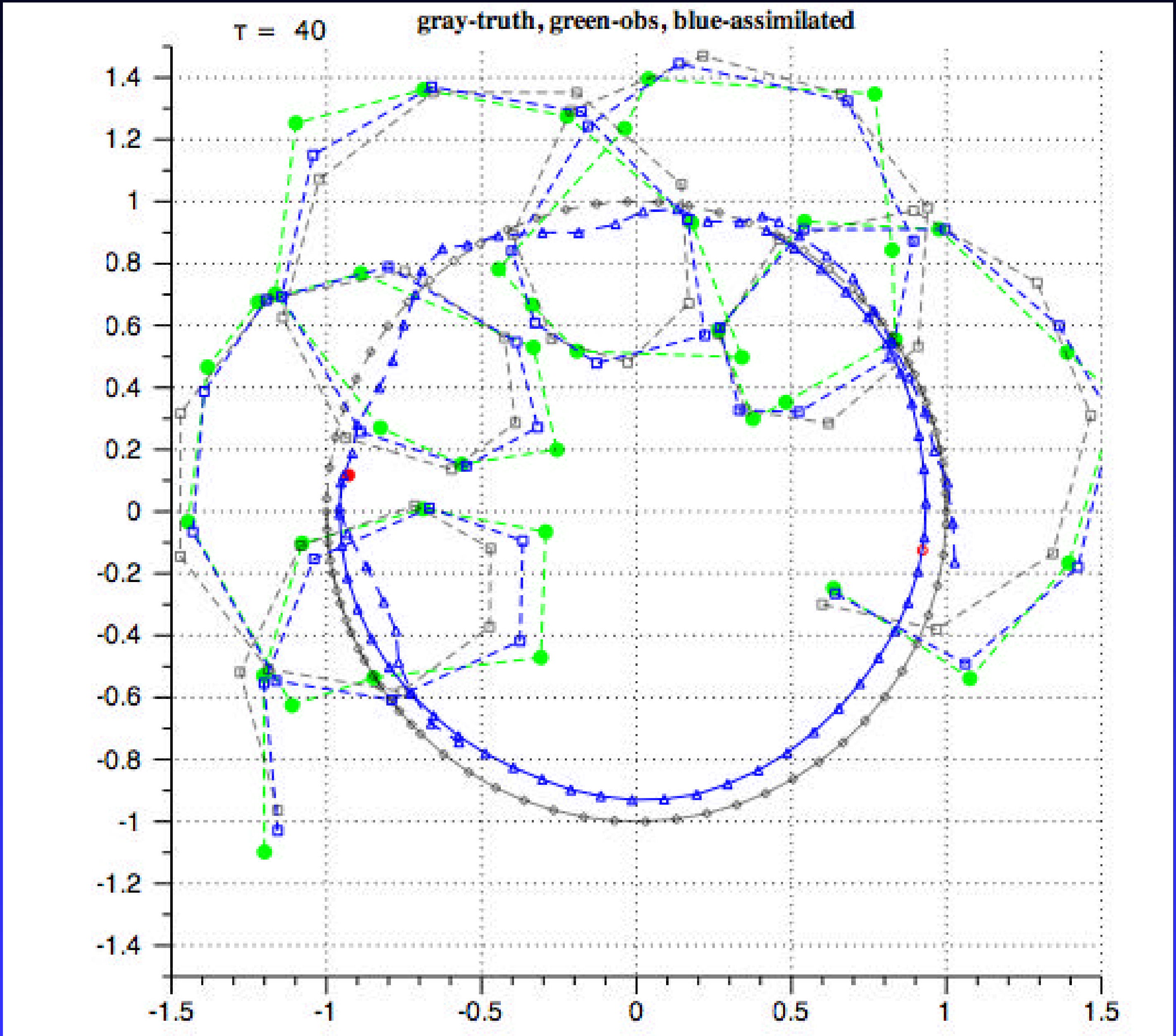

τ = 25
gray-truth, green-obs, blue-assimilated
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5

