Effect of near-inertial modes on the midlatitude double gyre problem

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OUTLINE

1. Motivation
2. Model Setup
3. Results
4. Conclusion

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Motivation: Balanced-to-Unbalanced Energy Transfers

- Simple example: take 2d turbulence as “balanced flow"; and 3d modes as “unbalanced"
- More involved: take QG turbulence as “balanced"; and inertia-gravity modes as “unbalanced" (e.g., Ngan *et al.*, 2007)

Motivation: Balanced-to-Unbalanced Energy Transfers

- Simple example: take 2d turbulence as “balanced flow"; and 3d modes as “unbalanced"
  - unstratified thin (hydrostatic) fluid
  - but allowing $\mathbf{u}$ to vary in z
  - double gyre forcing on depth averaged mode
  - large scale stochastic forcing on vertically varying mode

Model Equations

$$u_t \quad + \quad \nabla \cdot (\mathbf{v}u) - fv = -P_x + F^x - D^x$$

$$v_t \quad + \quad \nabla \cdot (\mathbf{v}v) + fu = -P_y - D^y$$

$$\mathbf{v} \quad \equiv \quad (u, v, w) \qquad \nabla_3 \cdot \mathbf{v} = 0 \qquad P_z = 0$$

- dynamics: hydrostatic (but unstratified)
- domain: 4000km square box ; $513 \times 513 \times 2$
- $f = f_0 + \beta y$ with $f_0 = 7.5 \times 10^{-5}$s^{-1}, $\beta = 2 \times 10^{-11}$m^{-1}s^{-1}
- hyperviscosity (∇^6) with slip conditions at boundaries
- Rayleigh drag on 2d mode
- Forcing: double gyre for 2d mode. Stochastic (and large scale) for 3d modes
- $\delta_i = (U_{Sv}/\beta)^{1/2} \sim 44$km $\sim$ 5.5dx

Model Equations

$$\nabla^2\psi_t + J(\psi, \nabla^2\psi + \beta y) = -\overline{\mathbf{v}' \cdot \nabla\zeta'} - \overline{\mathbf{v}'_x \cdot \nabla v'} + \overline{\mathbf{v}'_y \cdot \nabla u'} + F - D$$

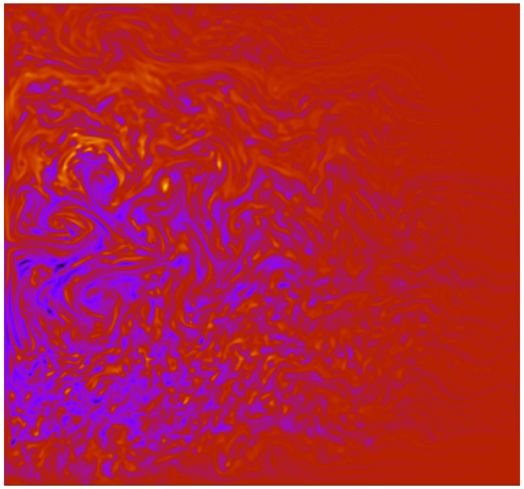
$$u'_t + \mathbf{v} \cdot \nabla u - \overline{\mathbf{v} \cdot \nabla u} - fv' \;=\; F^x - D$$

$$v'_t + \mathbf{v} \cdot \nabla v - \overline{\mathbf{v} \cdot \nabla v} + fu' \;=\; -D$$

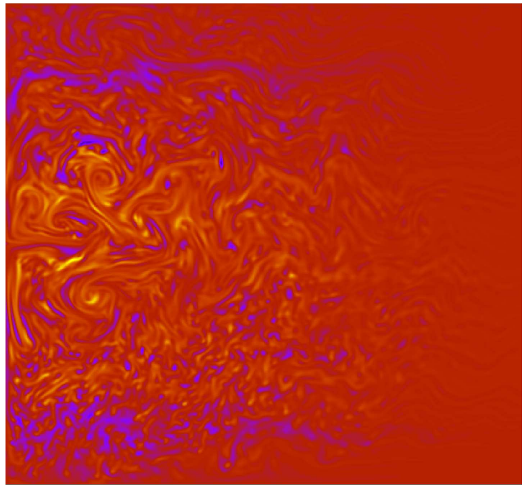
- C-grid
- Multigrid method for elliptic inversion (L-P Nadeau)
- $F^x = a(t)$sin(y/L)$e^{-(x^2+y^2)/(0.5L)^2}$
- a(t) given by an Ornstein-Ulenbeck process (with characterictic timescale of 1 day)

A day in the life of u'

