Parameterizing Submesoscale Eddies for Global Ocean Models

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NCAR Geophysical Turbulence Phenomena
IMAGe Theme of the Year: Turbulent Theory and Modeling
Boulder, CO; Wednesday 2/27/08, 12:15-12:30
Typical Ocean Stratification Permits
Two Types of Baroclinic Instability:

Mesoscale and SubMesoscale (Boccaletti et al., 2006)
Typical Ocean Stratification Permits
Two Types of Baroclinic Instability:

**Mesoscale** and **SubMesoscale** (Boccaletti et al., 2006)

Vertical fluxes are Submesoscale and tend to restratify

Horizontal fluxes are Mesoscale and tend to stir

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**Figure**

[Diagram showing growth rate vs. wavenumber with depth and amplitude scales.]

- **Mesoscale Eddies**
  - O(100km)
  - 1 month

- **SubMesoscale Mixed Layer Eddies**
  - O(1km)
  - 1 day
Having a Mixed Layer Counts!

The vertical buoyancy flux in the ML ($\langle w'b' \rangle$) without diurnal cycle is **not less** than with cycle (ML).
Having a Mixed Layer Counts!

The vertical buoyancy flux in the ML ($<w'b'>$) without diurnal cycle is less than with cycle (ML)
Having a Mixed Layer Counts!

The vertical buoyancy flux in the ML ($\langle w'b' \rangle$) without diurnal cycle is $2 \times$ less than with cycle (ML).
Having a Mixed Layer Counts!
The vertical buoyancy flux in the ML ($\langle w'\theta' \rangle$) without diurnal cycle is 2x less than with cycle (ML).
Having a Mixed Layer Counts!

The vertical buoyancy flux in the ML ($\langle w'b' \rangle$) without diurnal cycle is $3 \times$ less than with cycle (ML).
Having a Mixed Layer Counts!

The vertical buoyancy flux in the ML ($\langle w'b' \rangle$) without diurnal cycle is $4 \times$ less than with cycle (ML).
Prototype: Mixed Layer Front

Simple Spindown

Plus, Diurnal Cycle and KPP

Overturning Streamfunction

Mixed Layer

$z$ (m)

ML Base

Pycnocline

Eddy Buoy. Flux

$\Delta y$

$\Delta z$
Prototype: Mixed Layer Front

Simple Spindown

Plus, Diurnal Cycle and KPP
Prototype: Mixed Layer Front

Simple Spindown

Plus, Diurnal Cycle and KPP
Prototype: Mixed Layer Front

Simple Spindown

Plus, Diurnal Cycle and KPP
Prototype: Mixed Layer Front

Temperature on day:1

Simple Spindown

Plus, Diurnal Cycle and KPP

Overturning Streamfunction

Mixed Layer

z (m)

ML Base

Pycnocline

Eddy Buoy. Flux

y (km)
Prototype: Mixed Layer Front

Simple Spindown

Plus, Diurnal Cycle and KPP

Overturning Streamfunction

Mixed Layer

Eddy Buoy. Flux

Pycnocline

ML Base

y (km)
Prototype: Mixed Layer Front

Simple Spindown

Plus, Diurnal Cycle and KPP
Prototype: Mixed Layer Front

- Simple Spindown
- Plus, Diurnal Cycle and KPP

[3D diagrams showing temperature distribution and mixed layer streamfunction]
Prototype: Mixed Layer Front

Simple Spindown

Plus, Diurnal Cycle and KPP
Parameterization of Finite Amp. Eddies: Ingredients

Finite Amplitude

$N^2/f^2$ vs. time (days)
Parameterization of Finite Amp. Eddies: Ingredients

Finite Amplitude

Eddy Velocity Saturates Near Mean KE

- basin-avg. pert. KE
- linear predict. pert. KE
- initial mean KE^2, 1/2(M^2 H/f)^2
- avg. pert. v^2 in front
Parameterization of Finite Amp. Eddies: Ingredients

**Finite Amplitude**

Vert. Excursions 
\( \left( \frac{b_{\text{rms}}}{N^2} \right) \) scale with \( H \)

**Eddy Velocity Saturates**

Near Mean KE

- basin–avg. pert. KE
- linear predict. pert. KE.
- initial mean KE\(^2\): \( \frac{1}{2}(M^2 H/f)^2 \)
- avg. pert. \( v^2 \) in front

**Time (days)**
Parameterization of Finite Amp. Eddies: Ingredients

Finite Amplitude

Vert. Excursions \( \left( \frac{b'_{\text{rms}}}{N^2} \right) \) scale with \( H \)

Eddy Velocity Saturates

Near Mean KE

Eddy Fluxes are at nearly 1/2 the mean isopycnal slope
Parameterization of Finite Amp. Eddies: Ingredients

Finite Amplitude

Linear Solution $<w'b'>$ for vert. structure.
As in Branscome '83...

$(b'_{rms}/N^2)$ scale with $H$

Eddy Velocity Saturates
Near Mean KE

Eddy Fluxes are at nearly 1/2 the mean isopycnal slope
The Parameterization:

\[ \Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{z} \]

\[ \mu(z) = \left[ 1 - \left( \frac{2z}{H} + 1 \right)^2 \right] \left[ 1 + \frac{5}{21} \left( \frac{2z}{H} + 1 \right)^2 \right] \]

Fox-Kemper et al. (08)

- The horizontal fluxes are downgradient:

\[ u'_H b' = - \frac{C_e H^2 \mu(z)}{|f|} \frac{\partial b}{\partial z} \nabla_H \bar{b} \]

- Vertical fluxes always upward to restratify with correct extraction rate of potential energy:

\[ w'b' = \frac{C_e H^2 \mu(z)}{|f|} |\nabla \bar{b}|^2 \]
It works for Prototype Sims:

Red: No Diurnal

Blue: With Diurnal

>2 orders of magnitude!

