Spatially localized global Fourier analysis of 2D 3-vortex dynamics at $\mathsf{R} = 2 \times 10^4$ simulated by dynamically adaptive spectral elements

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The spectral-element method (SEM) for numerically solving partial differential equations combines the geometric flexibility and computational efficiency of the finite-element method with the high accuracy of the pseudo-spectral method (PSM). With noted exceptions, most SEM applications have been to compressible flow or to incompressible flow at low Reynolds number R; a new simulation code, the geophysical-astrophysical spectral-element adaptive refinement (GASpAR) code, recently introduced for 2D linear and nonlinear advection-diffusion simulations [5], is now extended to decaying incompressible Navier-Stokes flows at high R. GASpAR employs dynamic adaptive refinement (DARe) and coarsening, of non-conforming h type. GASpAR simulations converge w.r.t. element size (algebraically) and polynomial degree (exponentially) as quantified by comparison with 3 exact solutions. Dynamically adaptive GASpAR simulation of biperiodic flow at $\mathsf{R} = 2 \times 10^4$ initialized with 3 Gaussian vortices (Fig. 1) [1, 3, 4, 6] closely reproduces that by PSM with the same number of computational degrees of freedom, as well as w.r.t. energy and enstrophy wavenumber spectra. The high degree enables a recently introduced SEM-customized Fourier analysis [2] to find evidence of power-law-scaling regimes over about 1 wavenumber decade, with exponents between 3 and 4, and a new apparent association between exponent value and different flow features such as vortex filaments, that can be identified over different time intervals in certain spatial elements of various scales. Issues will be discussed, associated with isolating flow structures for individual Fourier analysis, without introducing discontinuities.

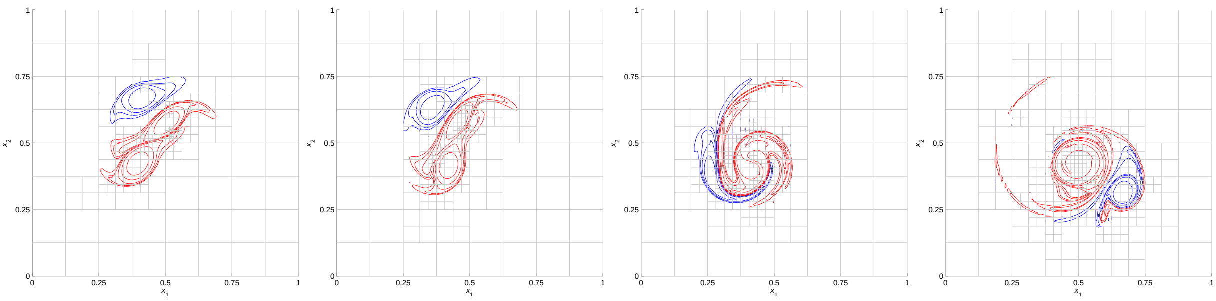

Figure 1: Vorticity $\zeta\,[t, \vec{x}]$ (contours $\log_2(\pm\zeta/\pi) = 0, \ldots 4$): $L = 226$ elements, $t = 1.82 t_{\mathrm{eddy}}$; $L = 256$, $t = 2.25 t_{\mathrm{eddy}}$; $L = 331$, $t = 3.96 t_{\mathrm{eddy}}$; $L = 409$, $t = 8.25 t_{\mathrm{eddy}}$.

Citations

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