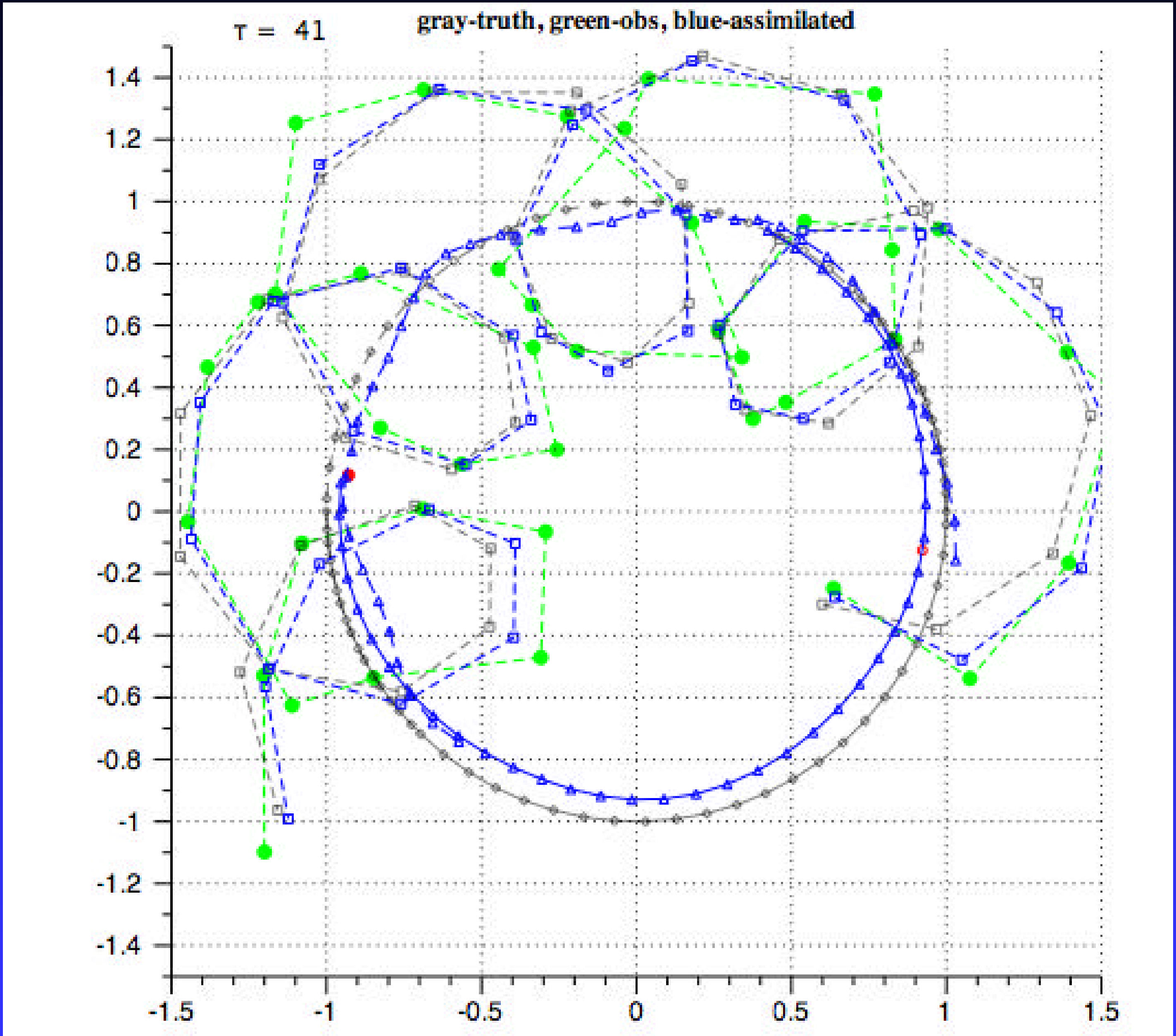


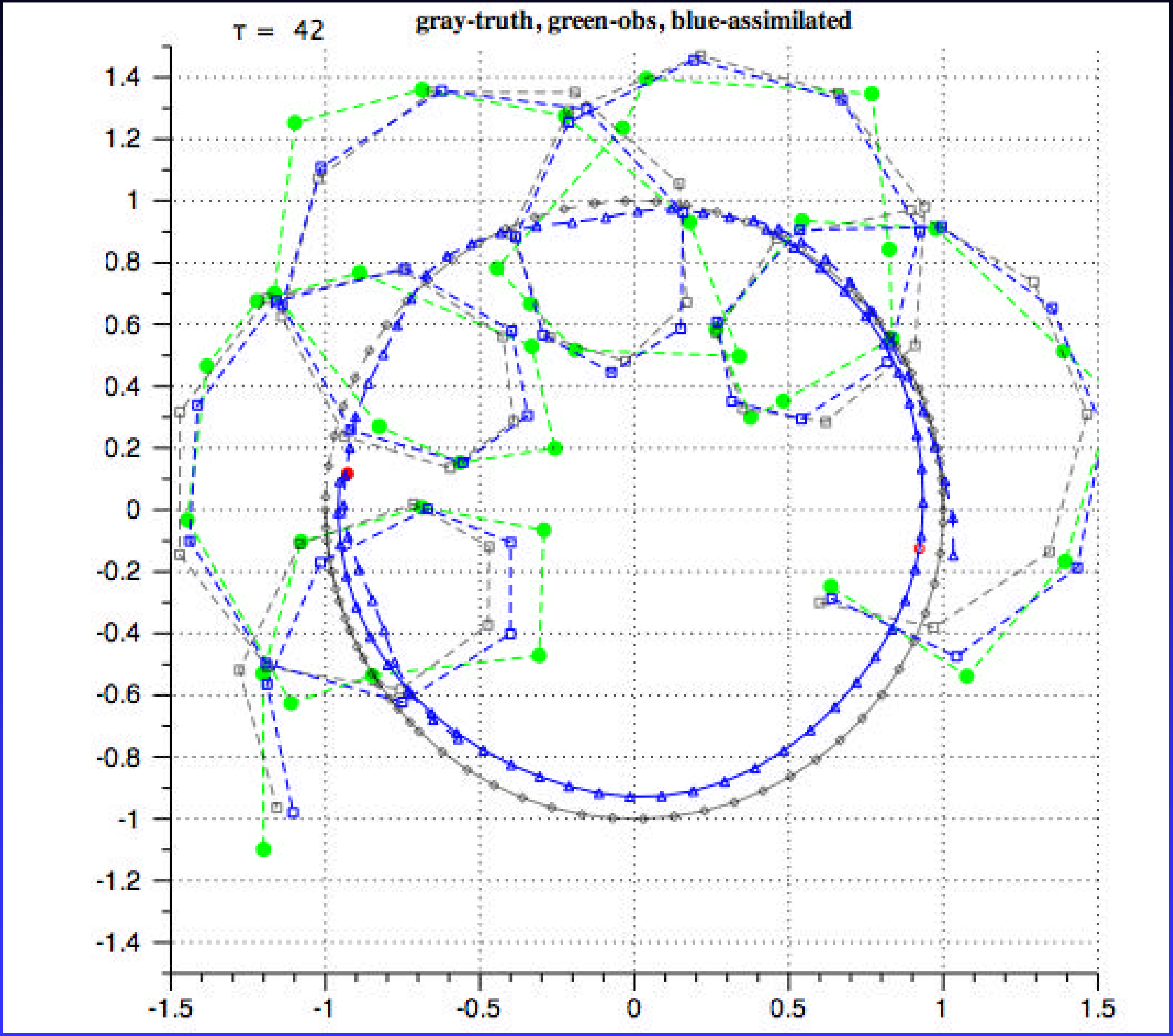
τ = 30
gray-truth, green-obs, blue-assimilated
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5

