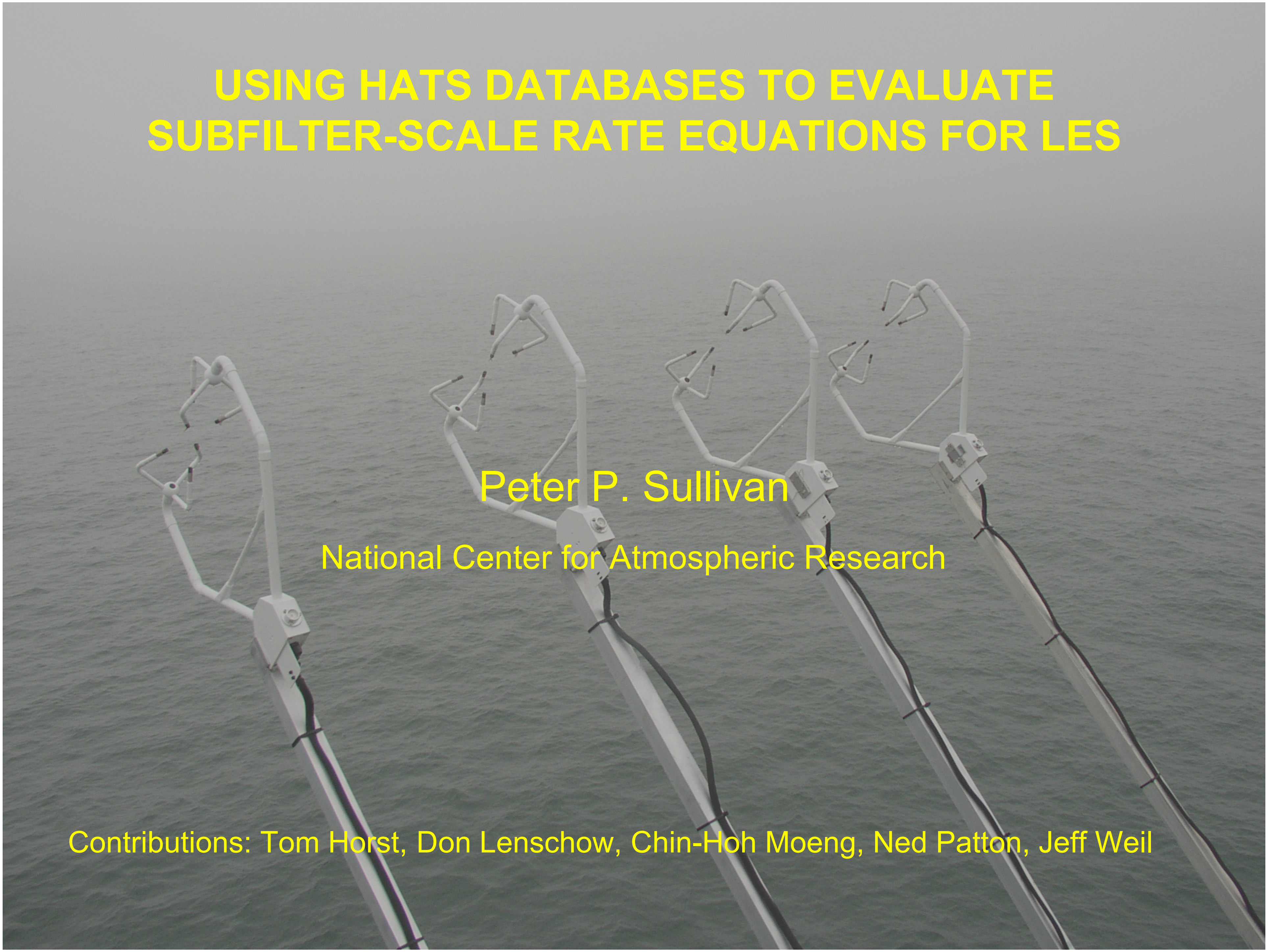

USING HATS DATABASES TO EVALUATE SUBFILTER-SCALE RATE EQUATIONS FOR LES

Peter P. Sullivan

National Center for Atmospheric Research

Contributions: Tom Horst, Don Lenschow, Chin-Hoh Moeng, Ned Patton, Jeff Weil

LARGE-EDDY SIMULATIONS AND OBSERVATIONS

Gulf of Tehuantepec U ~[20-25] m/s

Why LES of the PBL:

- Outdoor 4-D measurements are challenging
- Unsteady nature of the atmosphere and ocean
- Systematic investigation of the parameter space
- *Advances in parallel computing*

Validating/Improving LES with observations:

- Test the output
- Test the input subgrid-scale parameterizations

LES APPLICATIONS AND THE PBL

- Turbulence dynamics, stratification, entrainment
- Surface-atmosphere interactions
- Dispersion, chemistry
- Clouds
- ...

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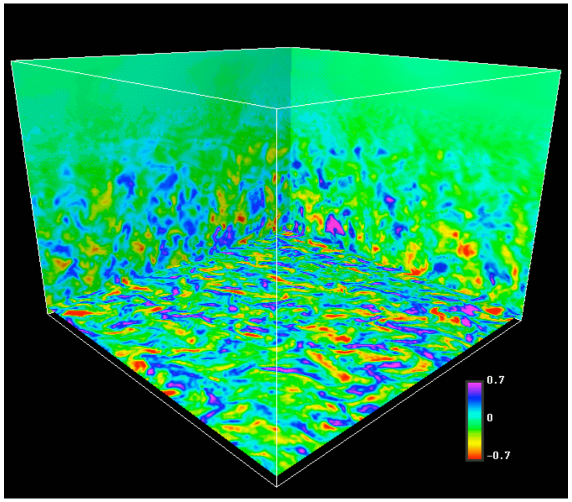
Stable boundary layers zi/L ~ 1.2



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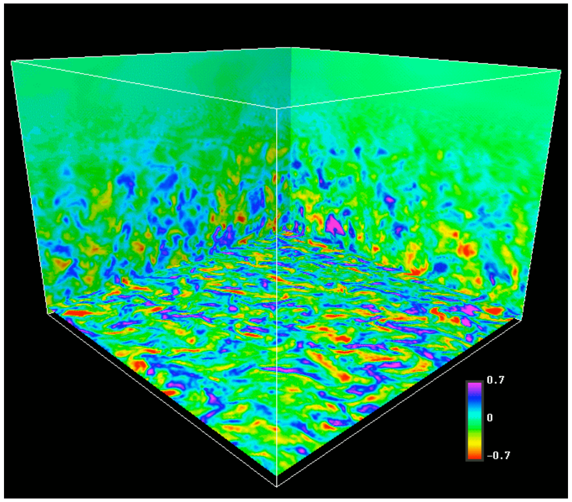

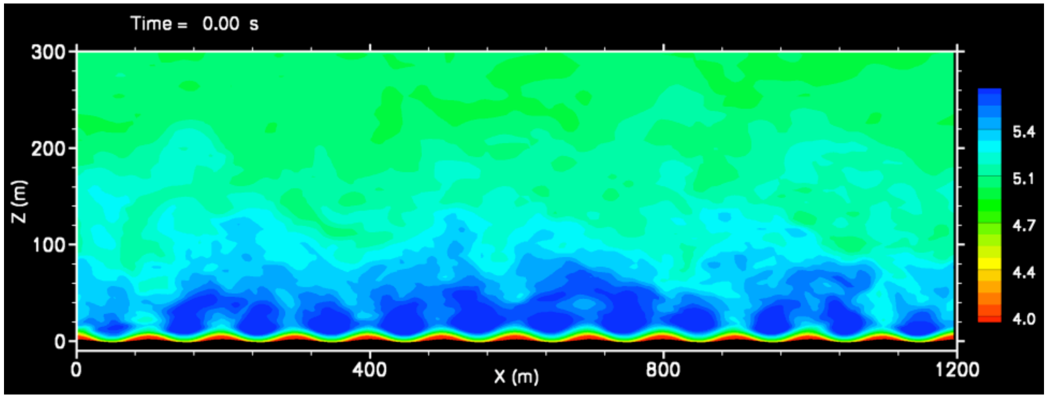

Non-equilibrium winds and waves

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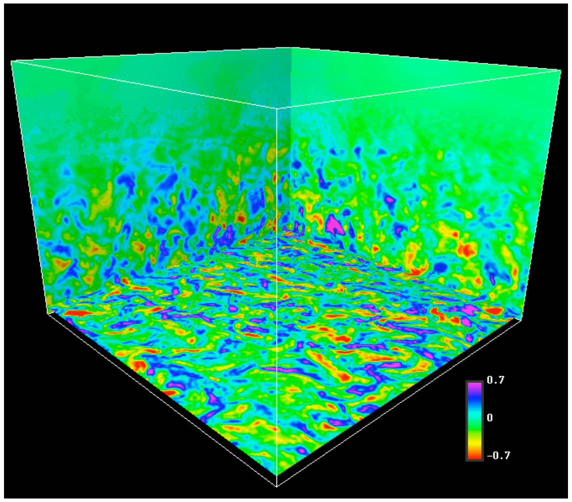
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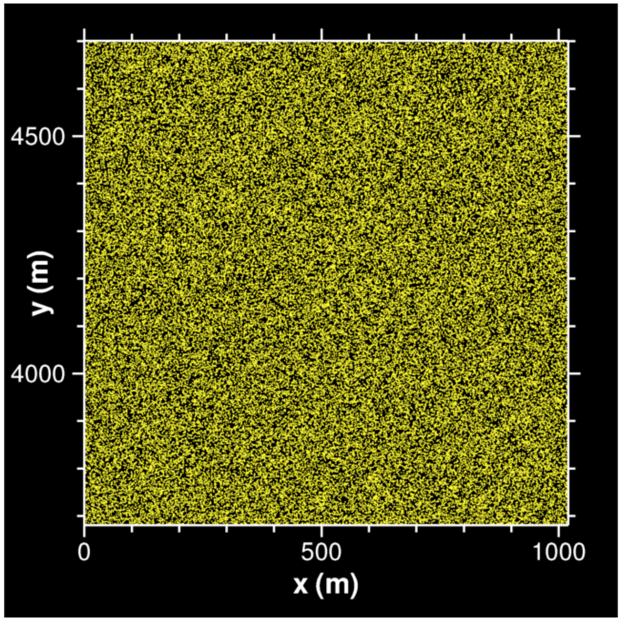
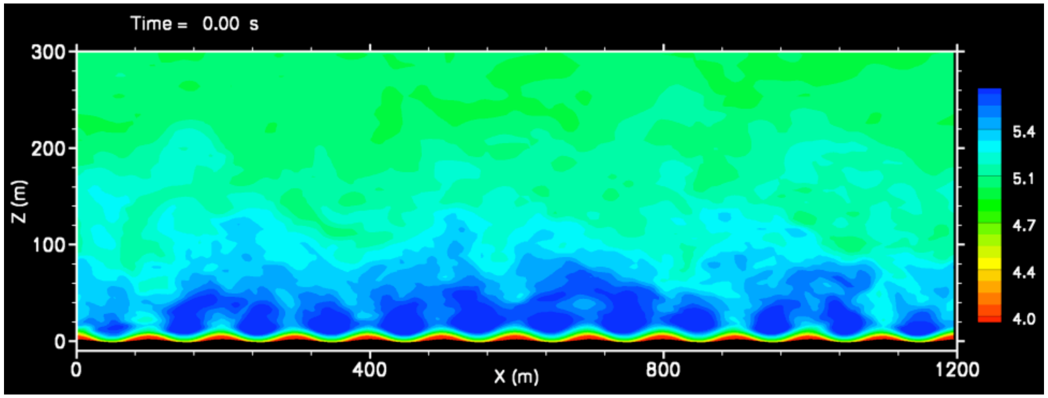