NOON



8PM



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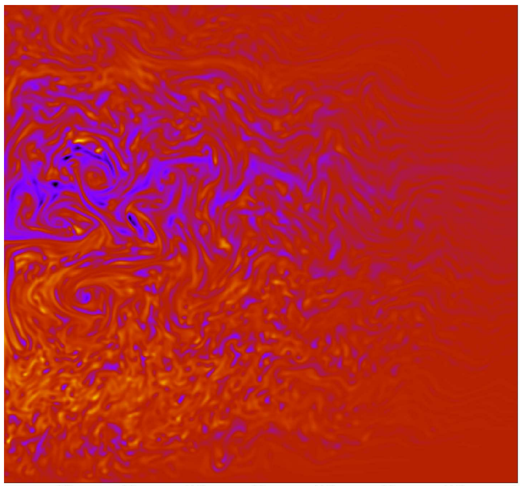
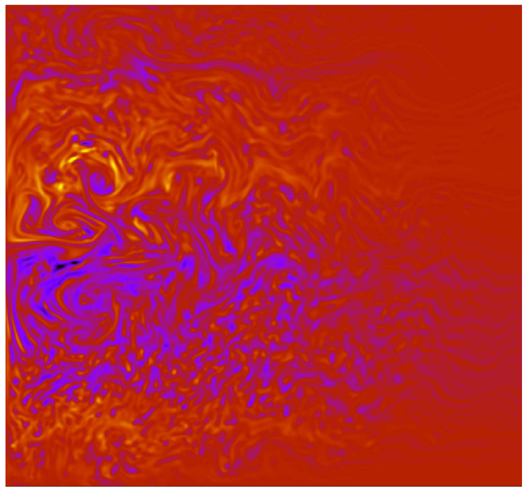
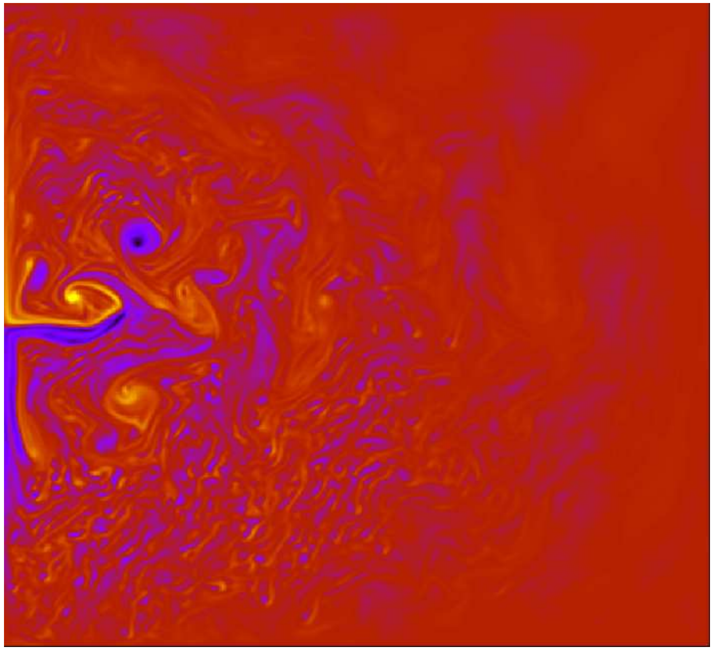


NOON (next day)

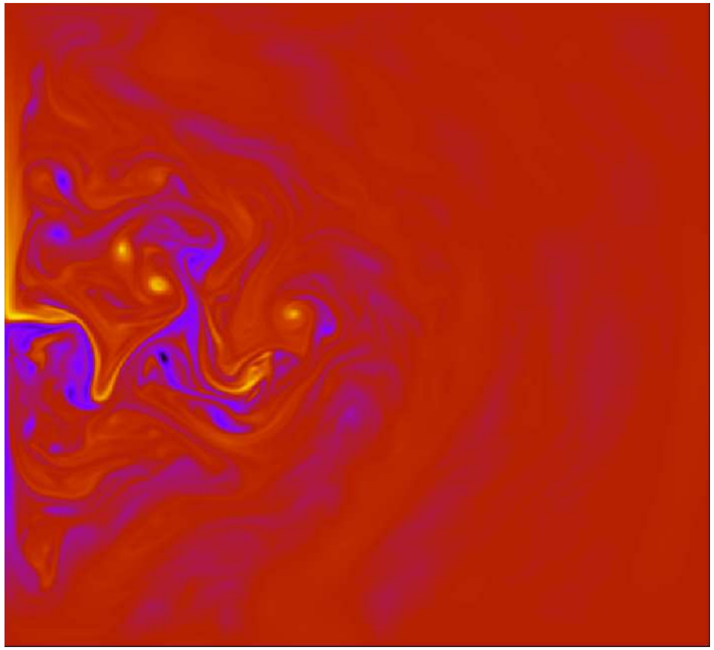


Vertical Vorticity ($r_{\text{Rayleigh}} = 5 \times 10^{-8}\text{s}^{-1}$)