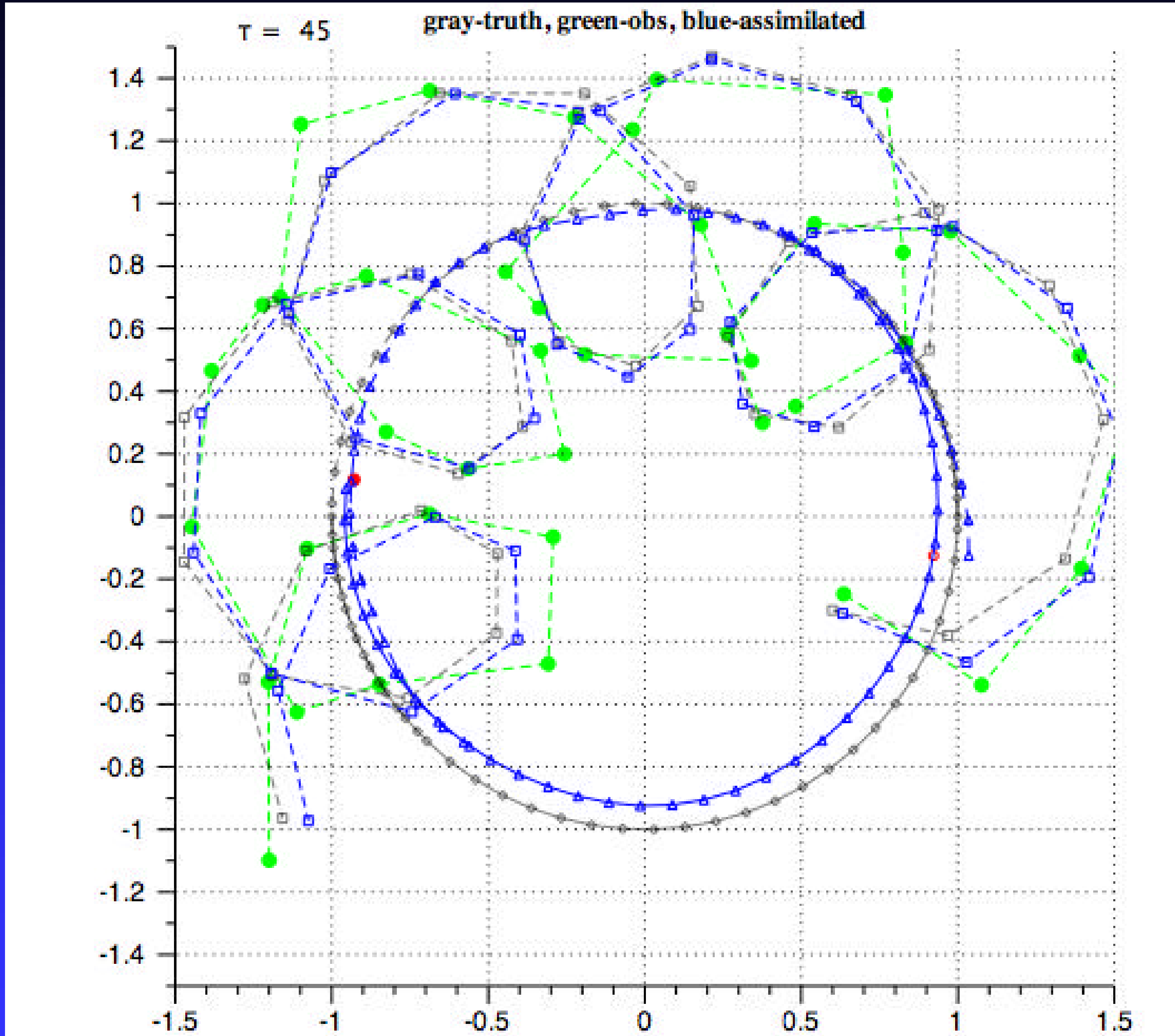


T = 35
gray-truth, green-obs, blue-assimilated

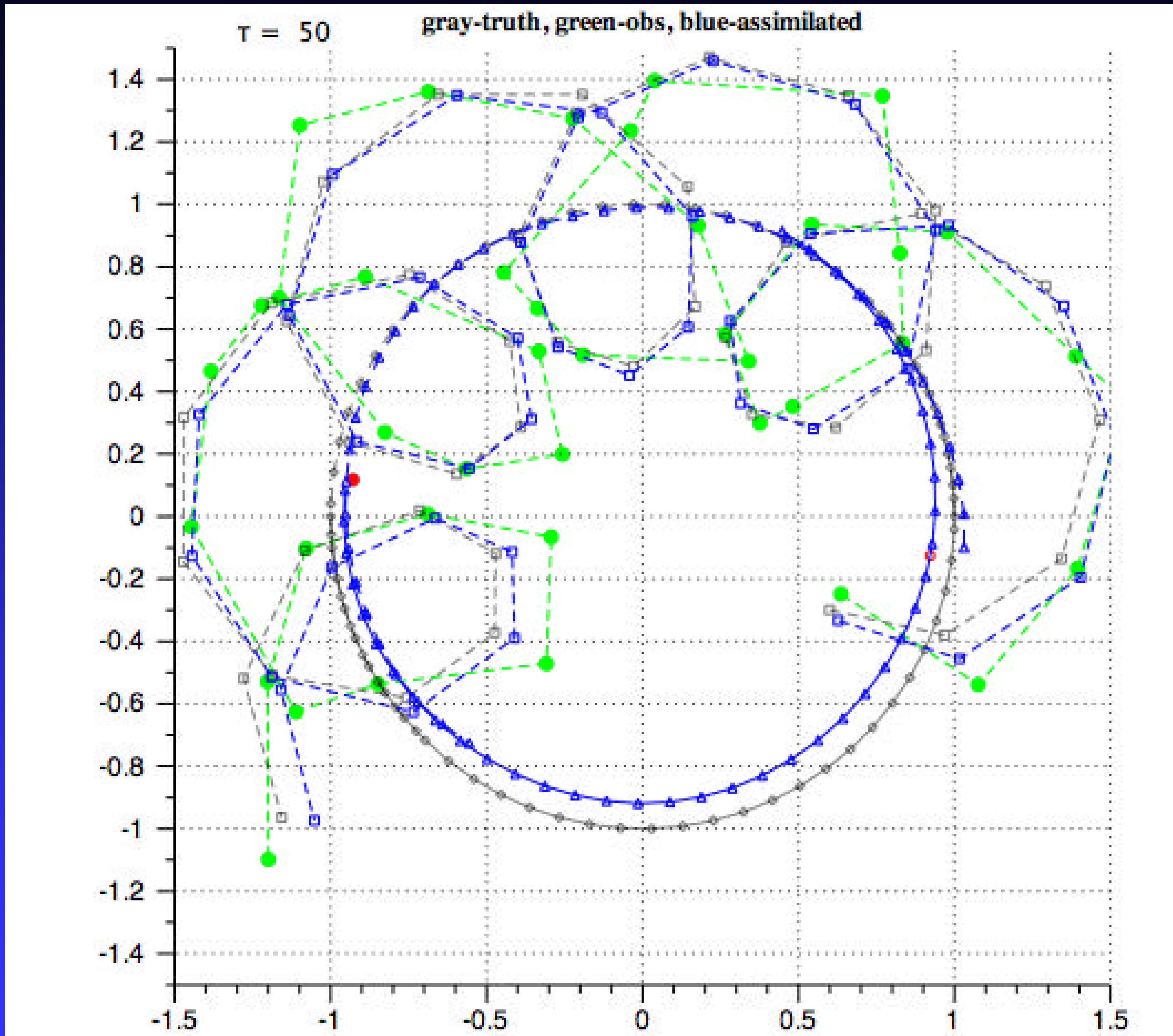

τ = 40
gray-truth, green-obs, blue-assimilated
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5

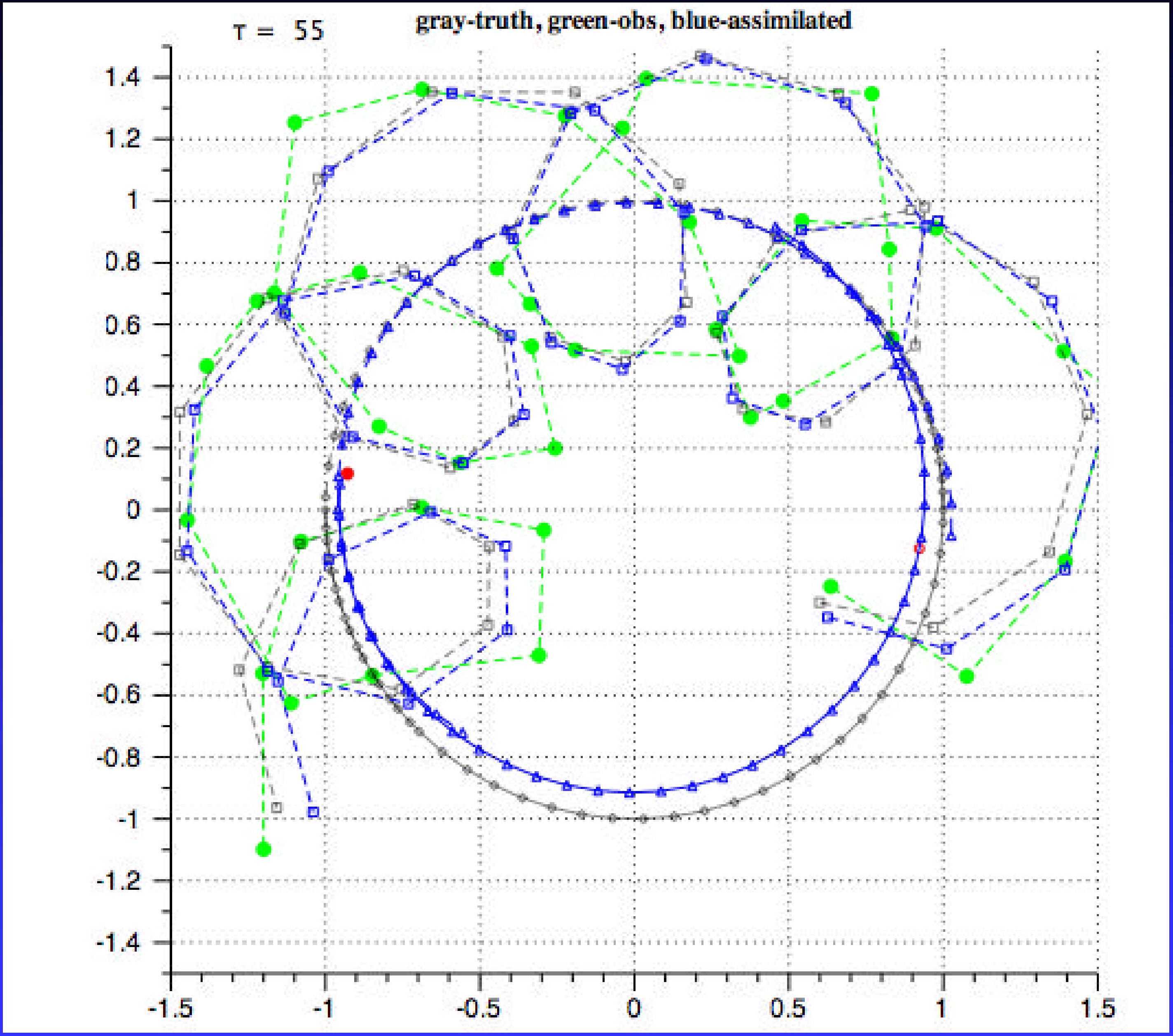

T = 41
gray-truth, green-obs, blue-assimilated
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5


T = 42
gray-truth, green-obs, blue-assimilated
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5