Plumes and dust-devils in a convective PBL 1024^3 simulation on 4096 cpus



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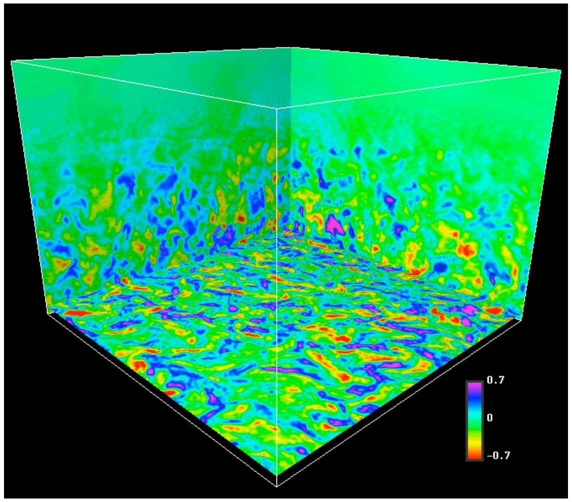
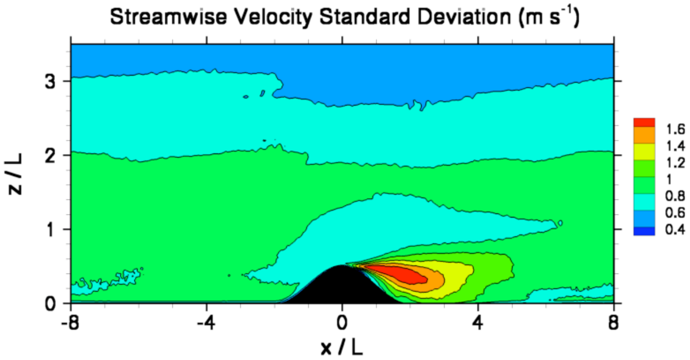
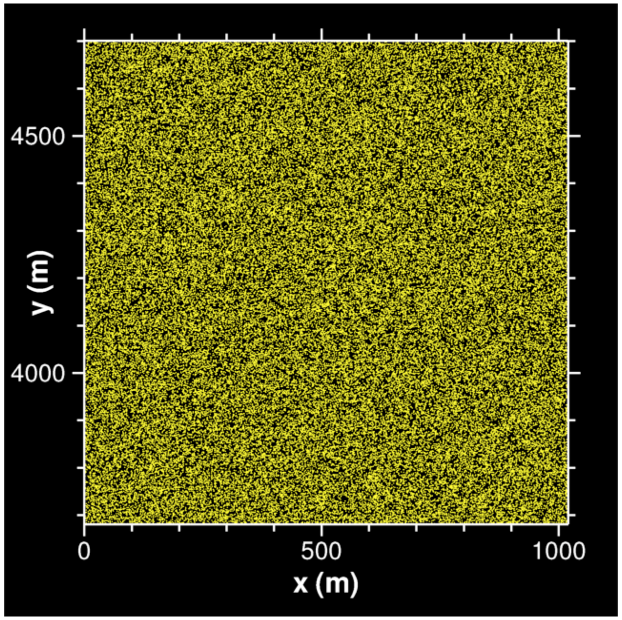


Idealized terrain





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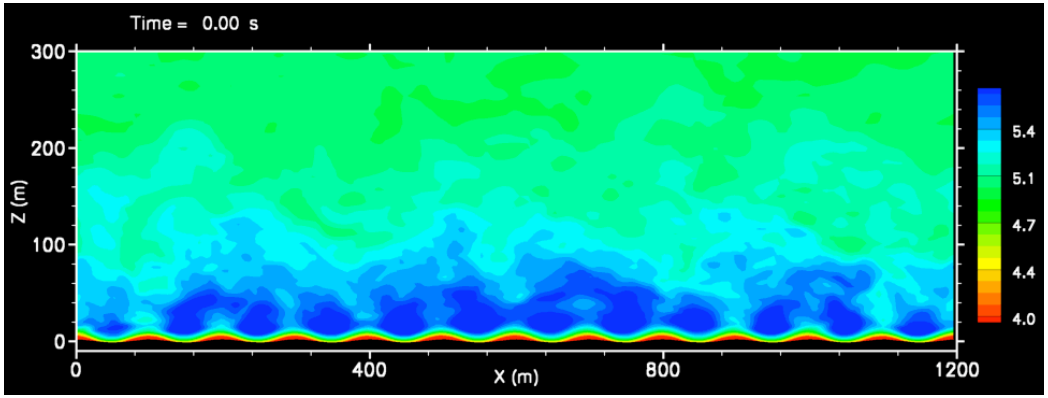


Non-equilibrium winds and waves

LES EQUATIONS FOR DRY ATMOSPHERIC PBL

Momentum

$$\frac{D\bar{\mathbf{u}}}{Dt} = -\,\mathbf{f}\times\bar{\mathbf{u}} - \nabla\pi + \hat{\mathbf{z}}g\frac{\bar{\theta}}{\theta_*} - \nabla\cdot\mathbf{T}$$

Scalar

$$\frac{D\bar{b}}{Dt} = -\nabla\cdot\mathbf{B}$$

TKE

$$\frac{De}{Dt} = -\mathbf{T}:\mathbf{S} + \mathbf{B}\cdot\hat{\mathbf{z}} - \mathcal{E} + \nabla\cdot(2\nu_t\nabla e)$$

Subgrid-scale momentum and scalar fluxes

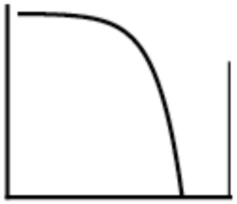
$$\begin{aligned}\mathbf{T} &= \overline{u_i\,u_j} - \overline{u_i}\,\overline{u_j}\\ \mathbf{B} &= \overline{u_i\,b} - \overline{u_i}\,\bar{b}\end{aligned}$$

Random variables, require a parameterization

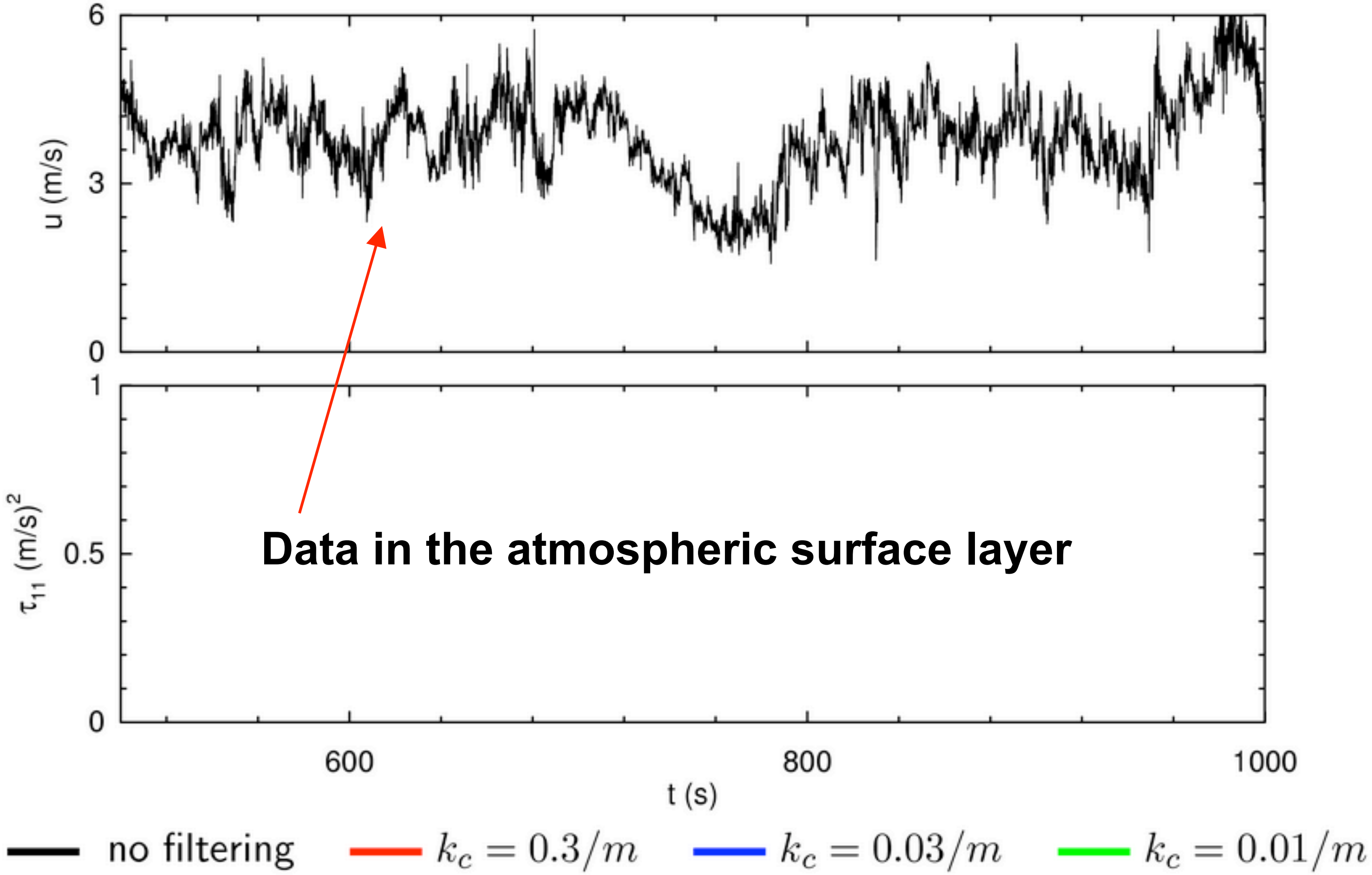
SIMPLE (CHEAP) FILTERING EXAMPLE …

$$\tau_{11} \;=\; \overline{u_1 u_1} - \overline{u}_1\,\overline{u}_1$$

MOVING BETWEEN DNS $\Longleftrightarrow$ LES $\Longleftrightarrow$ RANS

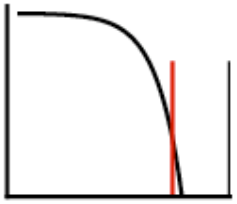


What happens to $\overline{u}_i$ and $\mathcal{T}_{ij}$ as we vary the filter cutoff k_c ?



