Customized Approximation with Radial Basis Functions

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Radial basis functions (RBFs) are a fundamental tool for approximating data sampled at scattered locations in two and higher dimensions. Since their discovery by R. Hardy in the early 1970s for cartographic purposes, both the knowledge about RBFs and their range of applications have grown tremendously. A survey of the present literature shows RBFs being used in, for example, geophysics, artificial intelligence, medical imaging, and finance.

Recently it has been shown that RBF approximants can be ``customized'' so that they preserve certain physical properties that may be known a priori about the sampled data. For example, if the data comes from a function whose integral over some region in space is known, then an RBF approximant can be constructed with this same integral. This is important for conserving mass. Another example is if the data comes from a vector field which is known to be divergence-free or curl-free (or both), then a vector-valued RBF approximant can also be constructed with either (or both) of these properties. This is important when approximating velocity, magnetic, or electric fields. Furthermore, all of these ideas can be extended to scalar or vector-valued data sampled on the sphere. We briefly describe how to customize RBFs for each of these examples and then present numerical results for some geophysical applications.