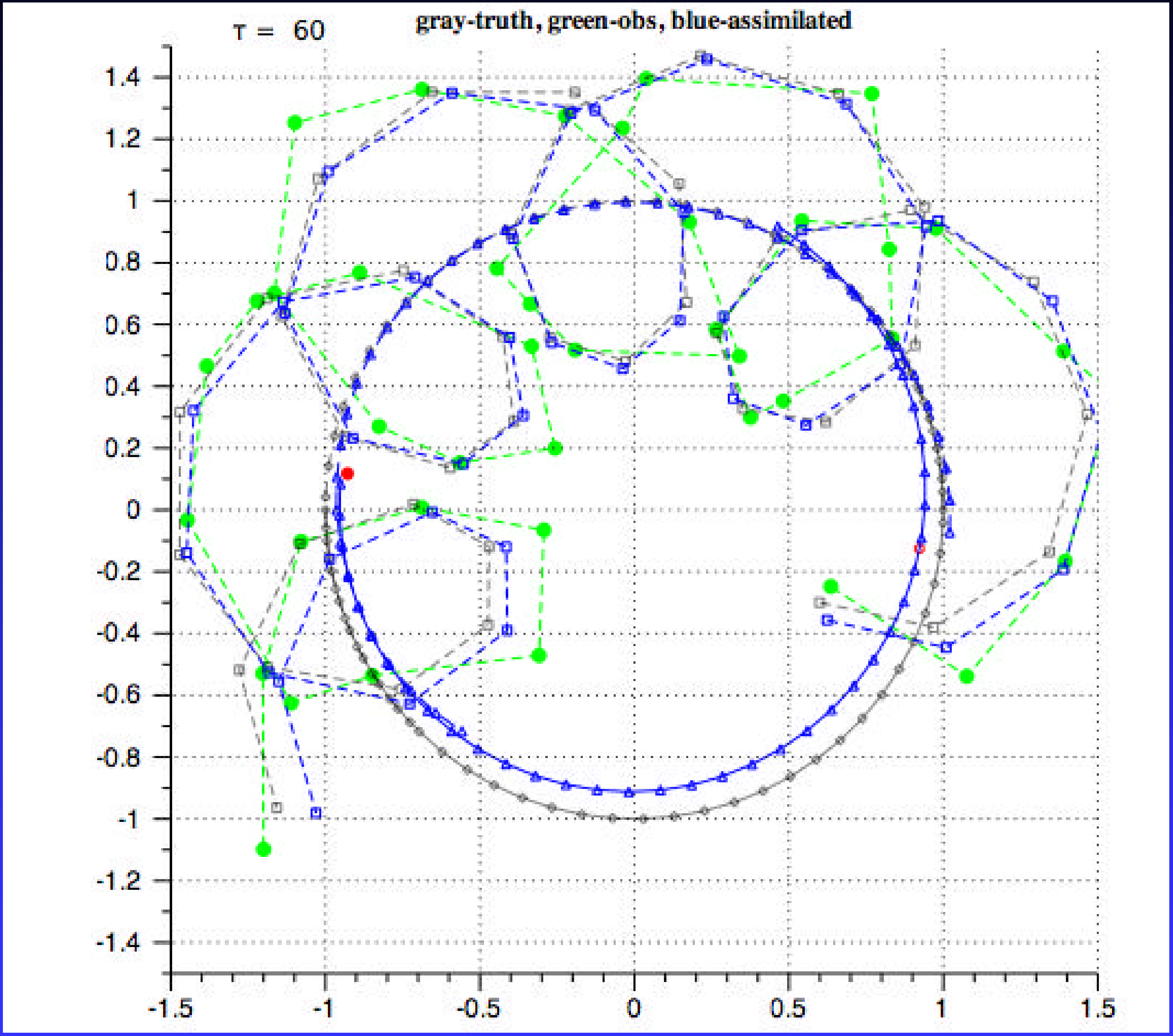


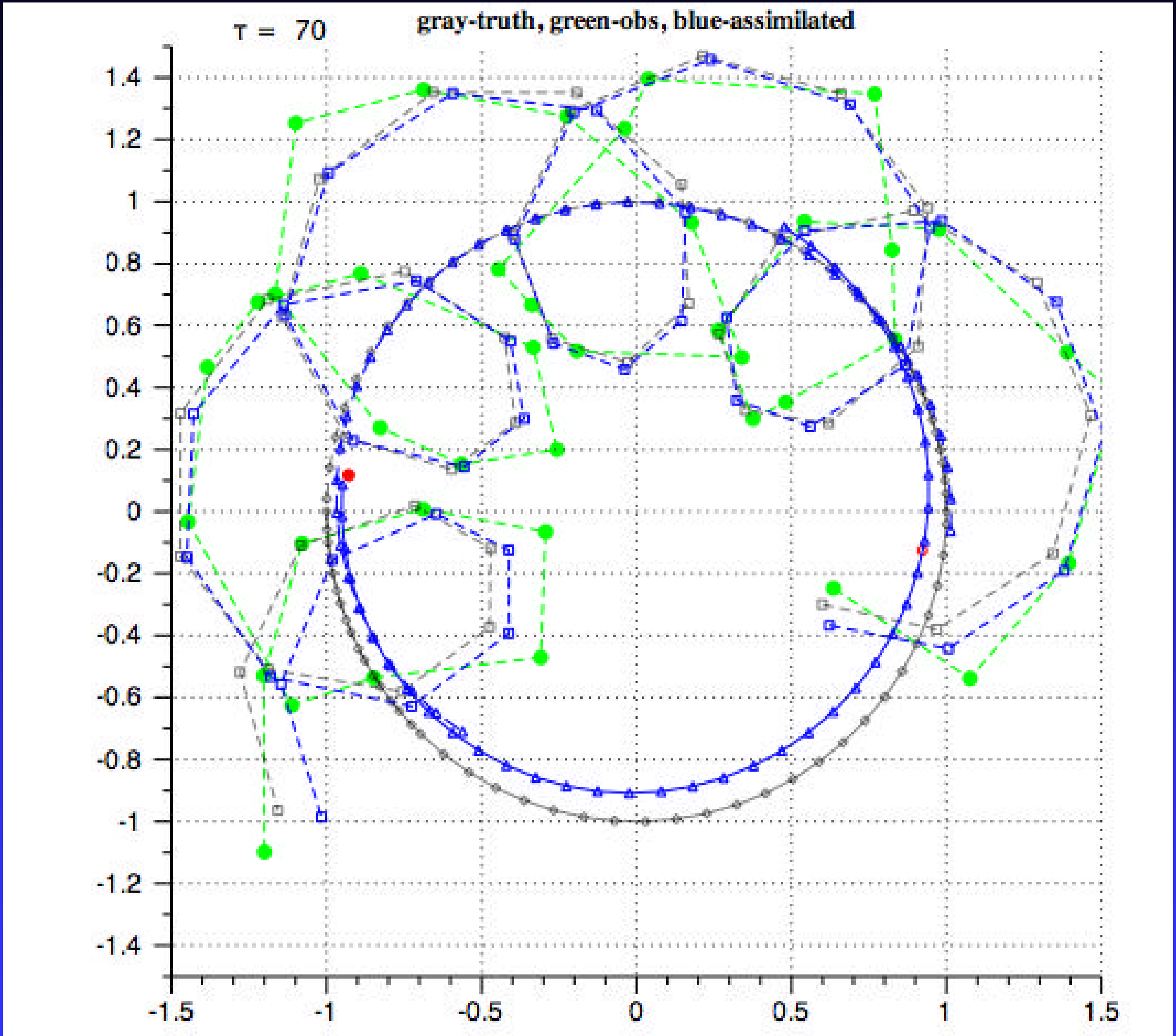
T = 45
gray-truth, green-obs, blue-assimilated
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5


τ = 50
gray-truth, green-obs, blue-assimilated
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5

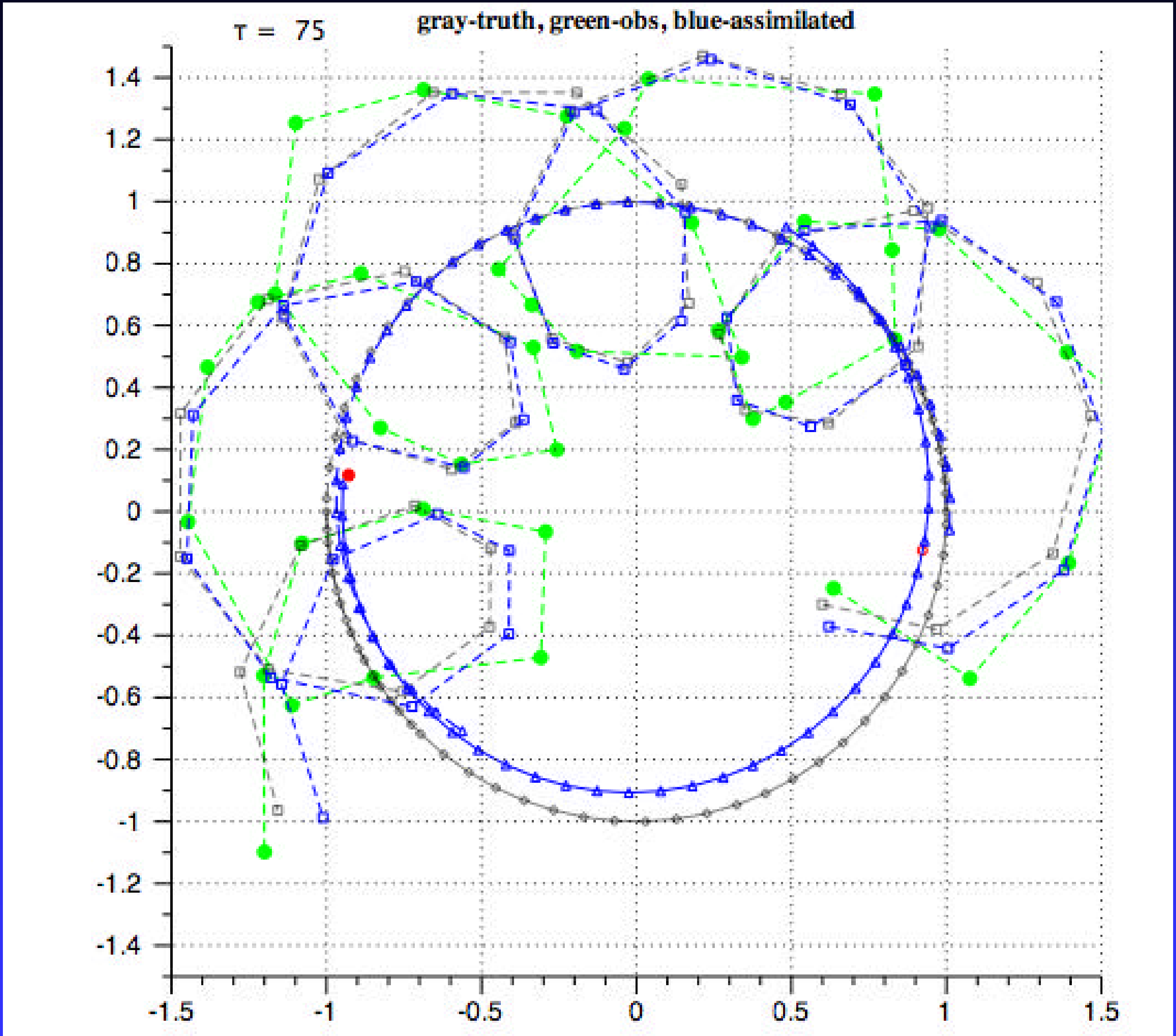


τ = 55
gray-truth, green-obs, blue-assimilated
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5