With 3d Forcing



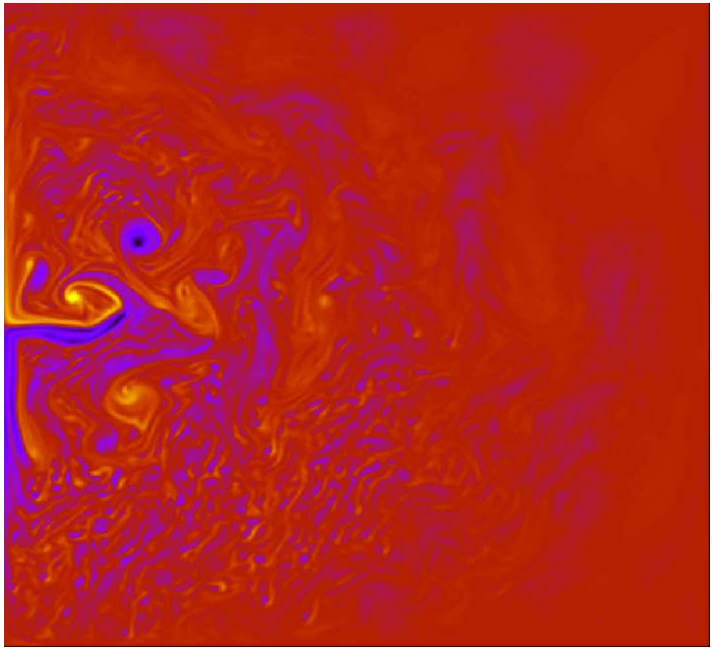
Without 3d Forcing

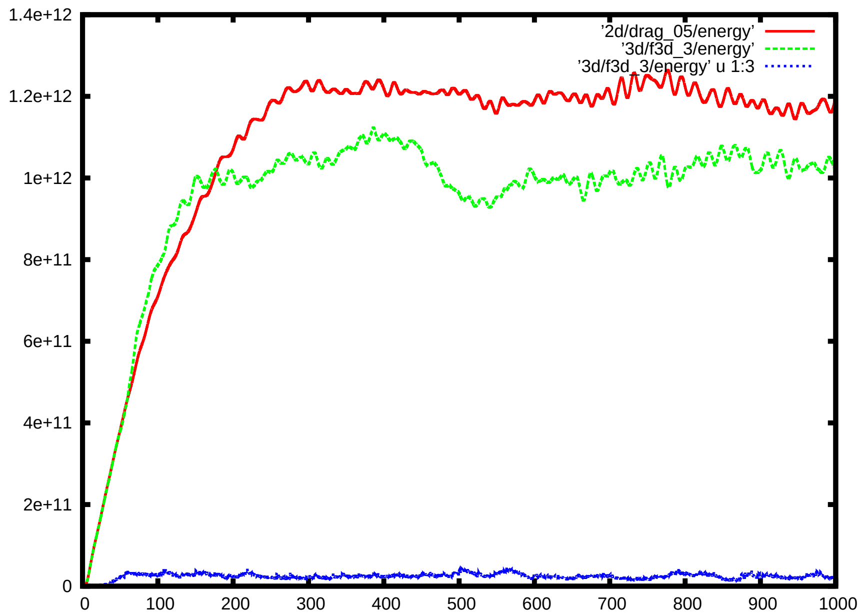


Vertical Vorticity and Energy Time Series

With 3d Forcing

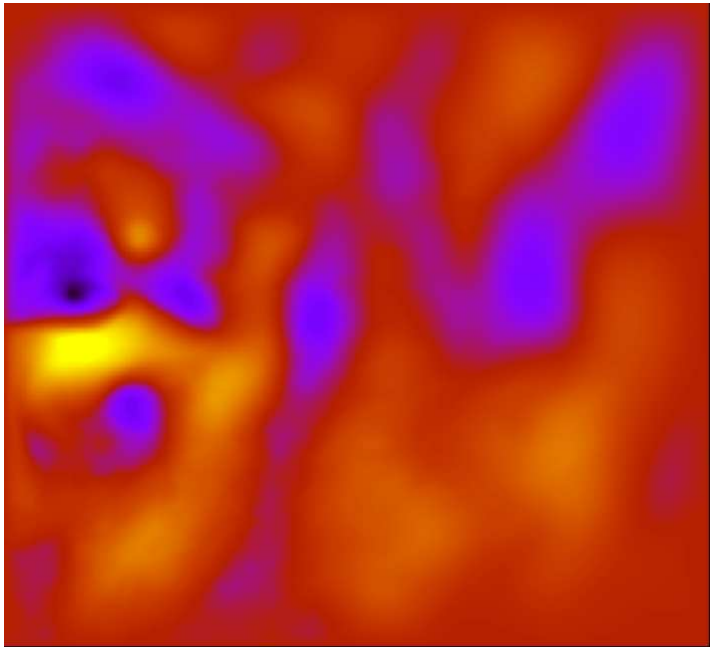
E(t)