Data in the atmospheric surface layer

MOVING BETWEEN DNS $\Longleftrightarrow$ LES $\Longleftrightarrow$ RANS



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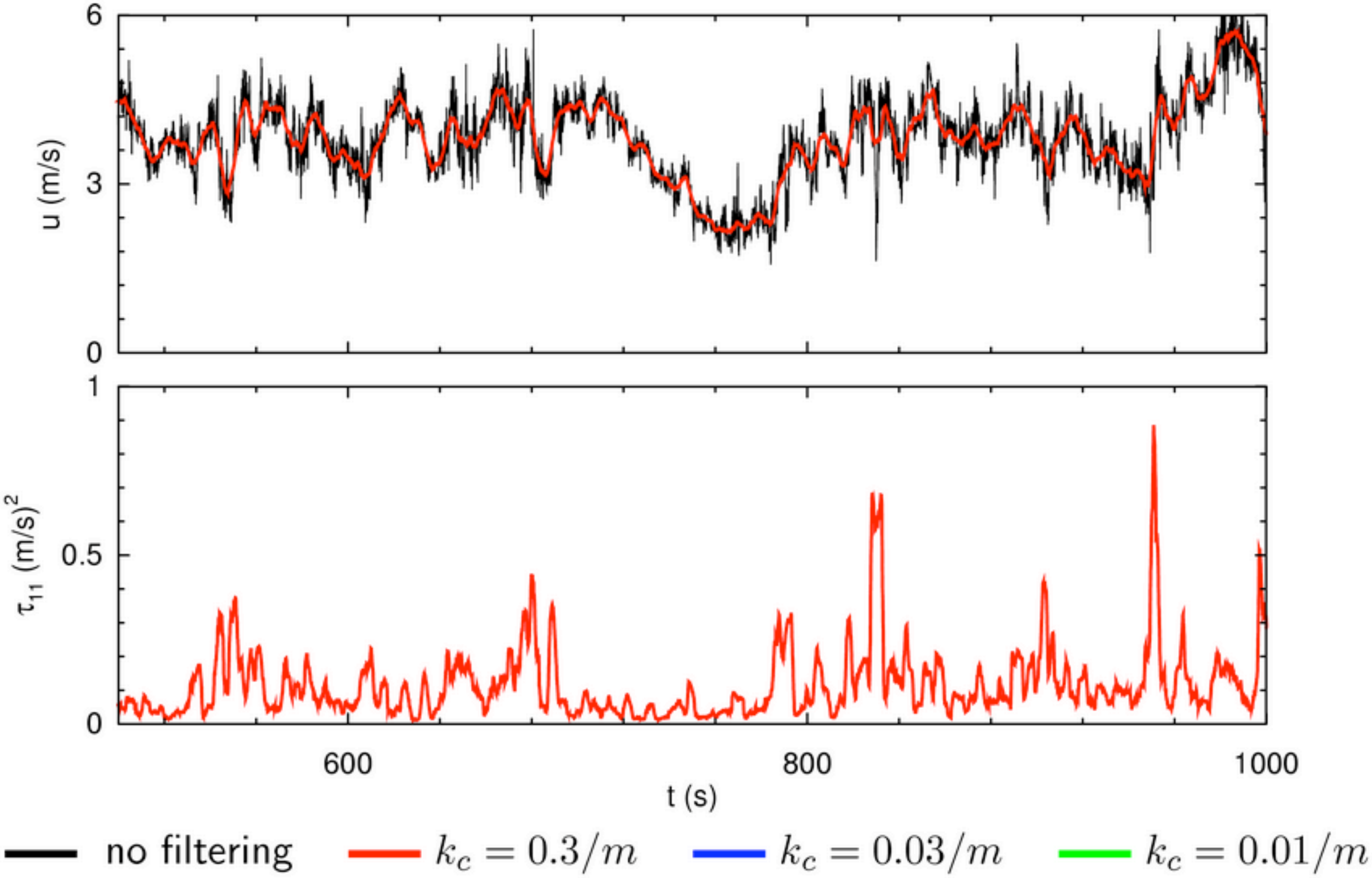

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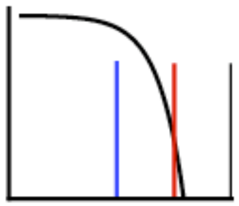

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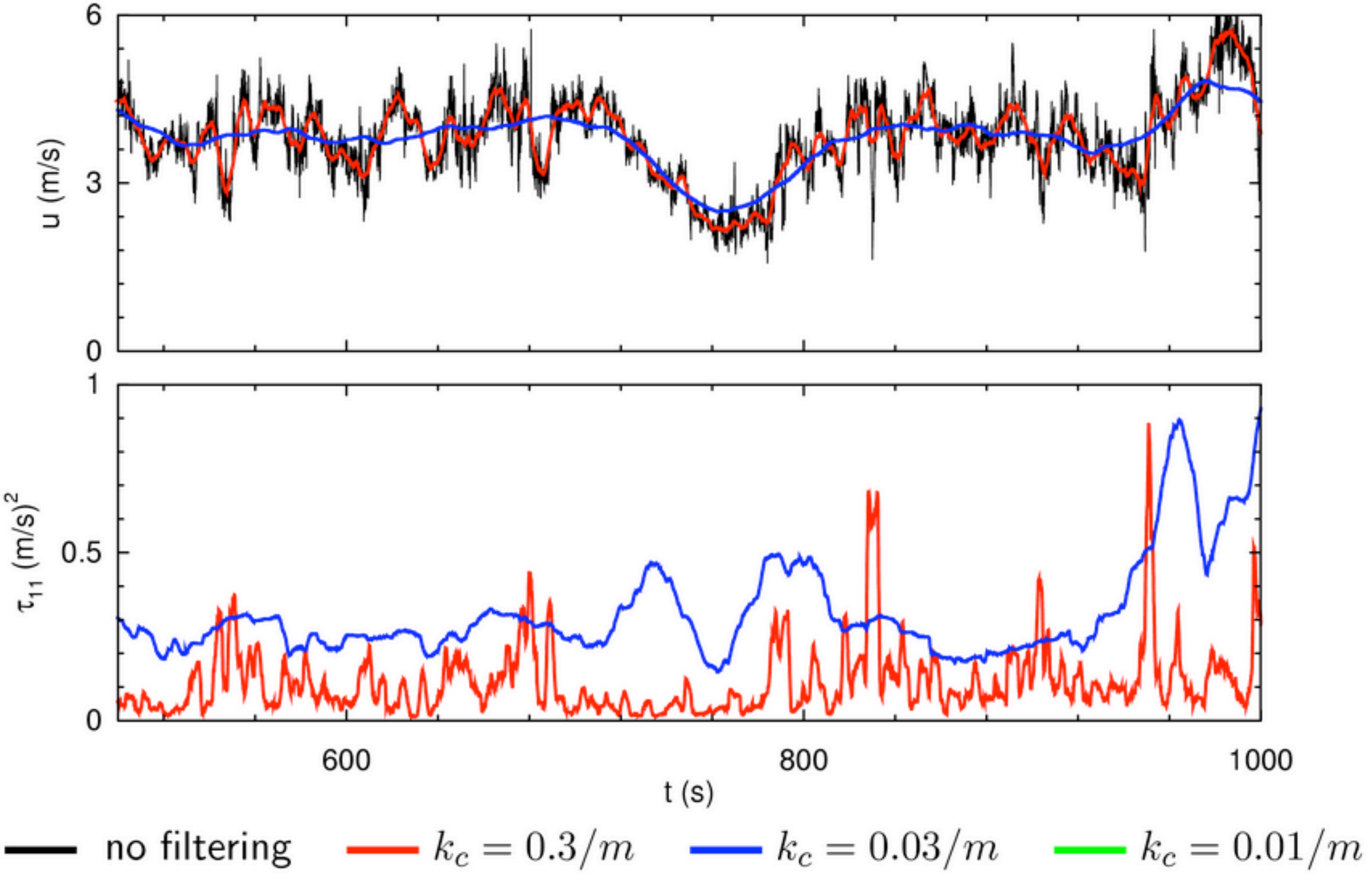

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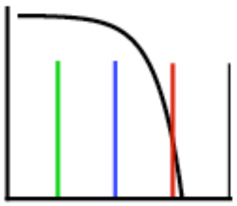

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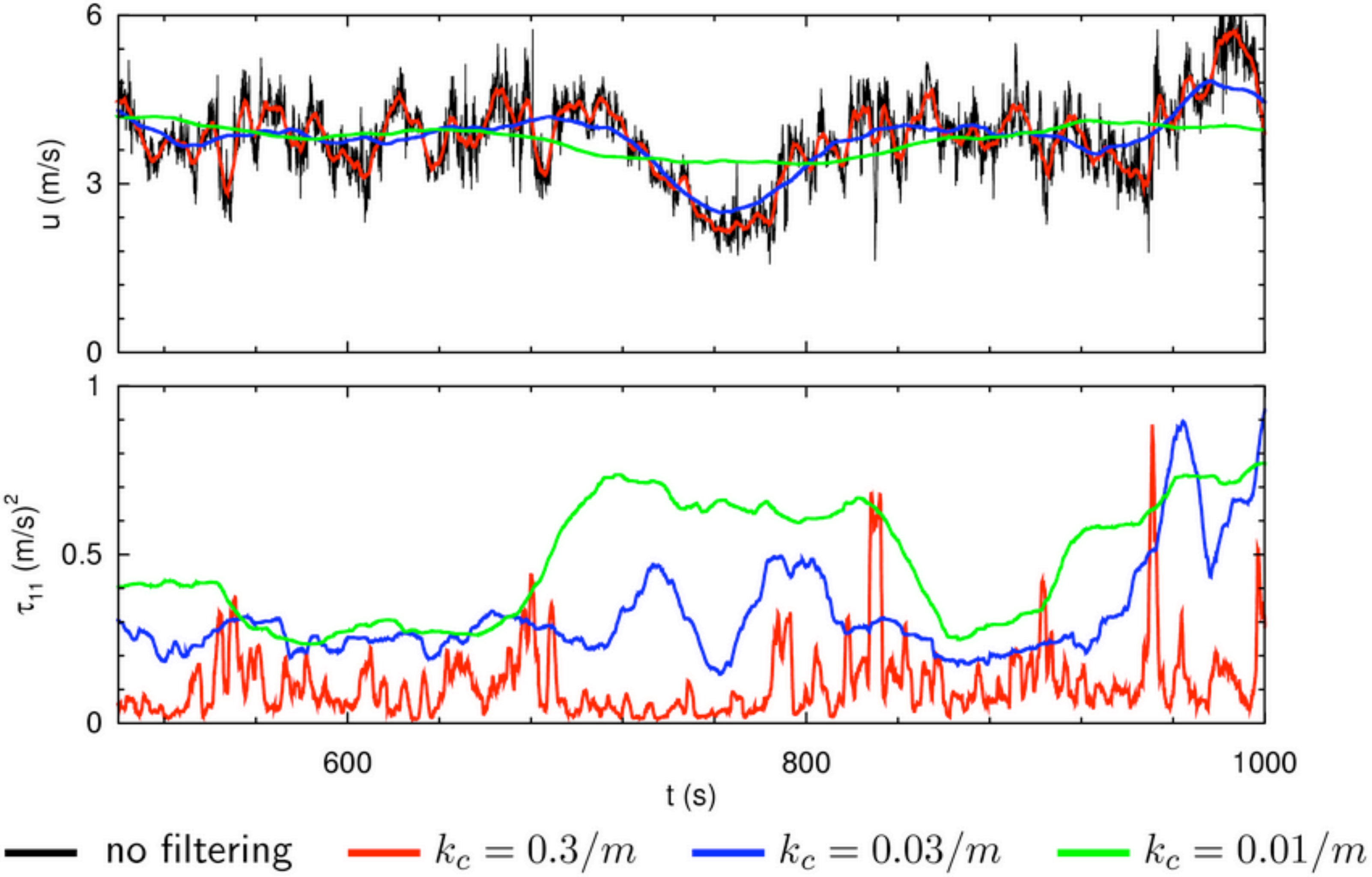