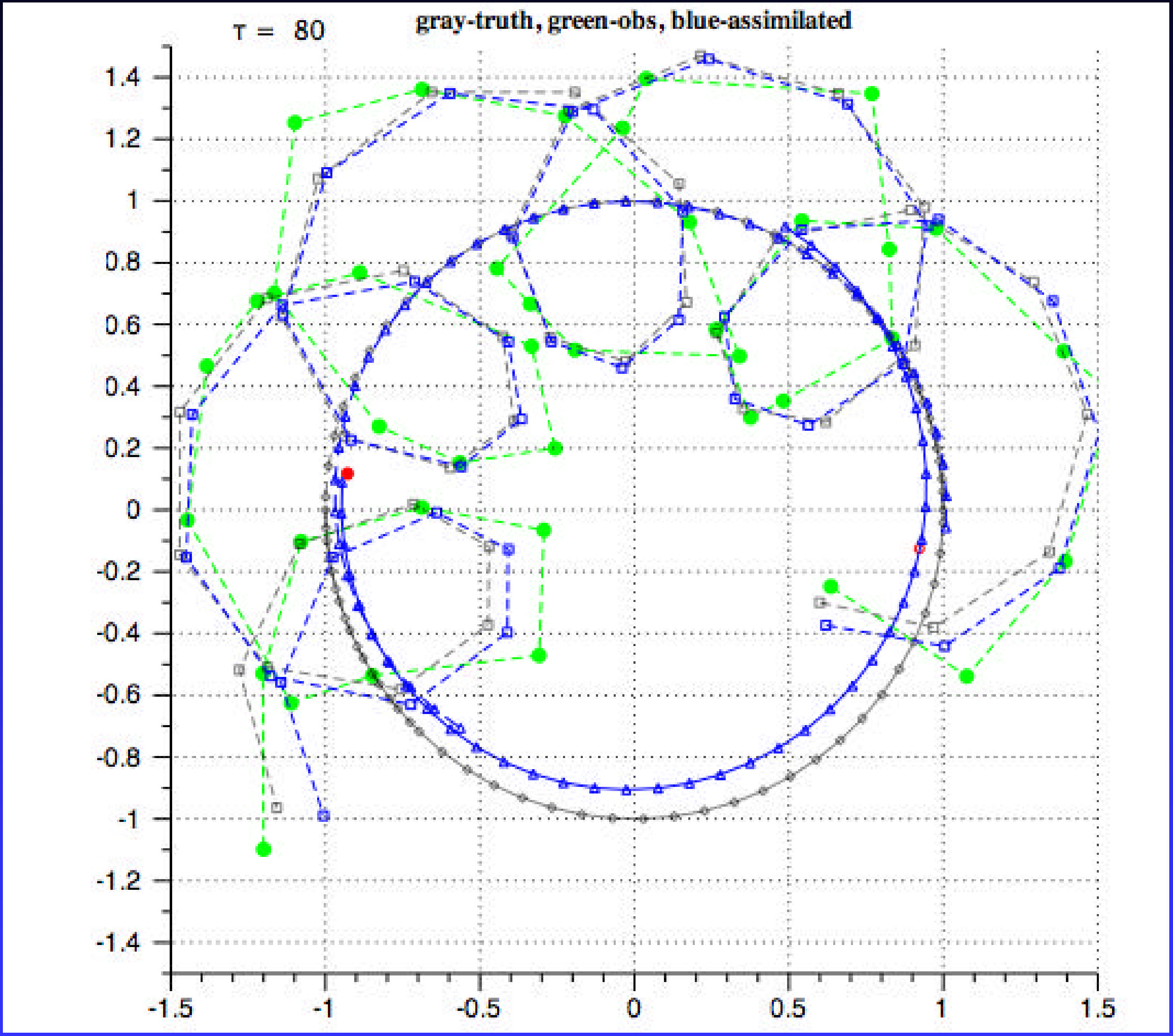

τ = 60
gray-truth, green-obs, blue-assimilated
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5



τ = 70
gray-truth, green-obs, blue-assimilated
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5

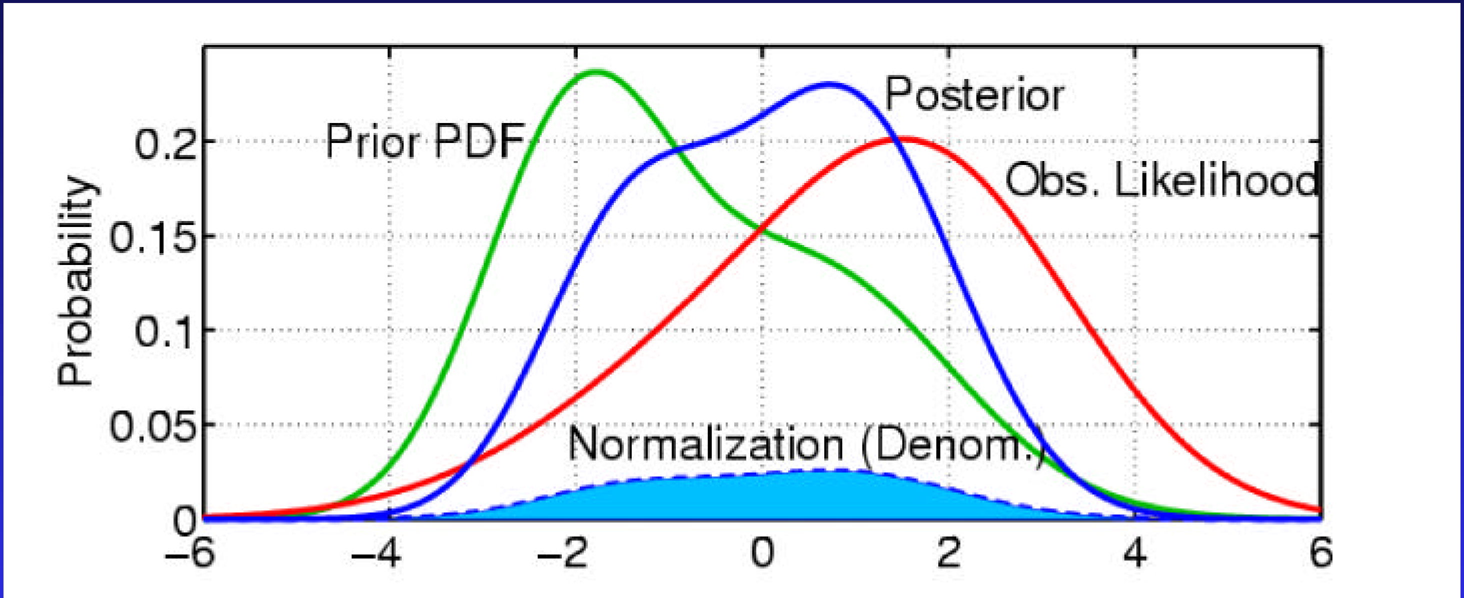


τ = 75
gray-truth, green-obs, blue-assimilated
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5


τ = 80
gray-truth, green-obs, blue-assimilated
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.5
-1
-0.5
0
0.5
1
1.5

11

Given the option, we would all exploit the results of "Perfect Bayes."
These are, of course, inaccessible even in the perfect model scenario.
Whatever we do, we aim to use probability calculus coherently,
but it is no longer clear we want to approximate the Perfect Bayes answer in a naïve Bayesian fashion! (balance prior, obs, dynamcis)



This is a very pretty picture:

Try computing the "normalization" from a ***sample*** of the true blue density (few ~50 in a 10^7-D space…) and see how much information from the prior and the likelihood is preserved…

Model error (better known as Model Inadequacy)

Data Assimilation:

From Observations to Model State(s) — One-to-Many ?sequential?

Simulation:

Current Model-state to future model-state — ?one-to-one?

Forecast:

Future model-state(s) to physical forecast — ?scenarios? ?joint? ?user-specific?

Nonlinearity Couples all of this!

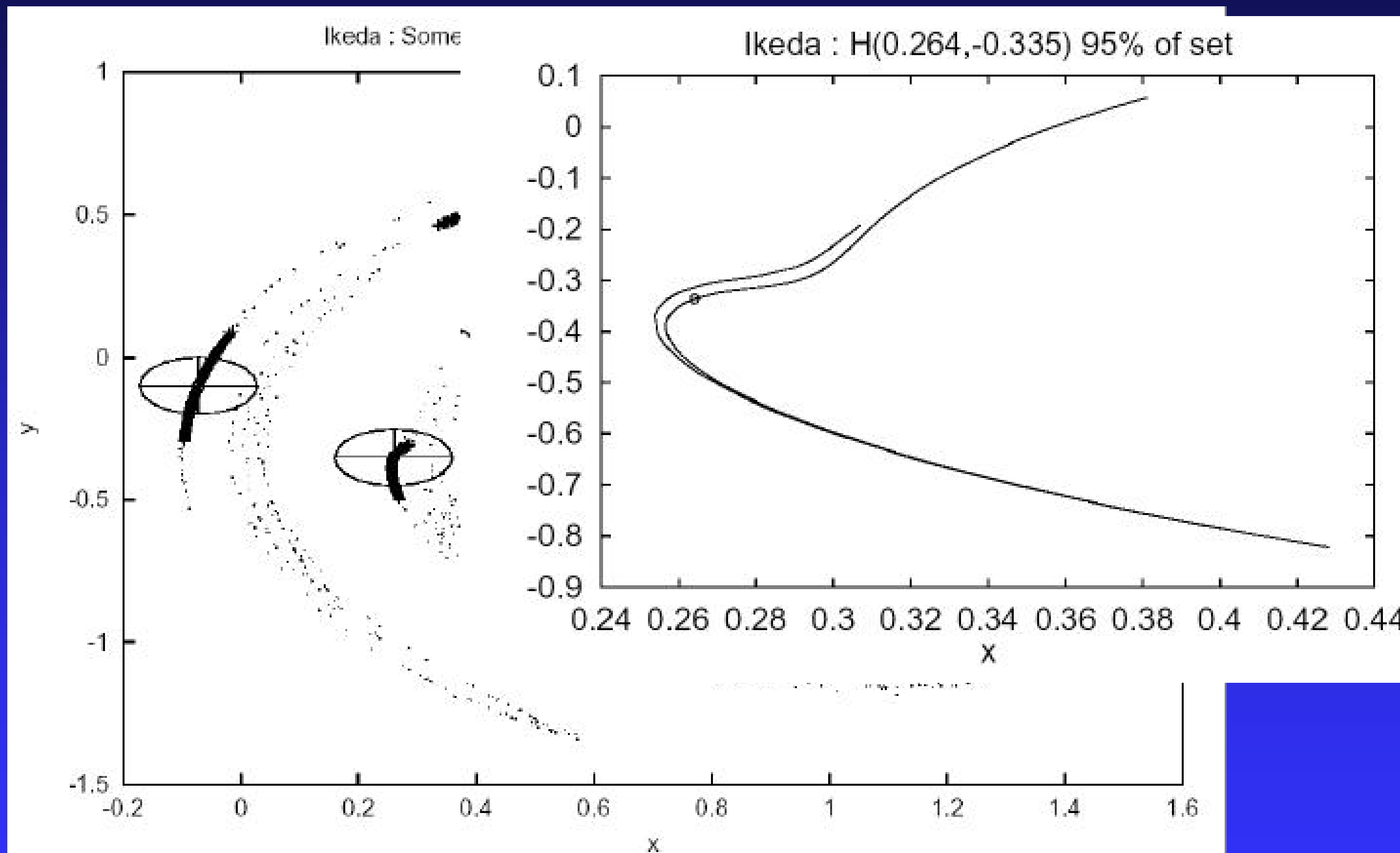
In its simplest terms, a "model" consists of:

A noise model (of the obs)
A dynamical model (deterministic or stochastic)
Two Projection operators

If the model is perfect, everything is well defined; if not, not.

Will being Bayesian buy me better probability forecasts?

Is there an alternative approach, which uses the same resource to find a much higher resolved estimate of (an inferior) PDF?



Probability Calculus still provides the ultimate goal: but given finite resources which approach is more valuable?

Traditional aims of state estimation:

$x(t_0)$	current model state
s_i	observations
$F_a(x)$	dynamical model
a	parameter values
n	obs noise model

$$P(x(t_0) \mid s_i, F_a(x), a, n)$$

Traditional aim of forecasting (in statistics)

$$P(x(t > t_0) \mid s_i, F_a(x), a, n)$$

In cases where $F_a(x)$ is imperfect (*i.e.* in practice), these two procedures may have different target different distributions for $P(x(t_0))$.

You will have understood the main point of this talk if you leave it unsure of the target in the second case

Non-Bayesian By Choice (even within PMS)

$$\mathrm{prob}(X \mid Y, I) = \frac{\mathrm{prob}(Y \mid X, I) \times \mathrm{prob}(X \mid I)}{\mathrm{prob}(Y \mid I)} \tag{1.3}$$

where ***I*** is the "relevant background information at hand".

D.S. Sivia

Even given a perfect model, if we want to move beyond (1.3) then we will be forced to sample the distributions.

Given the model, we can build an Accountable Bayes Importance Sampler (ABIS) which will yield weighted posterior ensembles which are accountable (suffer only from finite sample effects).

ISIS (Indistinguishable States Importance Sampler) ignore our ability to sample the prior, but arguably give more useful posterior ensembles than ABIS for any finite computational resource!

Even the EnKF is likely to beat ABIS if only small computational resource is available.

his week you have to make a choice on how/whether to model reality!

The Truth is out there **(Maths/Stat**

There is no spoon **(Physics)**

Prediction is very difficult, especially about the future.

Niels Bohr

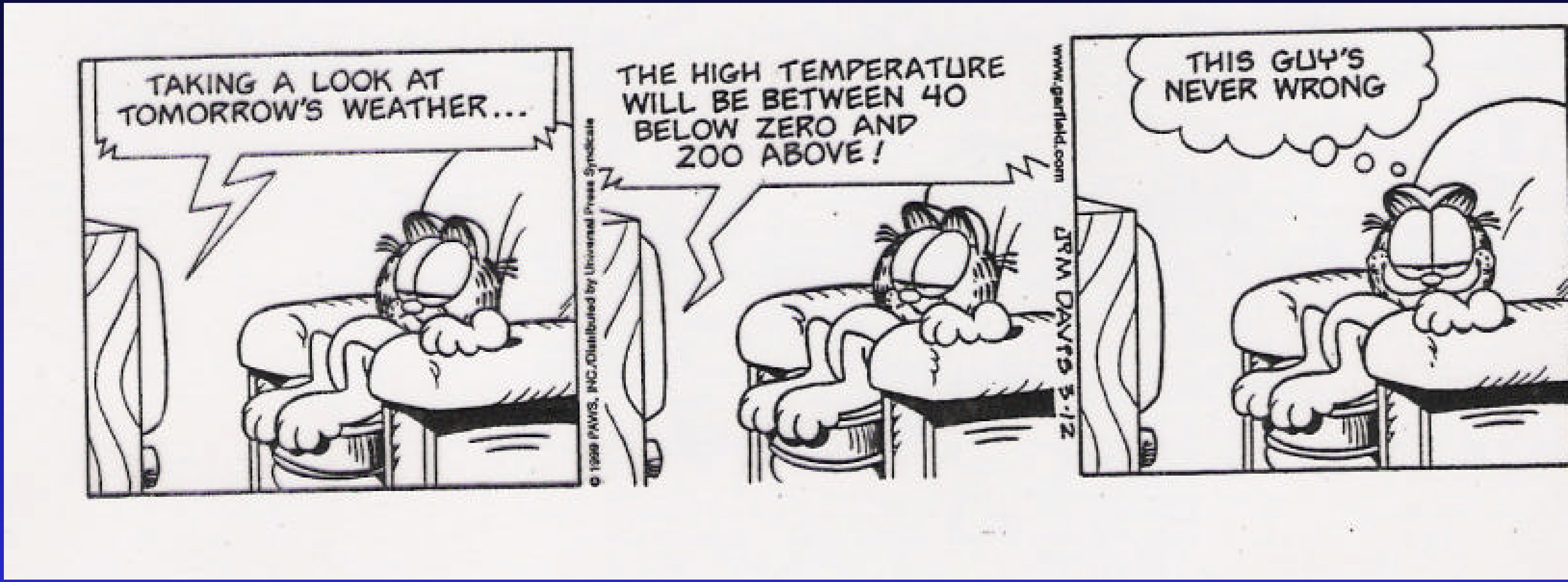
The future will be better tomorrow.

Dan Quayle

Maybe we oughta help him see,
the future ain't what it used ta be.

Tom Petty

Even inside PMS, the justified resource will depend on the user's aims



Aim: Deployable Probabilistic Forecasts with Accountability, Resolution, and Relevance

Informative

Assigns non-trivial probabilities (to what happened, no
what is the chance x happene

Suffers only from sampling finite N effects

A Method for Assimilation of Lagrangian Data

L. KUZNETSOV

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K. IDE

Department of Atmospheric Sciences and Institute of Geophysics and Planetary Physics, University of California, Los Angeles, Los Angeles, California

C. K. R. T. JONES

Department of Mathematics, University of North Carolina at Chapel Hill, Chapel Hill, North Carolina