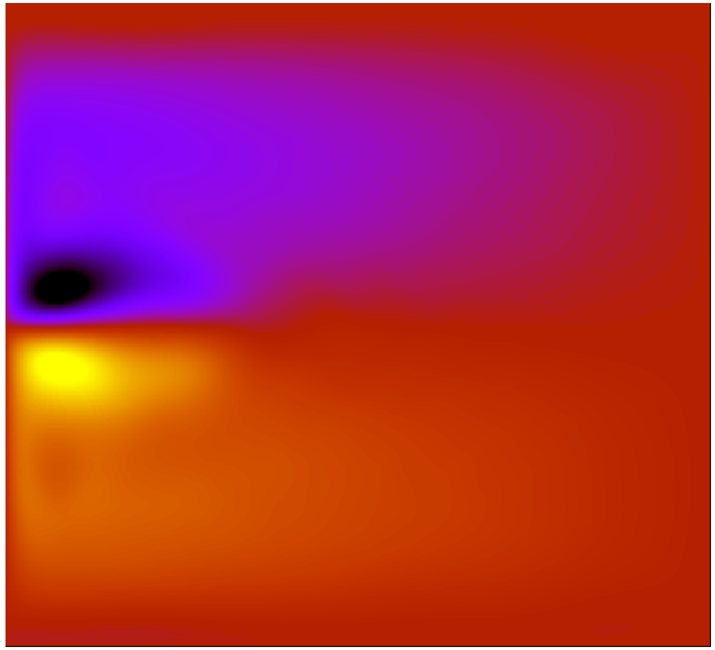


Streamfunctions

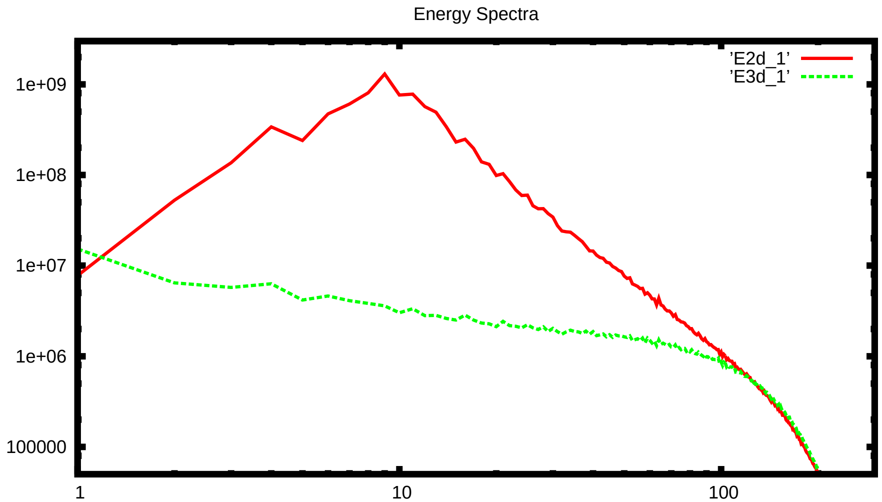
Typical Streamfunction

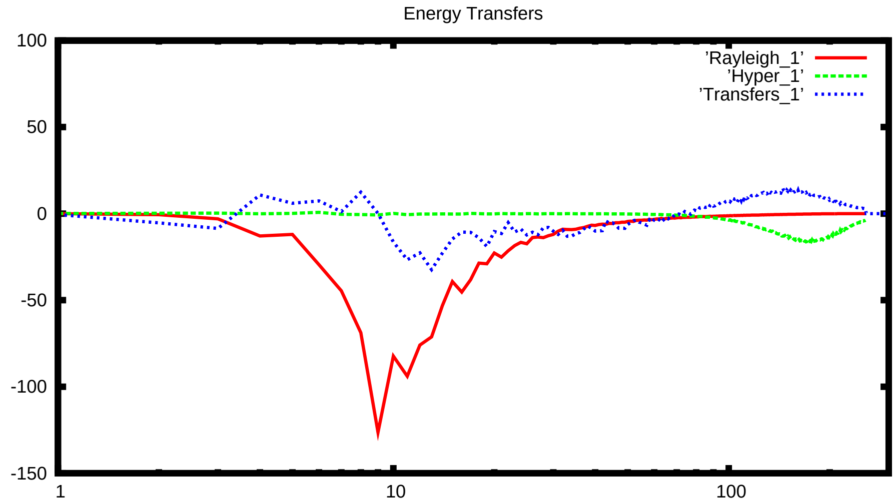


Time mean



Power, dissipation and transfer spectra





Red: $E_2d(k)$