Circles: Balanced Initial Cond.
Squares: Unbalanced Initial Cond.
Works In 'Real' Models!

- HIM/GOLD and CCSM/POP
- Soon to be in MITgcm & MOM

![Graph showing probability distribution](image1)

**RMS**: 16m→8m, Skew 2.4→0.6
Summary I:

Submesoscale features, and mixed layer eddies in particular, exhibit large vertical fluxes of buoyancy often ignored in climate models.

A parameterization of mixed layer eddy fluxes as an overturning streamfunction is proposed. The magnitude comes from extraction rate of potential energy.

Many observations are consistent, and model biases are reduced. Biogeochemical effects are likely, as vertical fluxes and mixed layer depth are changed.

In HIM & CCSM, soon to be in MITgcm and MOM.

4 Papers so far... fox-kemper.com/research
What if it’s not a **Surface Mixed Layer**?
e.g., Deep Convection (versus Jones & Marshall)

Param gives same scaling, but...
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Param gives same scaling, but...

Jones & Marshall 97
What if it’s not a **Surface Mixed Layer**?

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- Param gives same scaling, but...

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*Jones & Marshall 97*

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3d, 5d, 10d, 30d, 50d
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e.g., Deep Convection (versus Jones & Marshall)

Param gives same scaling, but...

Jones & Marshall 97
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Vertical structure is different...
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Vertical structure is different...

Jones & Marshall 97
What if it’s not a **Surface Mixed Layer**?

*e.g., Deep Convection (versus Jones & Marshall)*

Vertical structure is different...
What if it’s not a **Surface** Mixed Layer? e.g., Deep Convection (versus Jones & Marshall)

Vertical structure is different...
Consider SQG Eady + Mix

Unmixed Layers

\[ z = 0 \]
\[ z = -H_u \]
\[ z = -H_d \]

Move these around

Mixed Layer

1 T-like sheet

Setup

<table>
<thead>
<tr>
<th>Uniform PV&amp;Shear, Hi ( N^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform PV&amp;Shear, Lo ( N^2 )</td>
</tr>
<tr>
<td>Uniform PV&amp;Shear, Hi ( N^2 )</td>
</tr>
</tbody>
</table>

1 T-like sheet

2 T-like sheets

1 T-like sheet

Could make mix as cooling or stirring.

Instability amounts to 6th order poly...

Possibly 3 growing modes and 3 decaying.
Consider SQG Eady + Cool 0–100m

Growth Rates

Mesoscale

Other

Submesoscale
Consider SQG Eady + Cool 100–200m

Mesoscale

Other

Submesoscale
Consider SQG Eady + Cool 200–300m
Consider SQG Eady + Cool 300-400m

Mesoscale

Other

Submesoscale
Consider SQG Eady + Cool 400–500m

Mesoscale

Other

Submesoscale
A parameterization of mixed layer eddy fluxes as an overturning streamfunction is proposed.

The parameterization reduces model bias in GOLD and POP.

However, difficulties arise in parameterization of submesoscale features if restratification isolates reduced stratification away from boundaries.

Preliminary testing/linear instability analysis reveals that extending submesoscale may be relatively easy, but preventing mesoscale double-counting will not.

4 Papers so far... fox-kemper.com/research
The Global Parameterization:

\[ \Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla b \times \hat{z} \]

\[ \mu(z) = \left[ 1 - \left( \frac{2z}{H} + 1 \right)^2 \right] \left[ 1 + \frac{5}{21} \left( \frac{2z}{H} + 1 \right)^2 \right] \]

Account for equator by going to subinertial ML approx (Young 94)

\[ \Psi = \frac{C_e H^2 \mu(z)}{\sqrt{f^2 + \tau^{-2}}} \nabla b \times \hat{z} \]

Account for coarse res. by assuming

\[ E_b(k) \sim k^{-2} \rightarrow \Psi = \left[ \frac{L_f}{\Delta x} \right] \frac{C_e H^2 \mu(z)}{\sqrt{f^2 + \tau^{-2}}} \nabla b \times \hat{z} \]

Obs. reveal \( L_f \sim R_d \)
Estimate of Vert. Heat Flux from satellite data agrees
Changes To Mixing Layer Depth in Eddy-Resolving Southern Ocean Model
Changes To Mixing Layer Depth in Eddy-Resolving Southern Ocean Model
What does it look like?

Parameterization (2d, 10km grid)

7d0th from 2d parameterization

Submesoscale-Resolving (3d, 500m grid)

7d0th from 3d MITgcm (smoothed)

$N^2$

Overturning Streamfunction

Mixed Layer

ML Base

Pycnocline
What does it look like?

Parameterization (2d, 10km grid)

Submesoscale-Resolving (3d, 500m grid)

Overturning Streamfunction

Mixed Layer

ML Base

Pycnocline
What does it look like?

Parameterization (2d, 10km grid)
16d00h from 2d parameterization

Submesoscale-Resolving (3d, 500m grid)
16d00h from 3d MITgcm (smoothed)