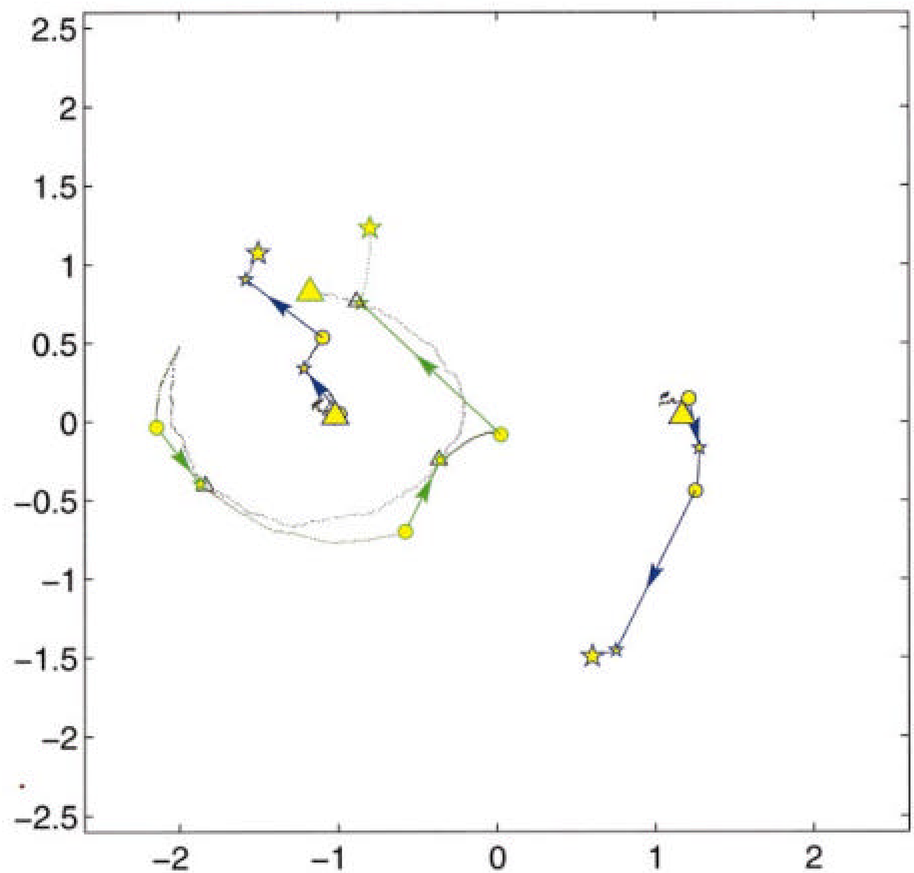
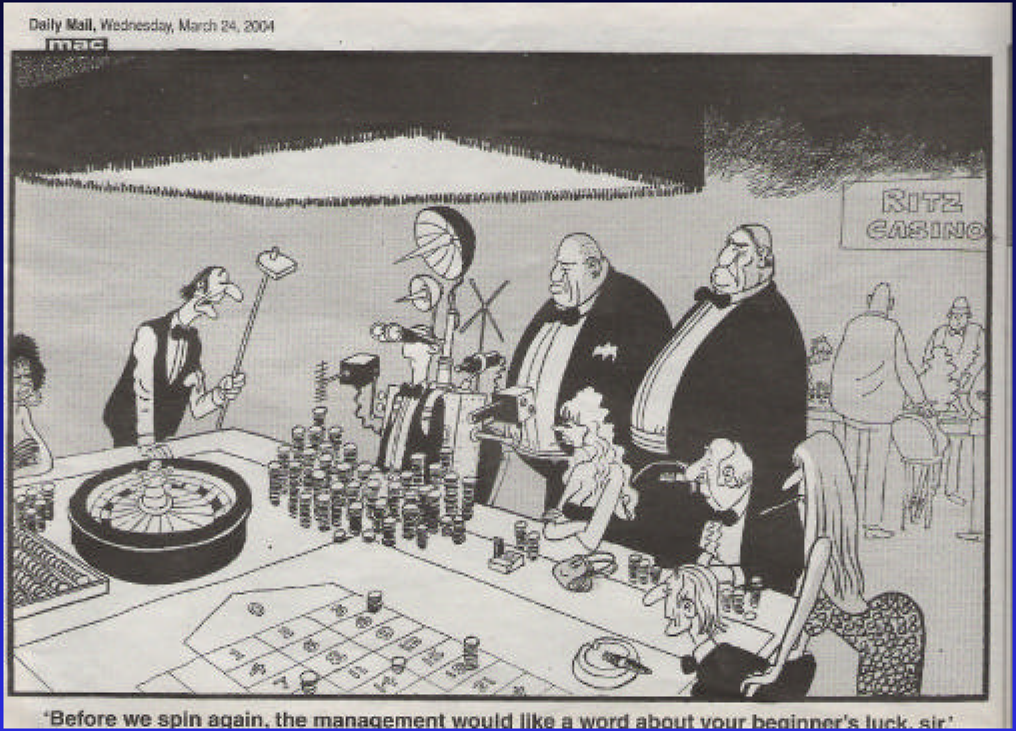


FIG. 5. Trajectories of the full system and the model in the corotating frame, shown for $8 < t < 12.4$; $\Delta T = 1.5$, $\sigma = 0.04$, $\rho = 0.02$, $N_F = 2$, $N_D = 1$. Small symbols: vortex and tracer positions at the observation times $t_j = 9, 10.5, 12$. Triangles: $\mathbf{x}_j^t$ (full system), circles: $\mathbf{x}_j^f$ (model before updates), stars: $\mathbf{x}_j^a$ (analysis state). Arrows: correction vectors. Large symbols: final positions at $t = 12.4$. Vortices are shown in blue, tracer in green.

If we had enough data, could we predict anything?



Alt: If we had enough data, could we predict more profitably?

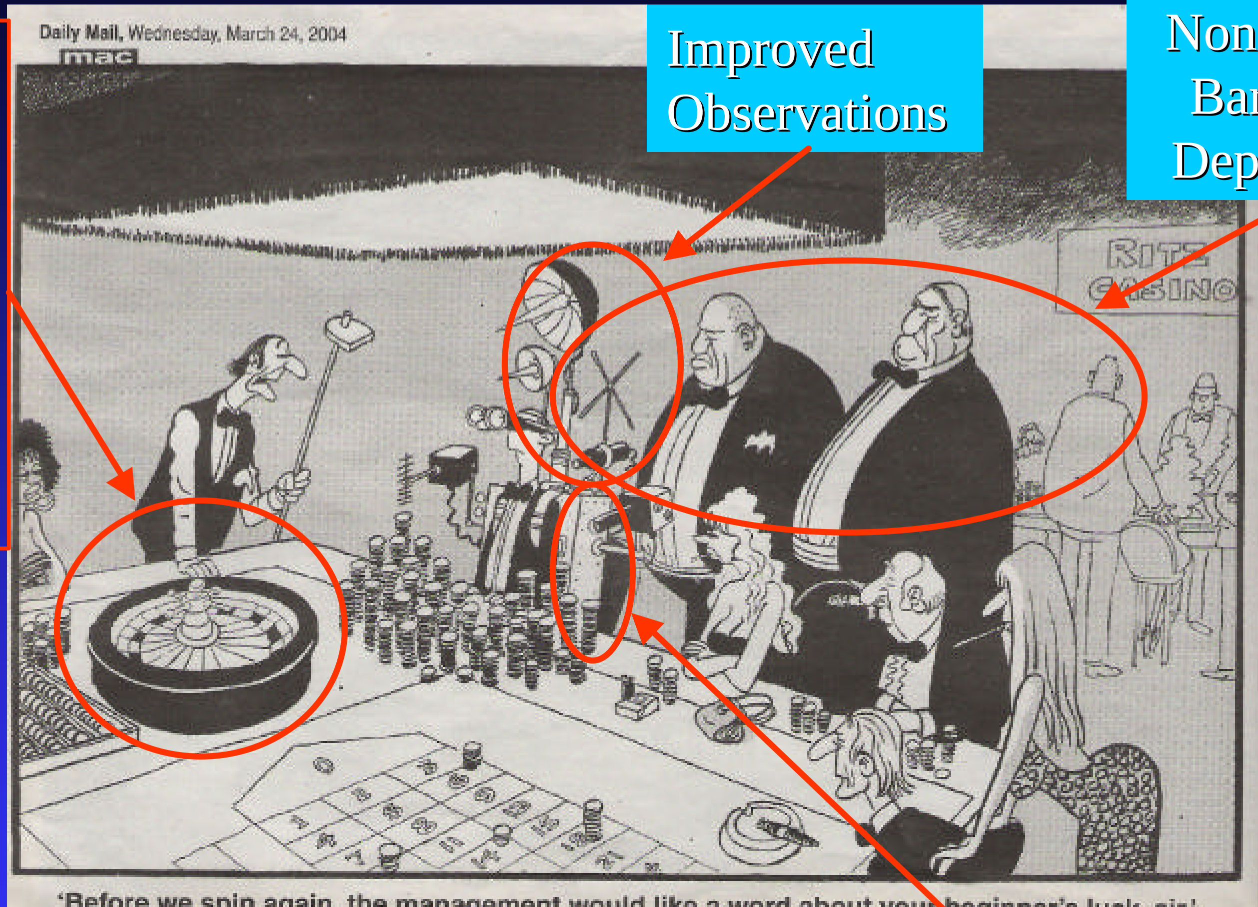
Resource allocation
(Identifying Weakest Links)

Theory of Stochastic Processes

Classical Dynamics

Improved Observations

Non-Science Barriers to Deployment



Bigger Computer

(In this particular case, obs were more valuable than theory)