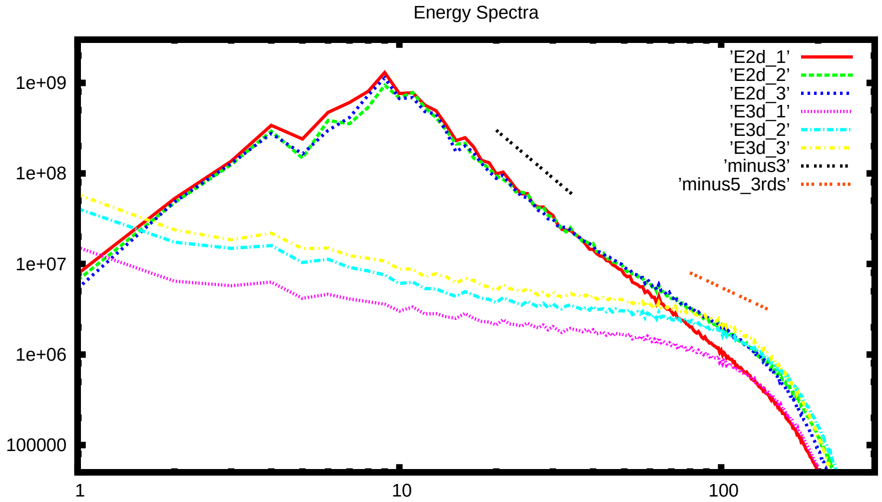
Green: $E_3d(k)$

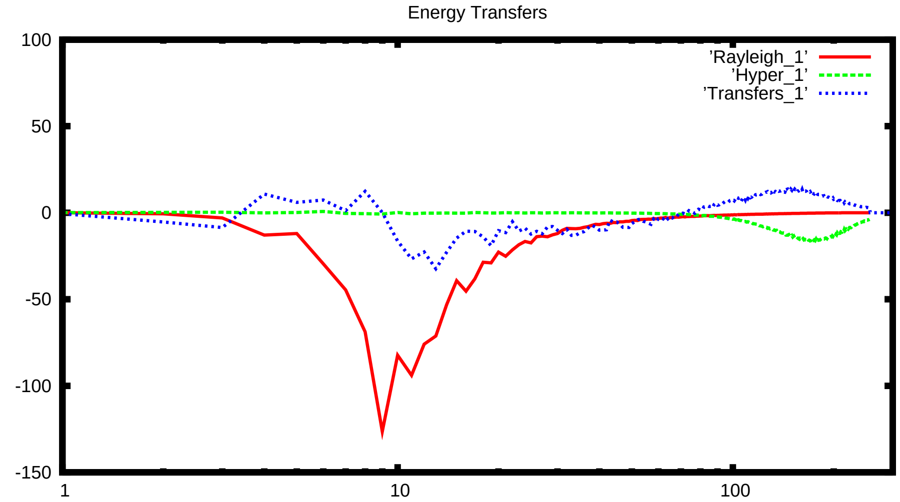
Red: Rayleigh Drag

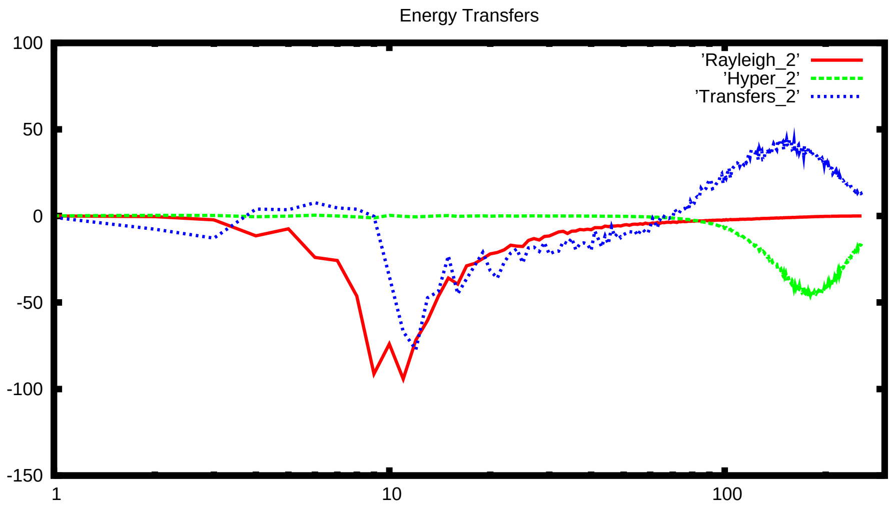
Blue: 2d-to-3d transfers

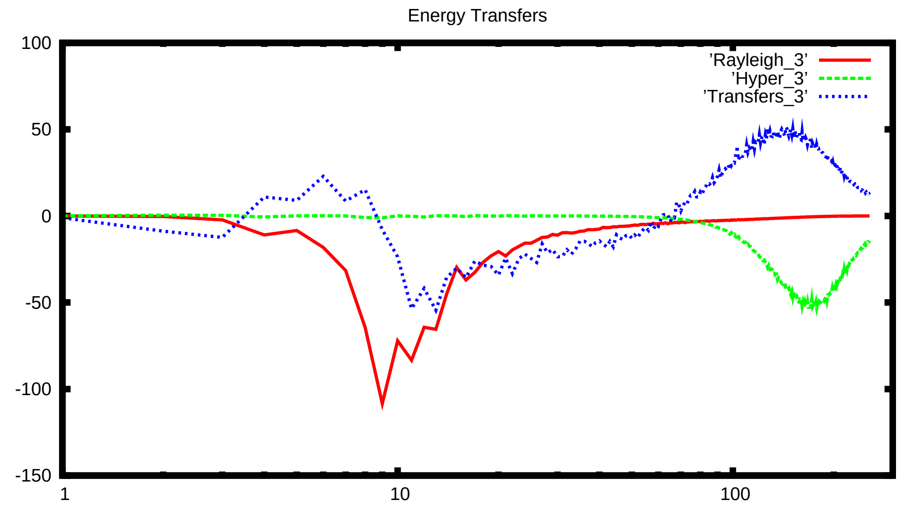
Green: Hyperviscosity