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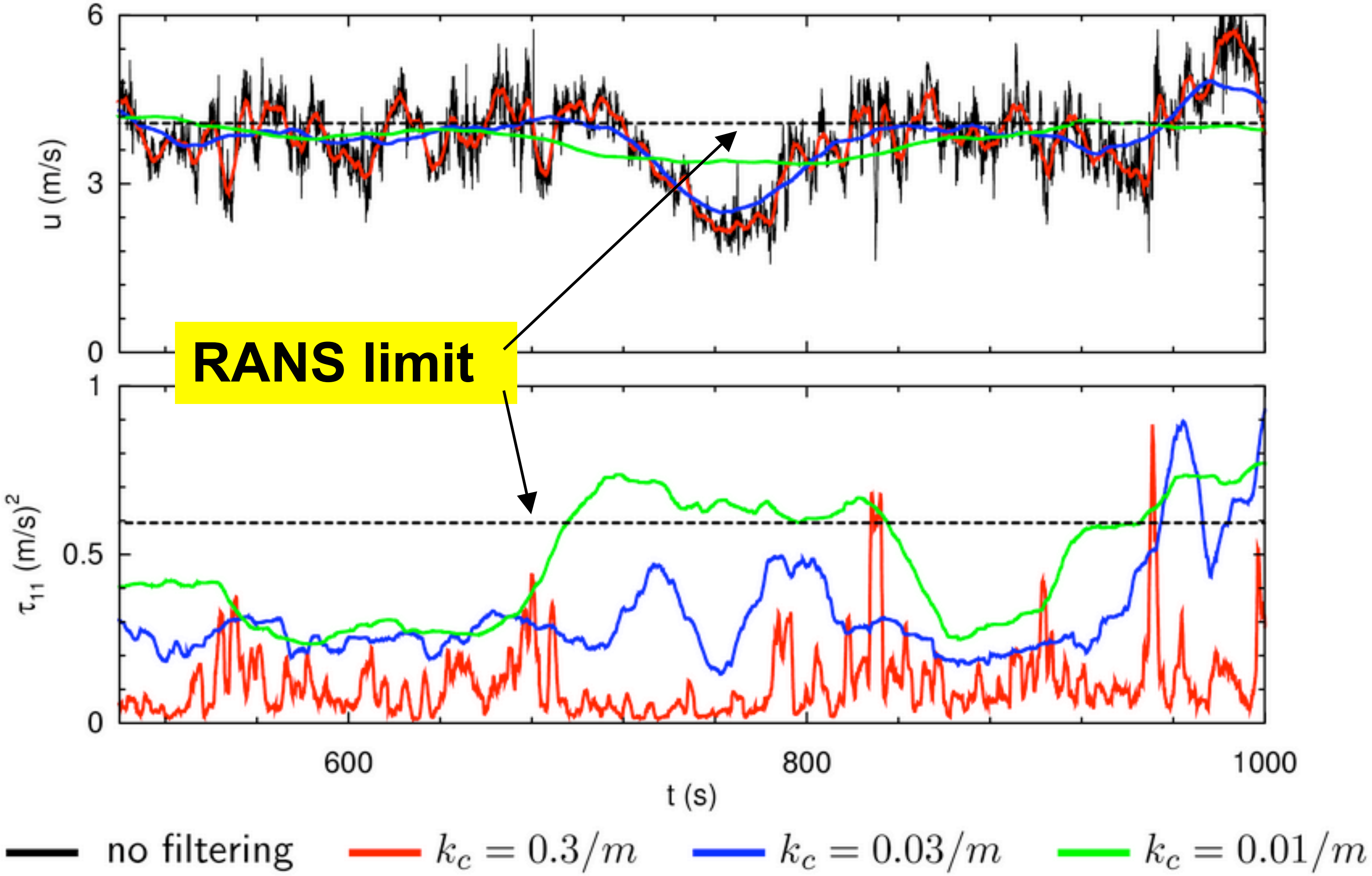
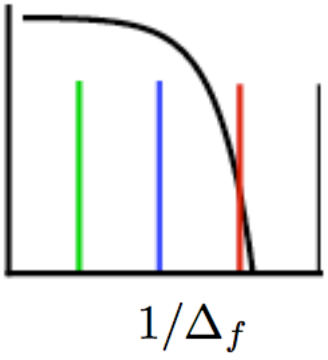


RANS limit

HIGH REYNOLDS NUMBER OBSERVATIONS AND LES

- **SINGLE-POINT MEASUREMENTS**
  - Cannot be used directly to improve LES
- **MULTI-POINT MEASUREMENTS**
  - Span a range of filter widths, $e.g.$, $\mathcal{O}$(m) to $\mathcal{O}$(100m)
  - Ideally 3-D, time varying "volume" of turbulence and scalars in canonical flows with shear, stratification, near boundaries, ...
  - Horizontal Array Turbulence Study field campaigns, HATS (2000), OHATS (2004), CHATS (2007), AHATS (2008)