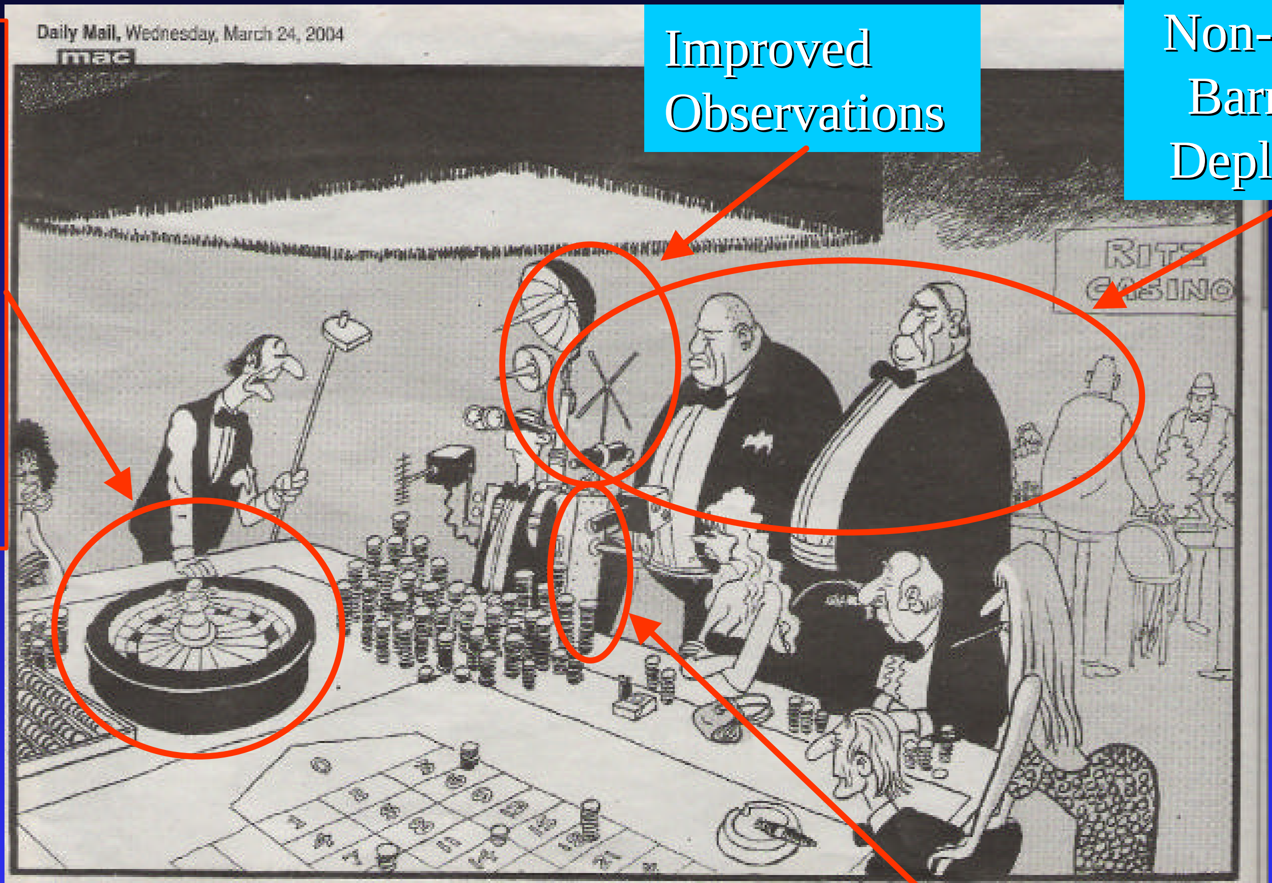
Resource allocation
(Identifying Weakest Links)

Theory of Stochastic Processes

Classical Dynamics

Improved Observations

Non-Science Barriers to Deployment



'Before we spin again, the management would like a word about your beginner's luck, sir.'

Bigger Computer

(In this particular case, obs are more valuable than theory)

In practice: Probability forecasts do not have to be accountable to be useful!

Wager £100 each day on the temperature at Heathrow, betting an amount proportional to your predicted probability of that outcome (Kelly Betting).

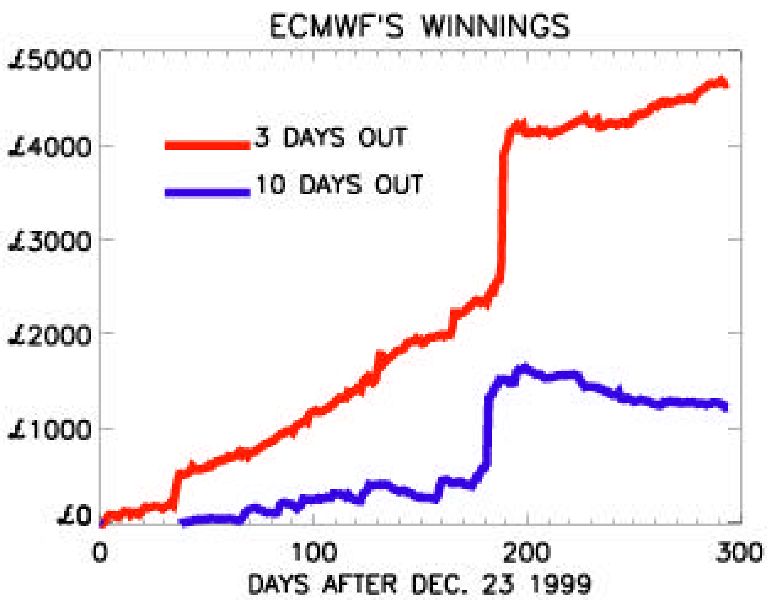
How would a probability forecast based on the ECMWF EPS fare against a house that set its odds using climatology?

WEATHER ROULETTE

TEMPERATURE AT HEATHROW

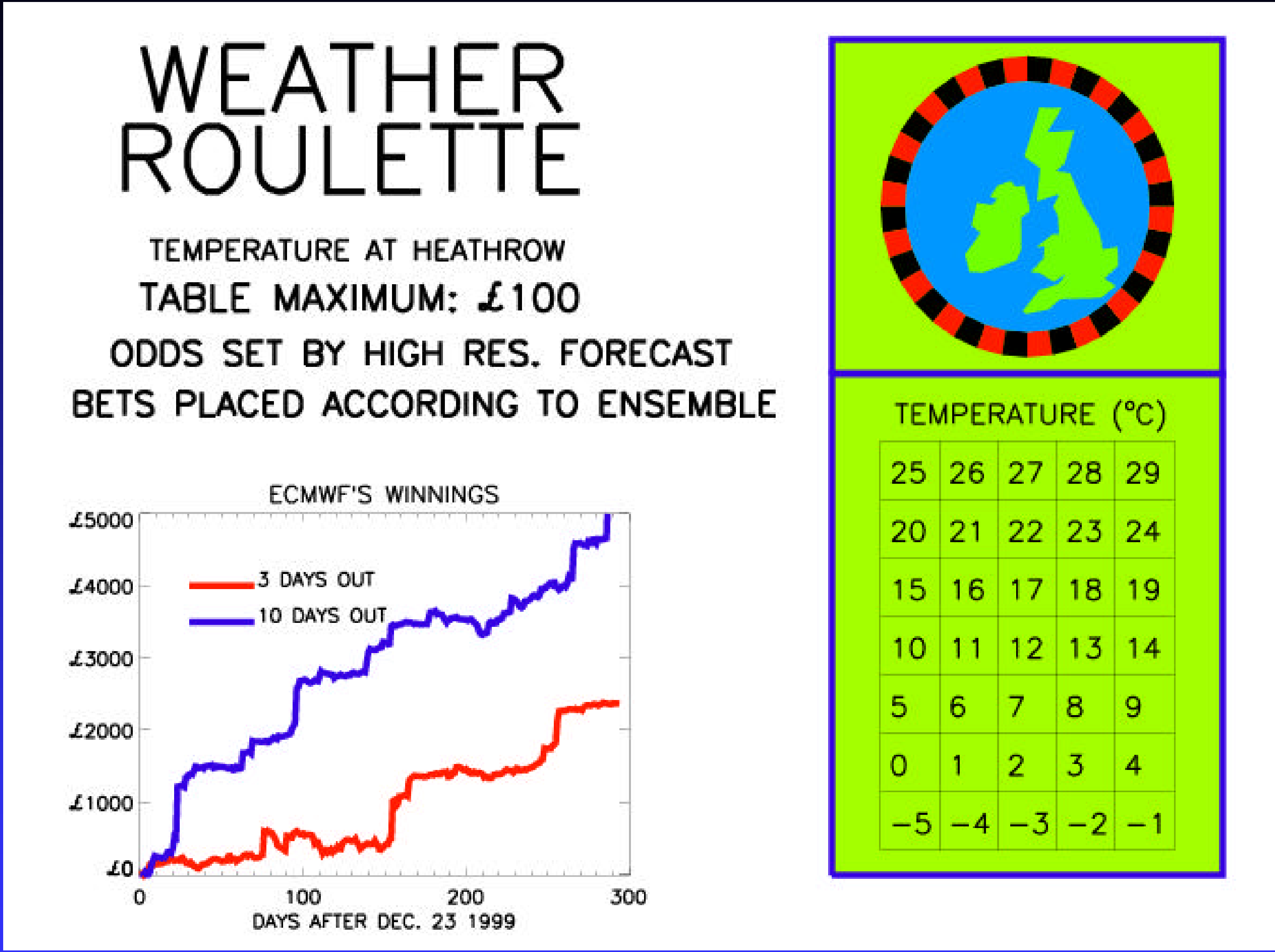
TABLE MAXIMUM: £100

1982–99 CLIMATOLOGICAL ODDS



TEMPERATURE (°C)

25	26	27	28	29
20	21	22	23	24
15	16	17	18	19
10	11	12	13	14
5	6	7	8	9
0	1	2	3	4
−5	−4	−3	−2	−1



Head to head comparisons of probability forecasts allow insight on resource allocation, at least for a subset of users…