Dependence on level of 3d forcing



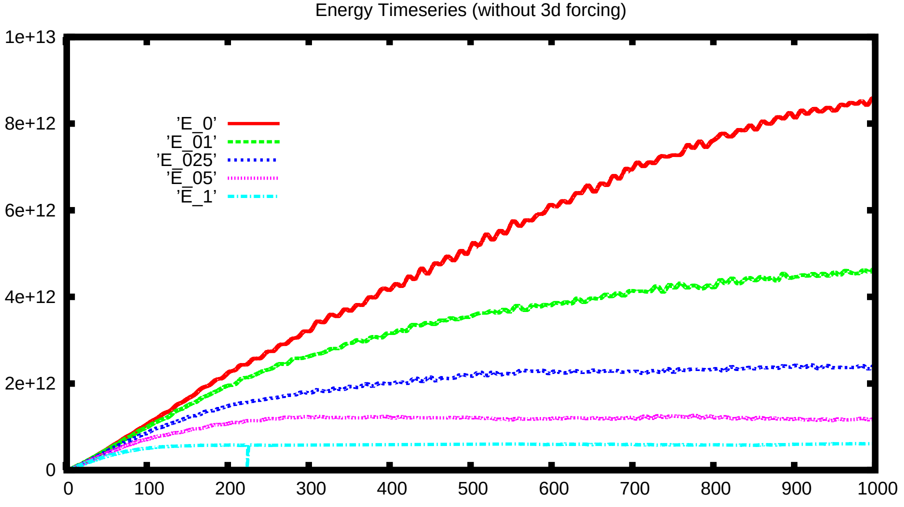




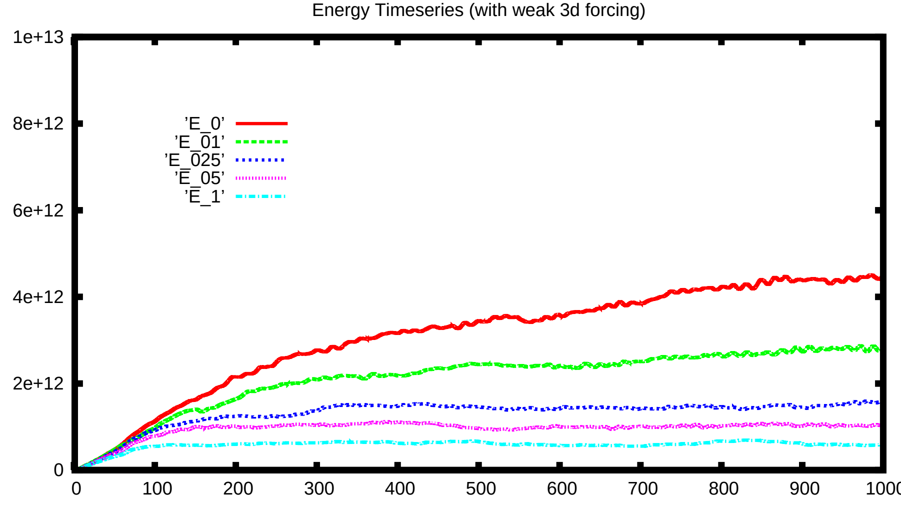


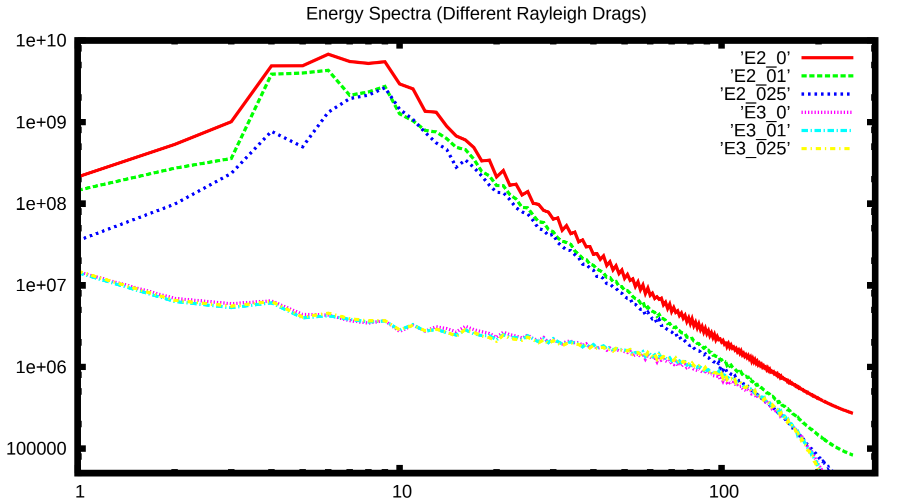
$$r_{\text{Rayleigh}} = (0, 1, 2.5, 5, 10) \times 10^{-8}\mathbf{s}^{-1}$$