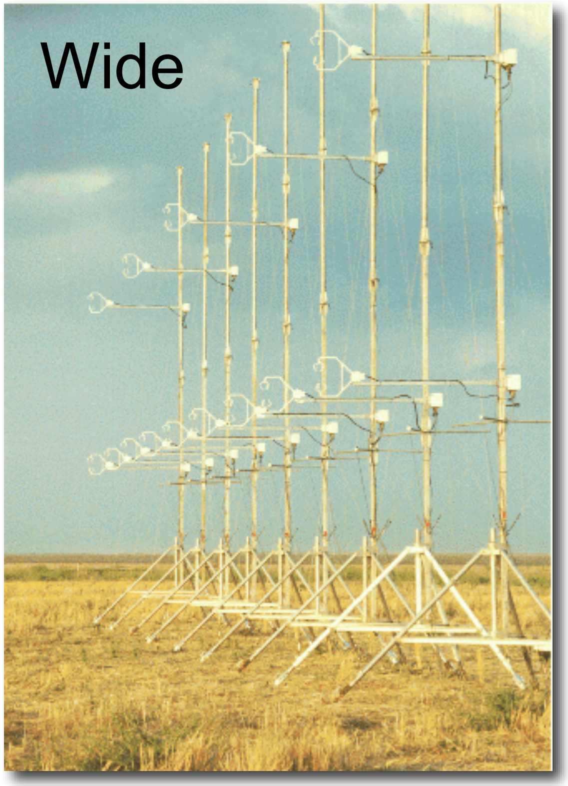


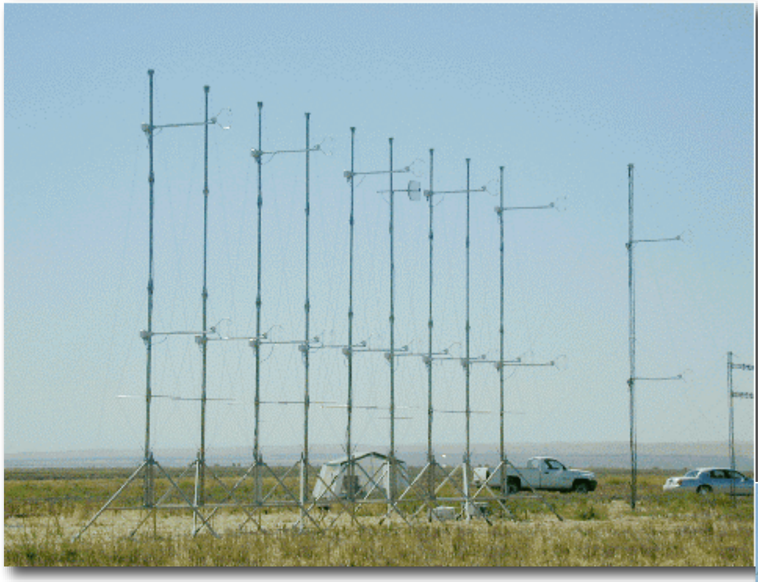
HATS CONFIGURATIONS

$\sim 36 \; cases$

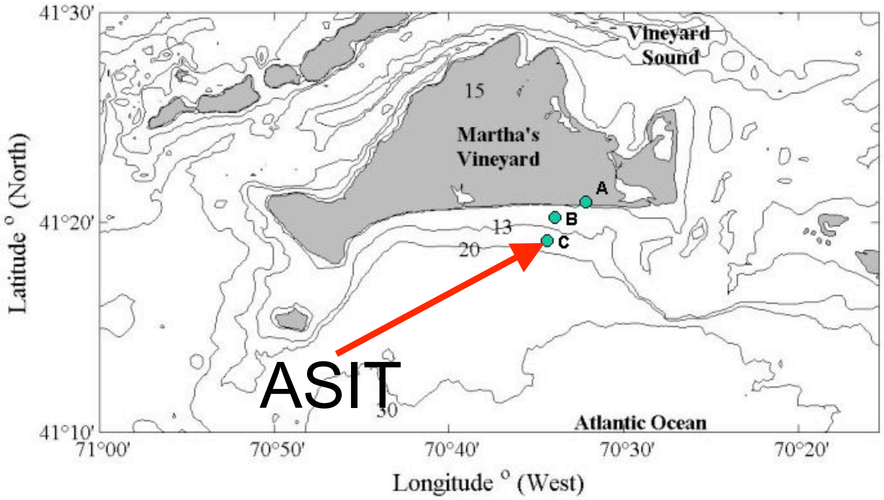
$-1.2 < z/L < 1.6$

$0.15 < \Lambda_w/\triangle_f < 15$





OHATS FIELD CAMPAIGN

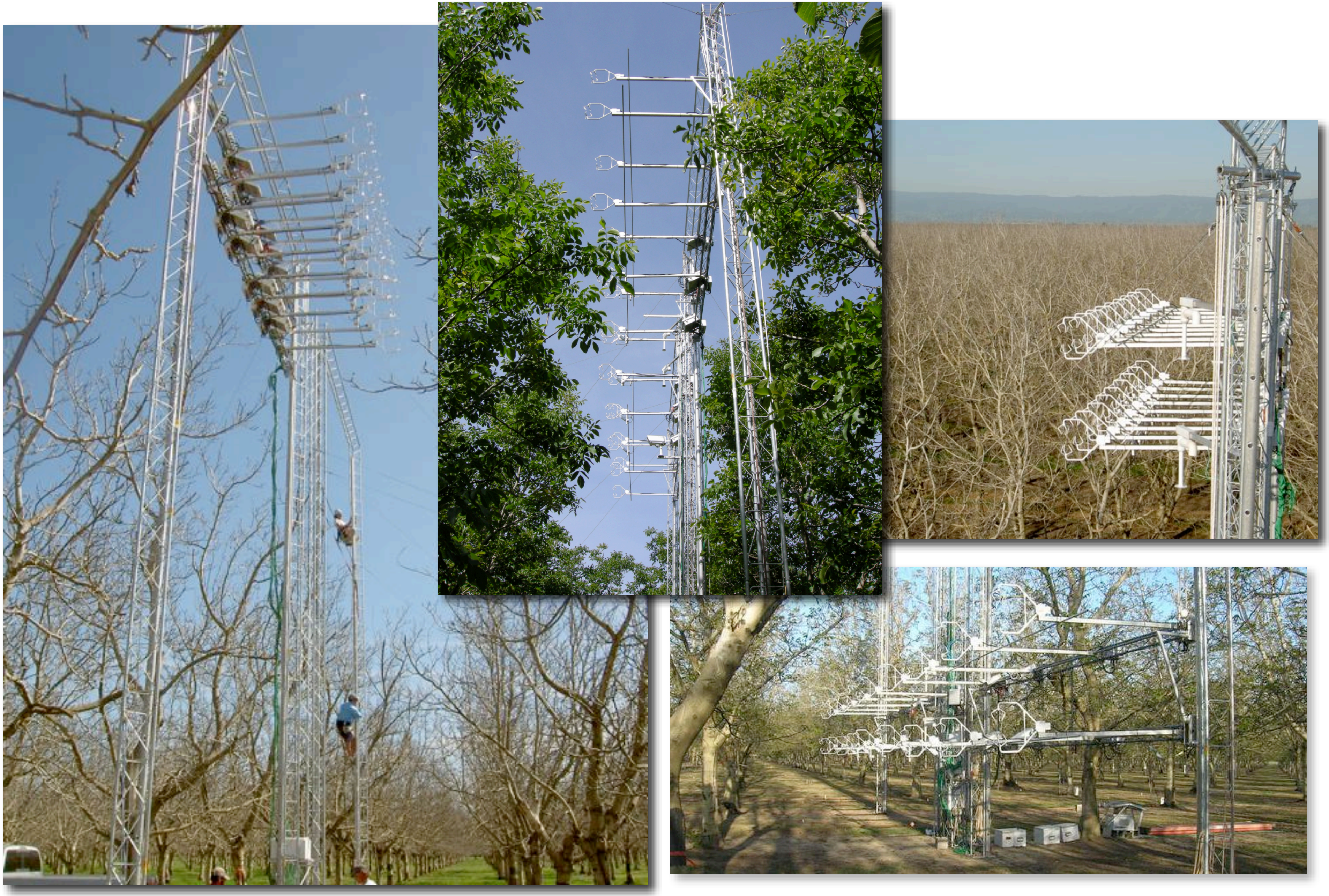


Laser altimeters

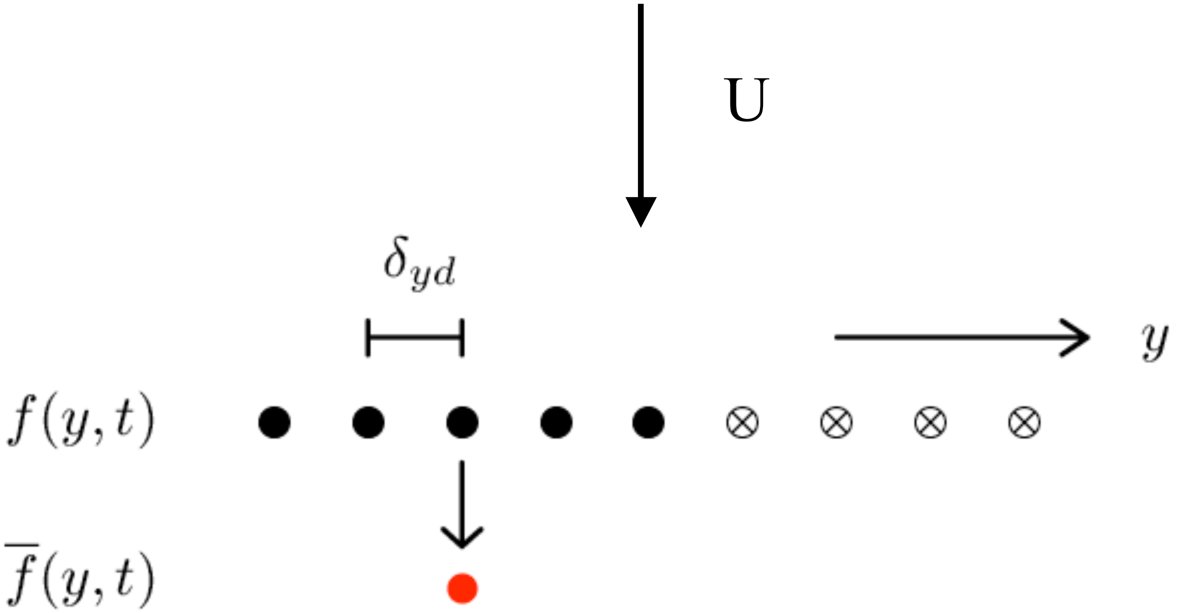
18 CSATS

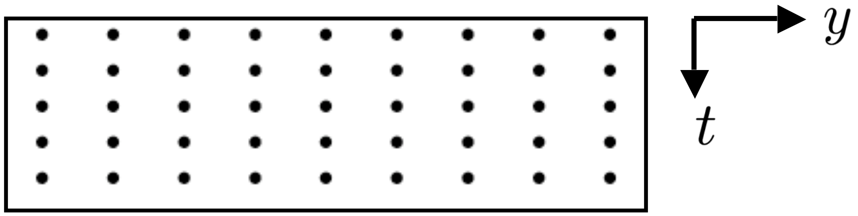
275 hours ``12 days of data” analyzed

CANOPY HORIZONTAL ARRAY TURBULENCE STUDY



AN EXAMPLE OF LATERAL (Y) FILTERING





``2D plane of turbulence”

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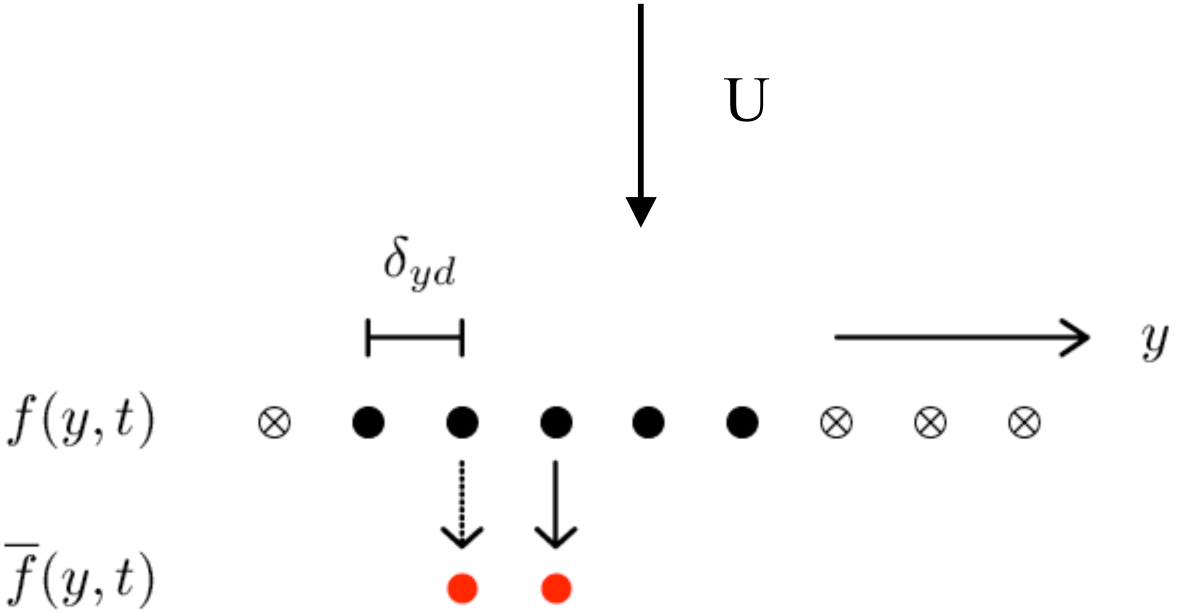

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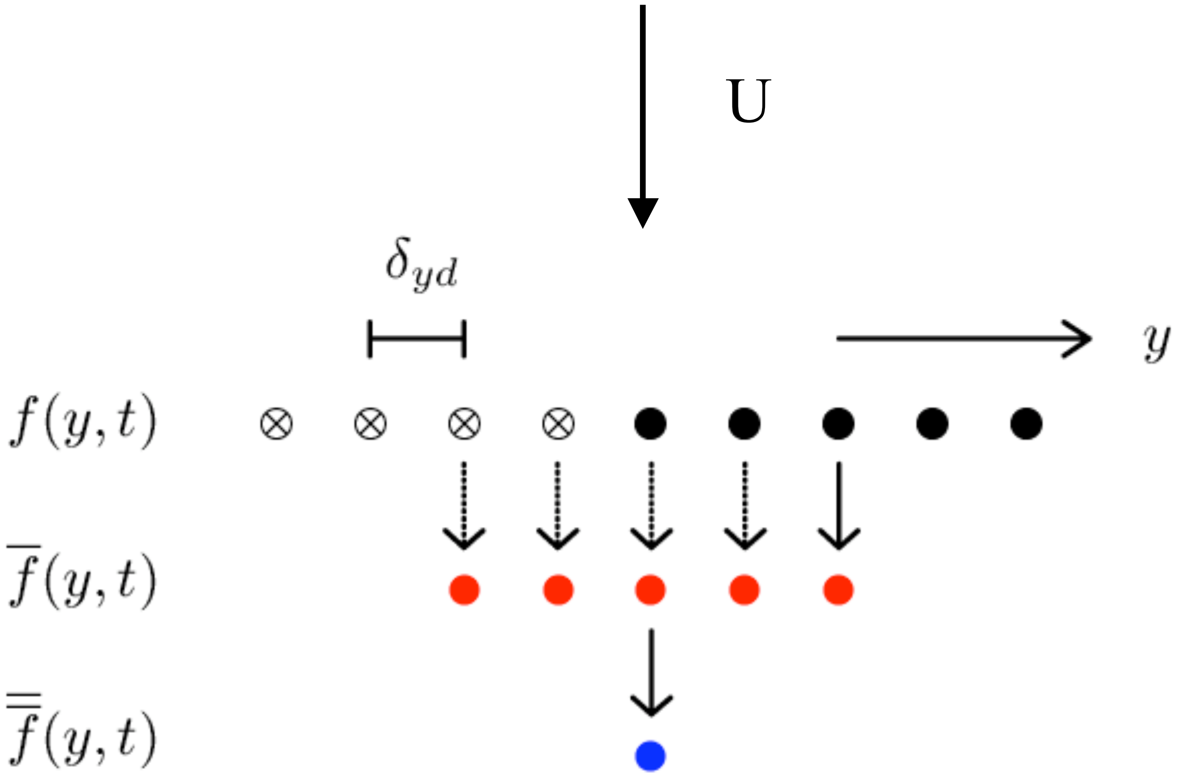
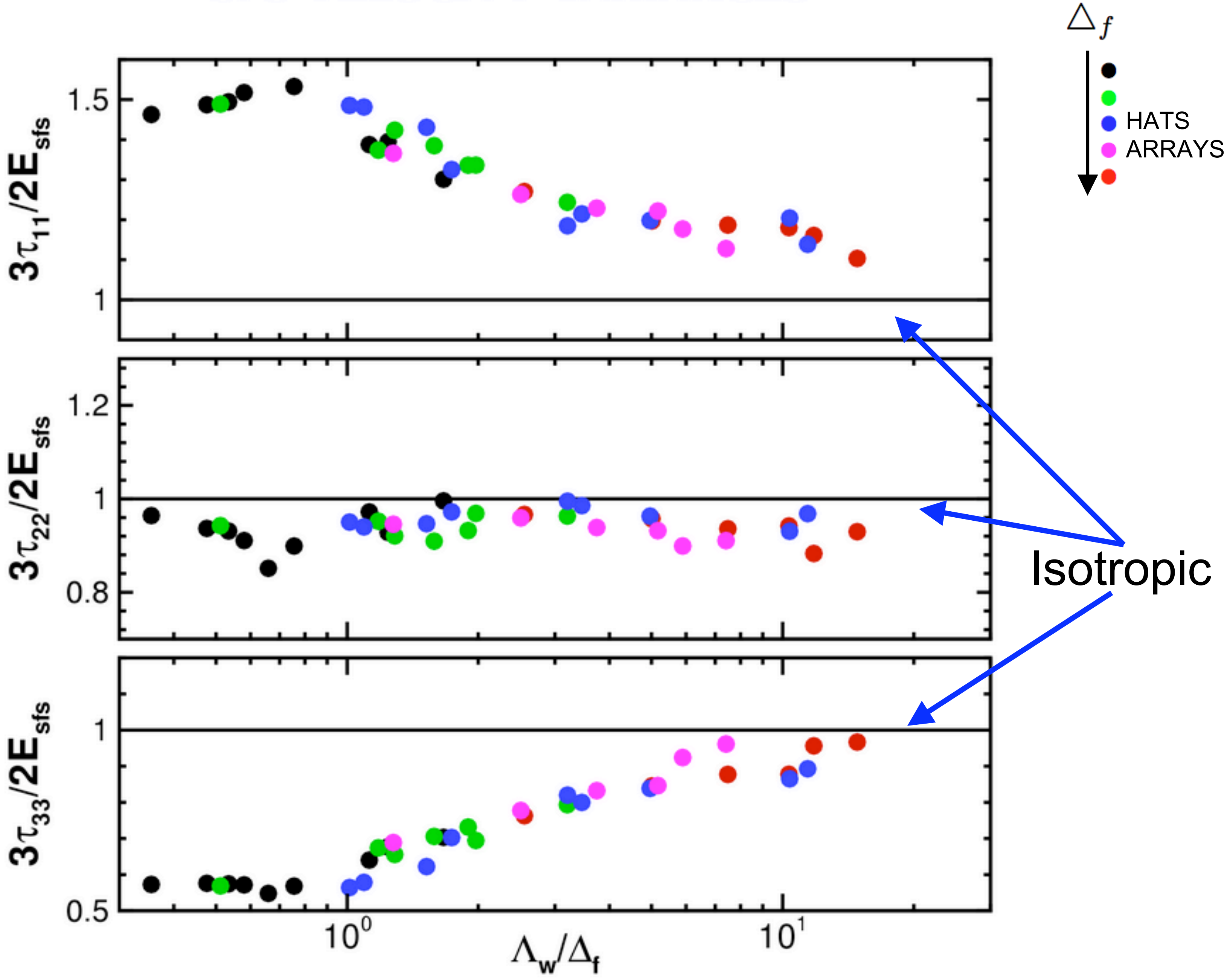
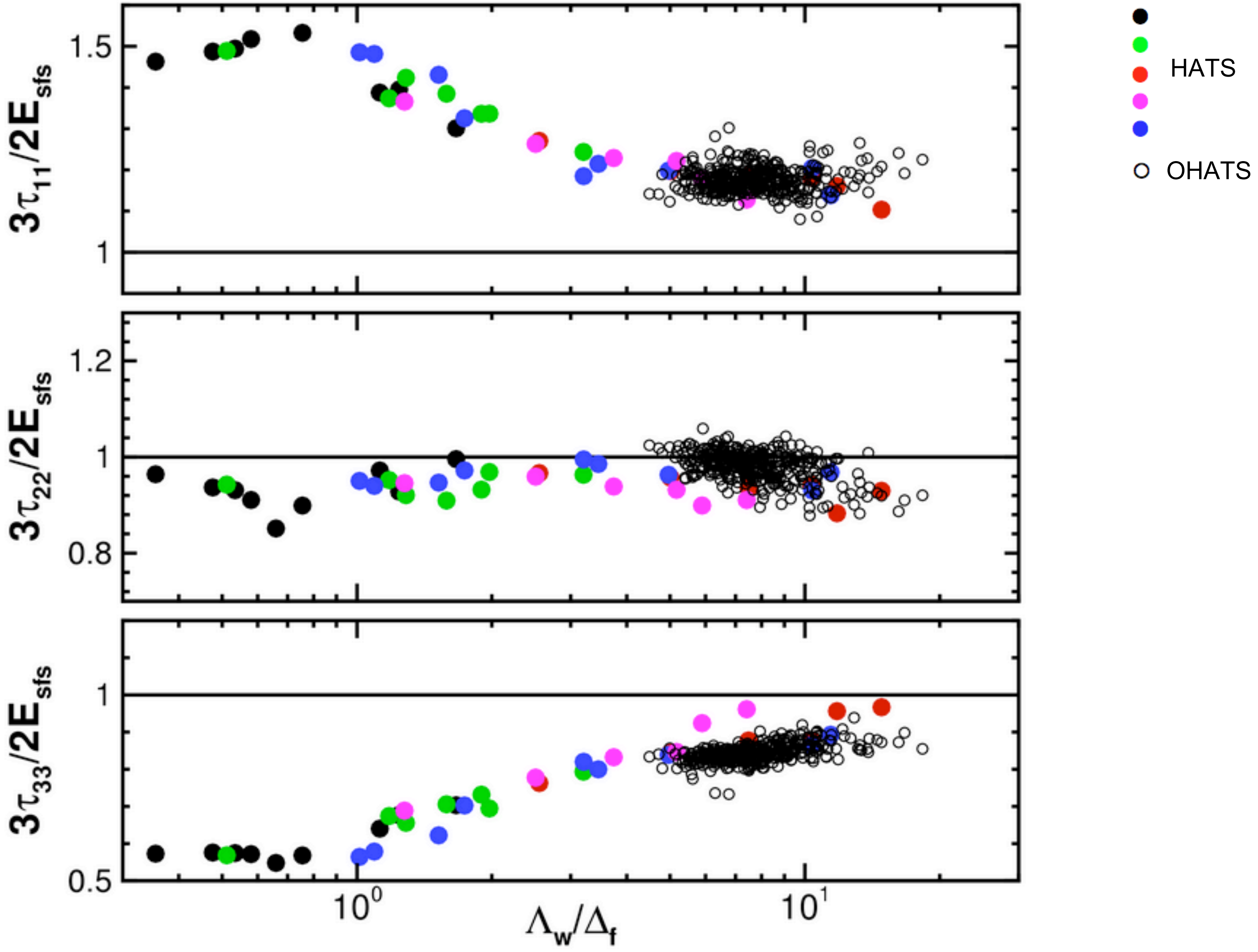


SFS VELOCITY VARIANCES



Isotropic

SFS VELOCITY VARIANCES



RATE EQUATIONS FOR SUBGRID DEVIATORIC STRESS

- What are the parent equations for the Smagorinsky model?

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  - Lilly (1967), Deardorff (1973), Wyngaard (2004), Hatlee & Wyngaard (2007)

Isotropic production

$$
\begin{aligned}
\frac{D\tau_{ij}}{Dt} &= \frac{2}{3}e\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) \\
&\quad - \left[\tau_{ik}\frac{\partial \overline{u}_j}{\partial x_k} + \tau_{jk}\frac{\partial \overline{u}_i}{\partial x_k} - \frac{1}{3}\delta_{ij}\tau_{kl}\left(\frac{\partial \overline{u}_k}{\partial x_l} + \frac{\partial \overline{u}_l}{\partial x_k}\right)\right] \\
&\quad - \frac{1}{\rho}\left[\overline{p\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)} - \overline{p}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right)\right] \\
&\quad + \text{transport} \; + \text{buoyancy production}
\end{aligned}
$$

Pressure destruction

Anisotropic deviatoric production

RATE EQUATIONS FOR SUBGRID DEVIATORIC STRESS

- What are the parent equations for the Smagorinsky model?
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$$
\begin{aligned}
\cancelto{0}{\frac{D\tau_{ij}}{Dt}} = {} & \frac{2}{3}e\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) \\
& - \cancelto{0}{\left[\tau_{ik}\frac{\partial \overline{u}_j}{\partial x_k} + \tau_{jk}\frac{\partial \overline{u}_i}{\partial x_k} - \frac{1}{3}\delta_{ij}\tau_{kl}\left(\frac{\partial \overline{u}_k}{\partial x_l} + \frac{\partial \overline{u}_l}{\partial x_k}\right)\right]} \\
& - \frac{1}{\rho}\left[\overline{p\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)} - \overline{p}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right)\right] \\
& + \cancelto{0}{\text{transport}} + \cancelto{0}{\text{buoyancy production}}
\end{aligned}
$$

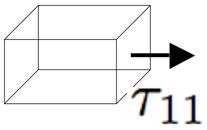
Rotta model

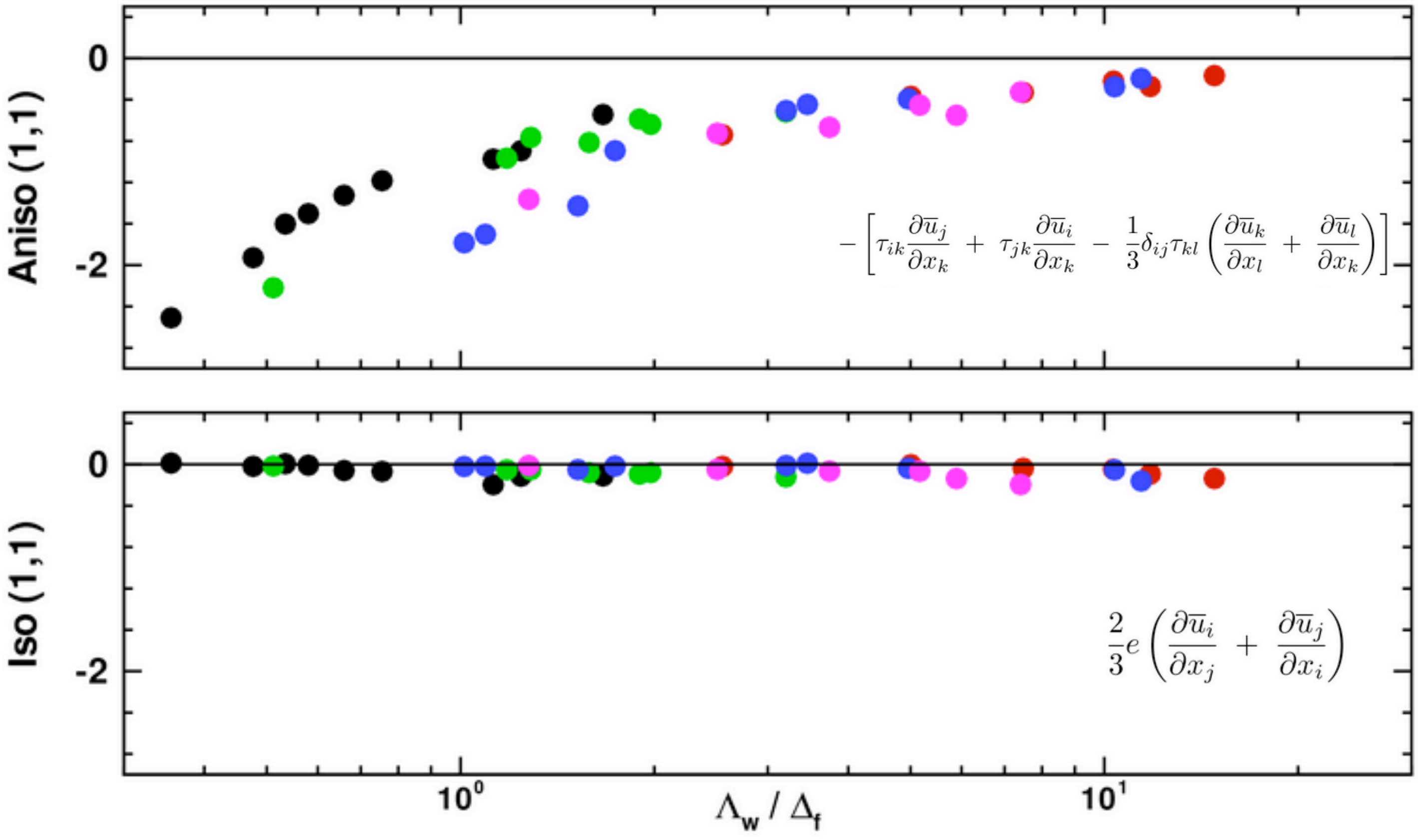
$$
\boxed{\frac{\tau_{ij}}{T} = \frac{2}{3}e\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right)}
$$