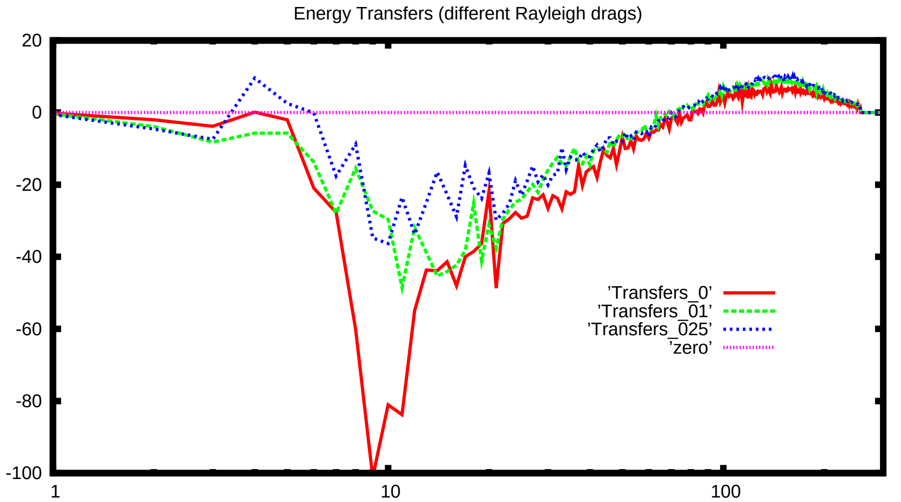
Without 3d forcing



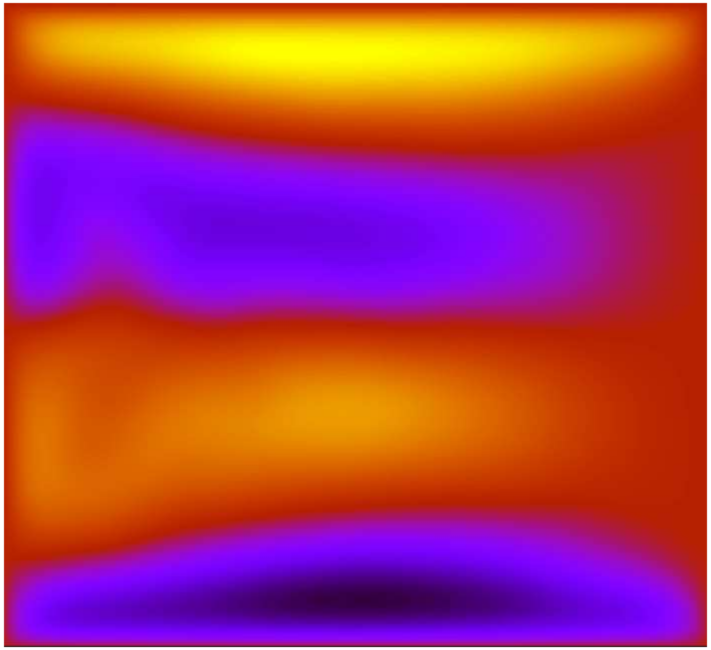
With (weak) 3d forcing



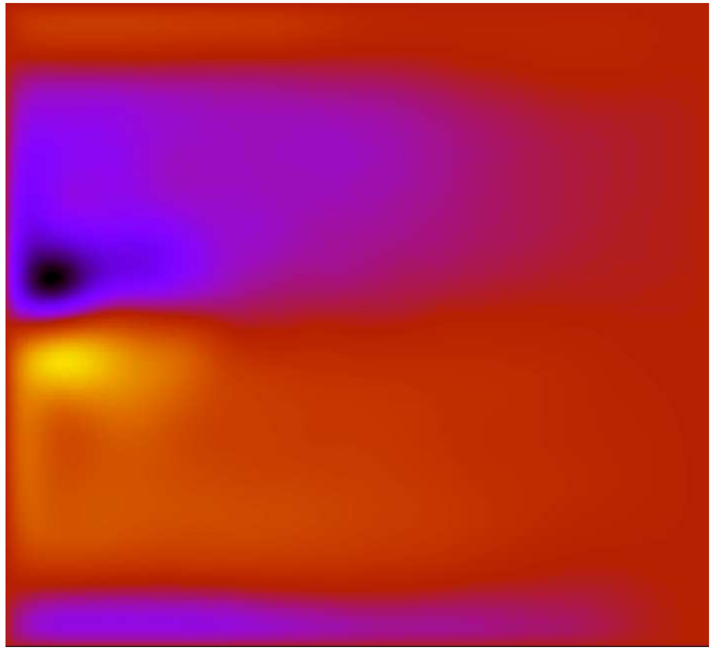




Zero Rayleigh Drag (mean streamfunctions)



Without 3d Forcing



With 3d Forcing

Conclusions

- When externally forced, geostrophic-to-inertial mode transfers can have a significant damping effect on the geostrophic motion.
- The effect is larger for stronger forcing of the inertial modes and weaker Rayleigh friction
- Stratified case: high vertical resolution is needed (i.e., $k_z > N/U$ or $Nh/fL \leq O(1)$ for $O(1)R_0$)

Rotating / Stratified Case

- thin aspect ratio $\longrightarrow$ hydrostatic
- Horz. PGF $\sim g\tilde{\rho}k_h/\rho_0 k_z$
- HPGF small if

$$N^2 \ll k_z^2 U^2 \quad \text{(assuming } T \sim L/U\text{)}$$

or

$$N^2 k_h^2 \ll f^2 k_z^2 \quad \text{(assuming } R_0 \sim O(1)\text{)}$$

Model Equations

$$\nabla^2\psi_t + J(\psi, \nabla^2\psi + \beta y) = -\overline{\mathbf{v}' \cdot \nabla\zeta'} - \overline{\mathbf{v}'_x \cdot \nabla v'} + \overline{\mathbf{v}'_y \cdot \nabla u'} + F - D$$

$$u'_t - (f + \nabla^2\psi)v' - \zeta'\overline{v} \;\; = \;\; -(\overline{u}u' + \overline{v}v')_x + F^x - D^x$$

$$v'_t + (f + \nabla^2\psi)u' - \zeta'\overline{u} \;\; = \;\; -(\overline{u}u' + \overline{v}v')_y - D^y$$

- C-grid
- Multigrid method for elliptic inversion (L-P Nadeau)
- $F^x = a(t)$sin(y/L) or $F^x = a(t)$sin(y/L)$e^{-0.25(x^2+y^2)/L^2}$
- a(t) given by an Ornstein-Ulenbeck process (with characterictic timescale of 1 day)