Time scale

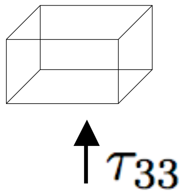
$$
T = c\,\frac{\Delta_f}{\sqrt{e}}
$$

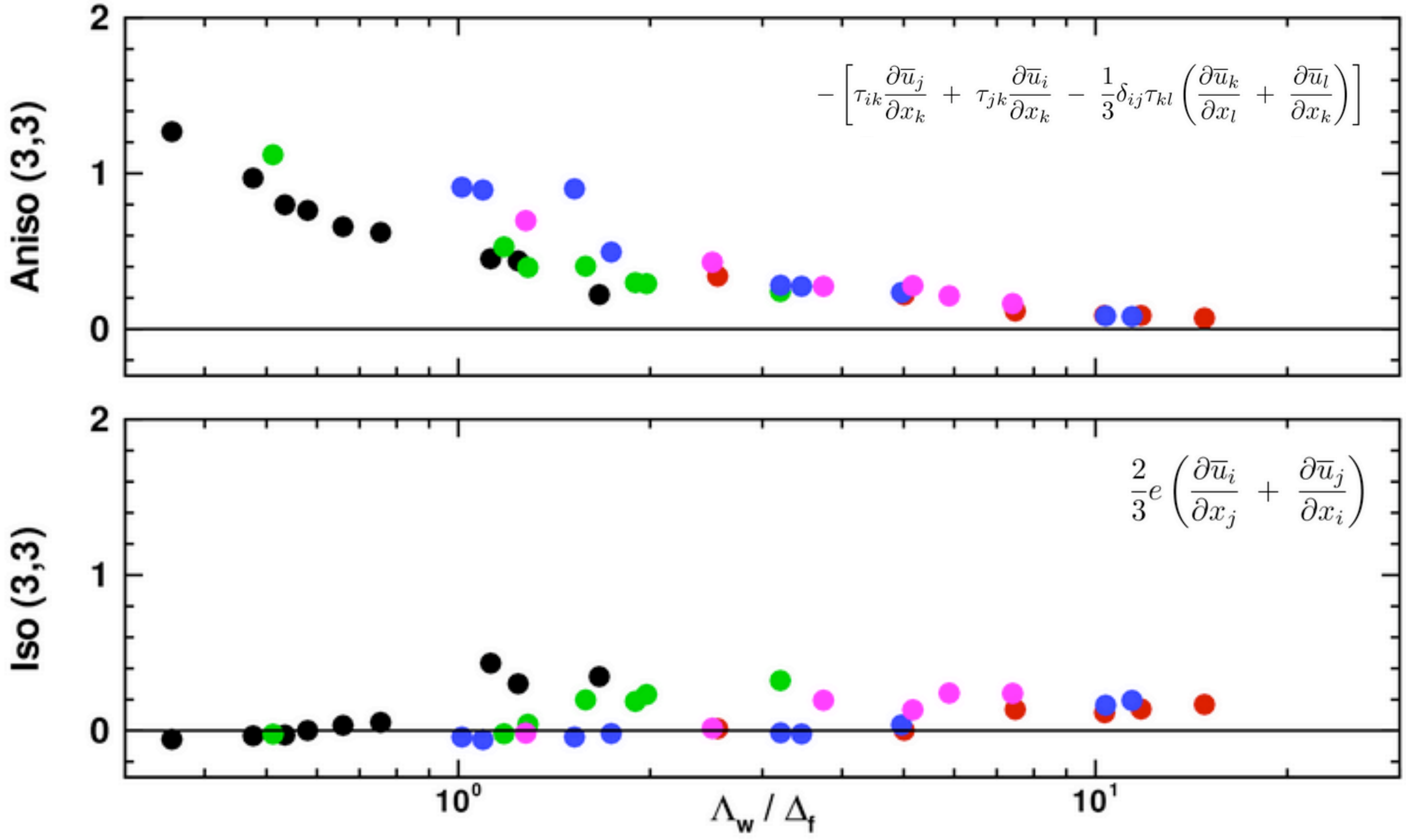
PRODUCTION OF SUBFILTER SCALE FLUX τ_{11}



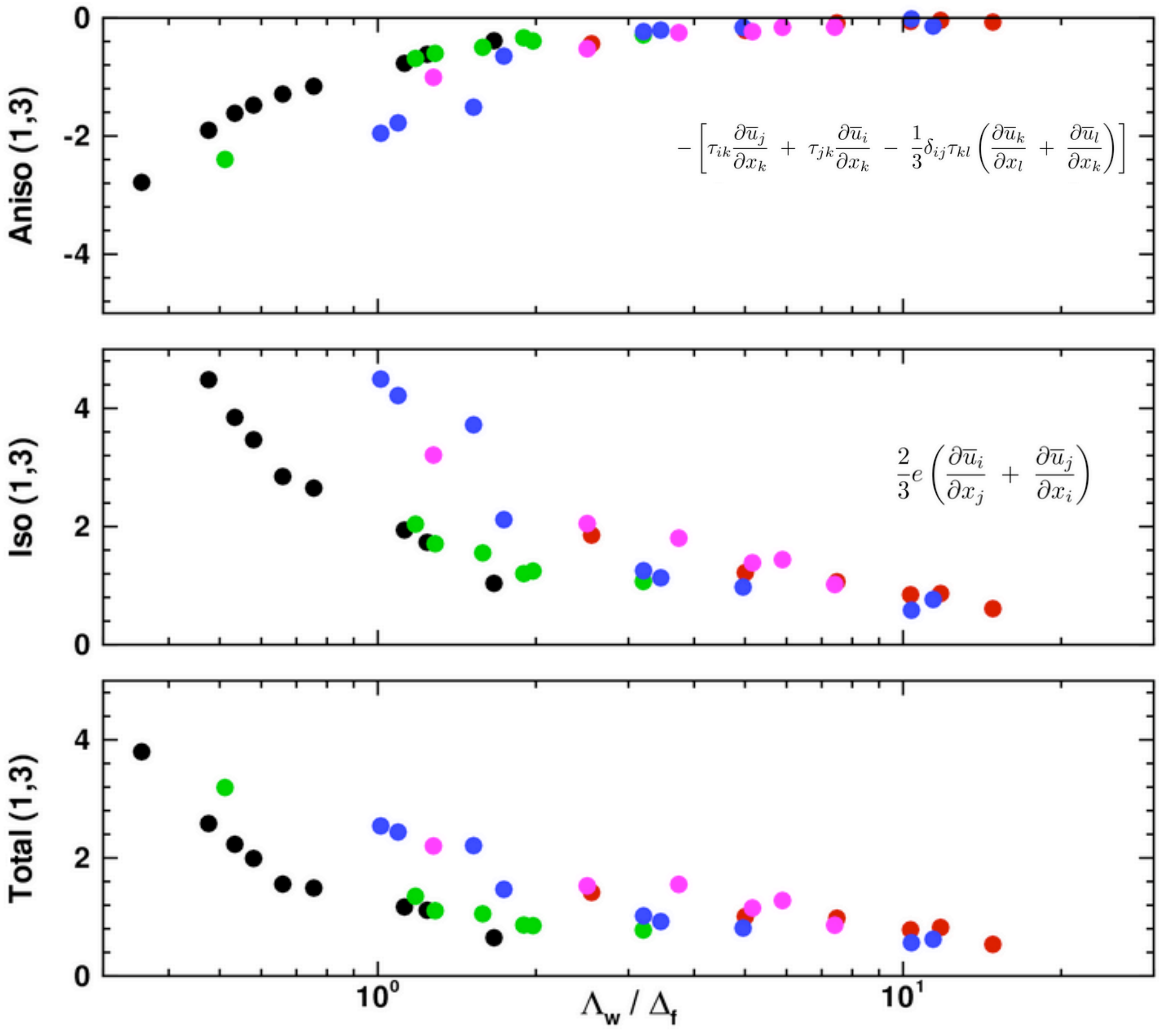


PRODUCTION OF SUBFILTER SCALE FLUX τ_{33}



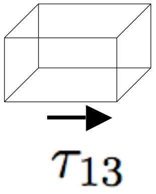


PRODUCTION OF SUBFILTER SCALE FLUX τ_{13}

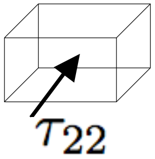


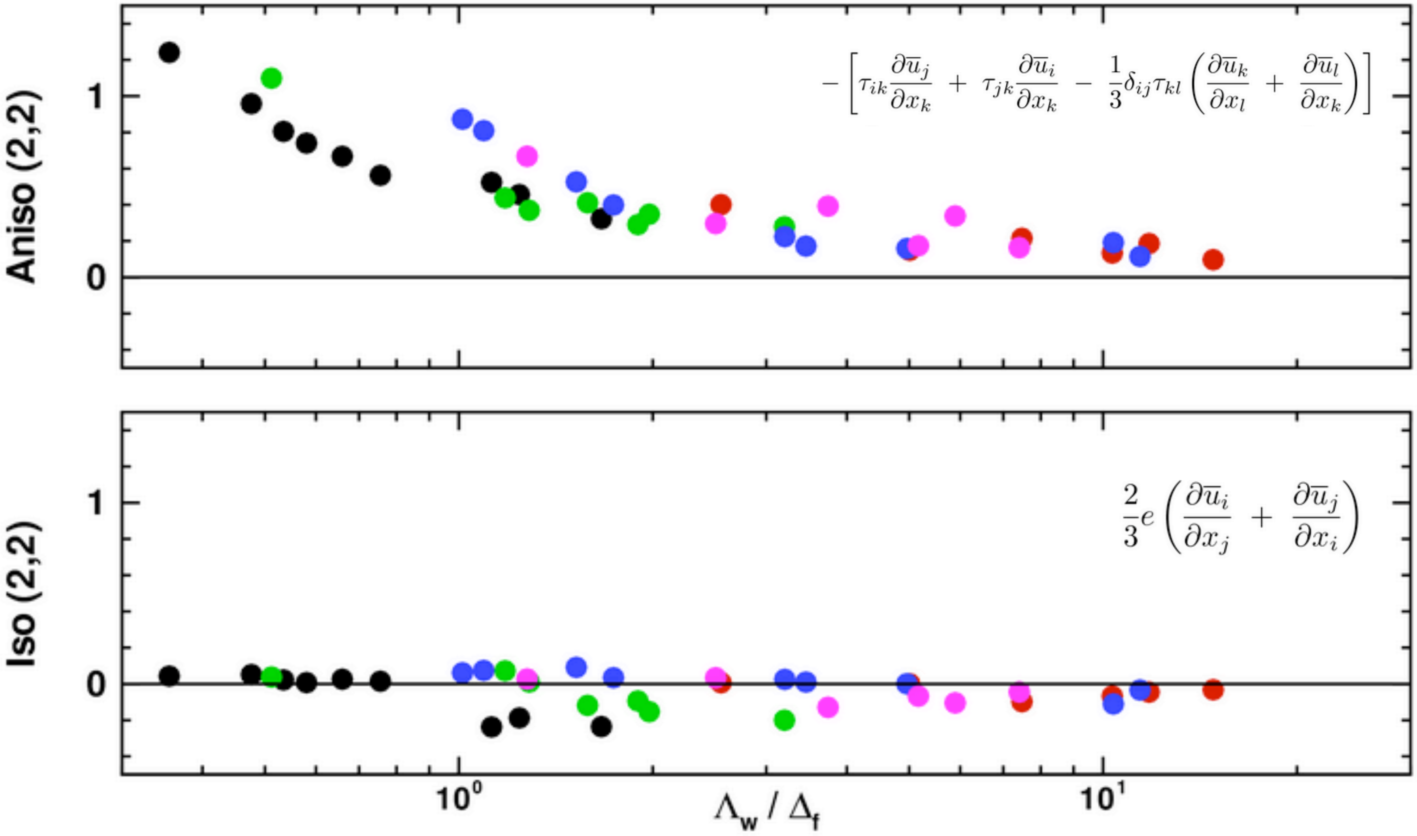
$$-\left[\tau_{ik}\frac{\partial\overline{u}_j}{\partial x_k} + \tau_{jk}\frac{\partial\overline{u}_i}{\partial x_k} - \frac{1}{3}\delta_{ij}\tau_{kl}\left(\frac{\partial\overline{u}_k}{\partial x_l} + \frac{\partial\overline{u}_l}{\partial x_k}\right)\right]$$

$$\frac{2}{3}e\left(\frac{\partial\overline{u}_i}{\partial x_j} + \frac{\partial\overline{u}_j}{\partial x_i}\right)$$



PRODUCTION OF SUBFILTER SCALE FLUX τ_{22}





VARIATION OF DEVIATORIC STRESS IN LIMIT $\Lambda_w/\triangle_f \to 0$

$\langle\tau_{11}\rangle \;=\; T\left(-2\langle\tau_{13}\rangle\dfrac{\partial U}{\partial z}+\dfrac{2}{3}\epsilon\right)$	$\langle\tau_{11}\rangle \;=\; 0$
$\langle\tau_{22}\rangle \;=T\left(\dfrac{2}{3}\epsilon\right)$	$\langle\tau_{22}\rangle \;=\; 0$
$\langle\tau_{33}\rangle \;=T\left(\dfrac{2}{3}\epsilon\right)$	$\langle\tau_{33}\rangle \;=\; 0$
$\langle\tau_{13}\rangle \;=T\left(\dfrac{2}{3}e\dfrac{\partial U}{\partial z}-\langle\tau_{33}\rangle\dfrac{\partial U}{\partial z}\right)$	$\langle\tau_{13}\rangle \;=T\left(\dfrac{2}{3}e\dfrac{\partial U}{\partial z}\right)$

Steady-state rate equations

Smagorinsky model

WHAT ABOUT SCALARS?

RATE EQUATIONS FOR SUBGRID SCALAR FLUX

- What are the parent equations for subgrid-scale scalar flux?

$$f_i = \overline{u_i c} - \overline{u}_i \overline{c}$$

$$\begin{aligned} \frac{Df_i}{Dt} = & -\frac{2}{3} e \frac{\partial \overline{c}}{\partial x_i} \\ & - f_j \frac{\partial \overline{u}_i}{\partial x_j} + \tau_{ij} \frac{\partial \overline{c}}{\partial x_j} \\ & + \frac{1}{\rho} \left(\overline{p \frac{\partial c}{\partial x_i}} - \overline{p} \frac{\partial \overline{c}}{\partial x_i} \right) \\ & + \text{transport} + \text{buoyancy} \end{aligned}$$

Isotropic production

Pressure destruction

Anisotropic production

RATE EQUATIONS FOR SUBGRID SCALAR FLUX

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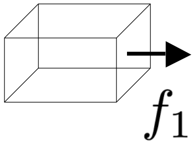
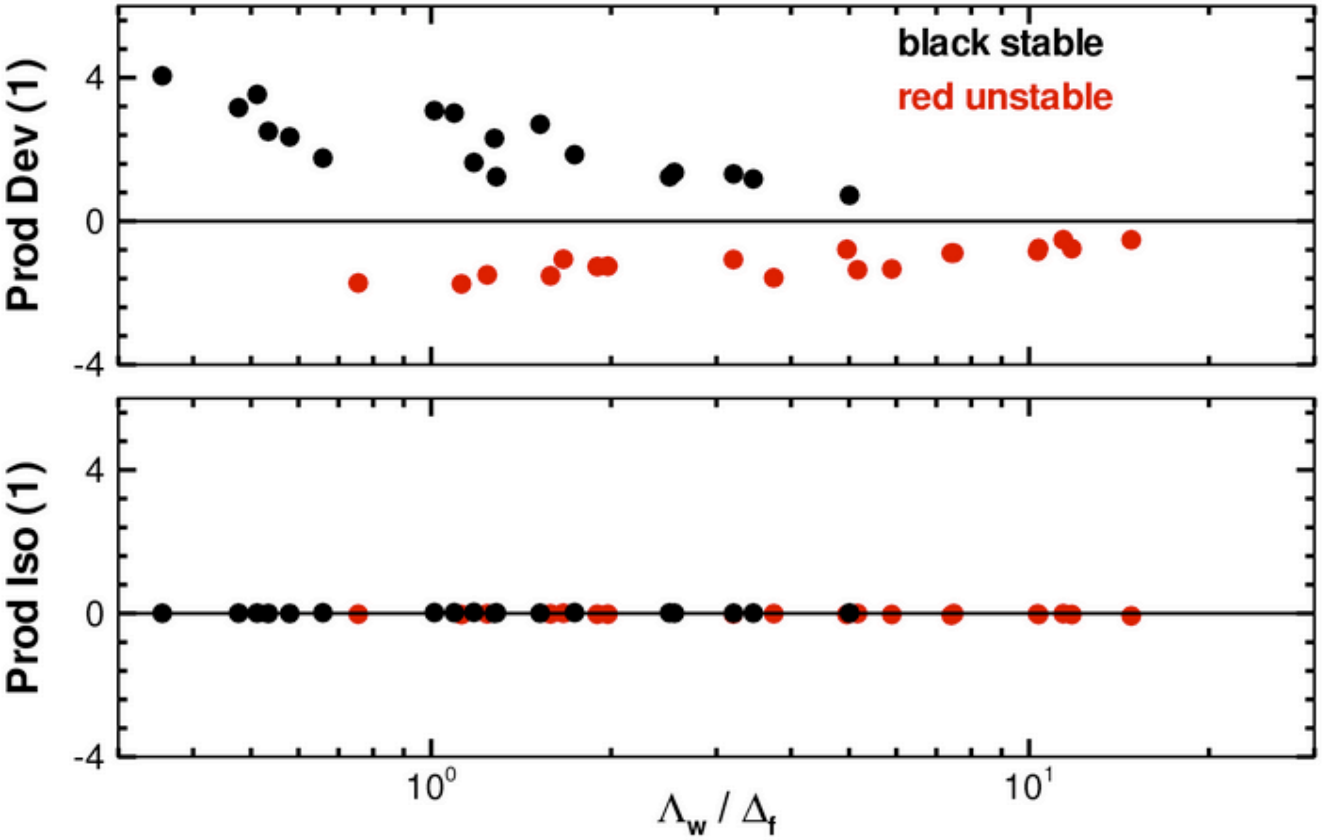
& \cancelto{0}{-f_j \frac{\partial \overline{u}_i}{\partial x_j} + \tau_{ij} \frac{\partial \overline{c}}{\partial x_j}} \\
& + \frac{1}{\rho}\left(\overline{p \frac{\partial c}{\partial x_i}} - \overline{p} \frac{\partial \overline{c}}{\partial x_i} \right) \\
& + \cancelto{0}{\text{transport}} + \cancelto{0}{\text{buoyancy}}
\end{aligned}$$

Eddy viscosity model

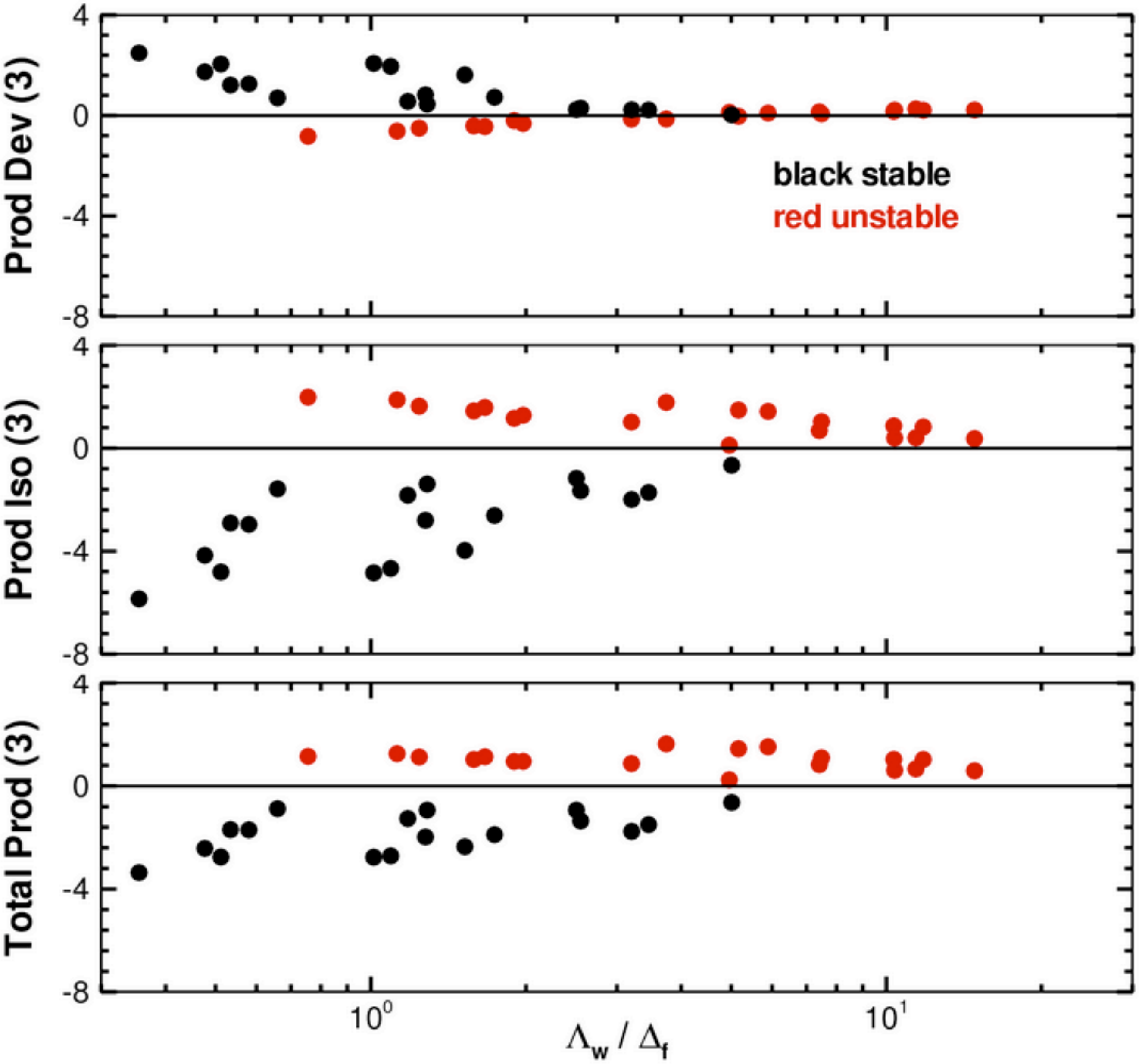
$$\boxed{f_i = -\nu_h \frac{\partial \overline{c}}{\partial x_i} \qquad \nu_h = \frac{2 c_h \triangle_f \sqrt{e}}{3}}$$

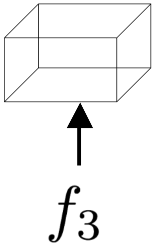
PRODUCTION OF SUBFILTER SCALE SCALAR FLUX f_1





PRODUCTION OF SUBFILTER SCALE SCALAR FLUX f_3





SUBGRID-SCALE SCALAR FLUX

Comments:

- Net horizontal scalar flux $f_1 = \langle \overline{uc} - \overline{u}\,\overline{c} \rangle \neq 0$ even horizontally homogeneous PBLs, $i.e.,\ \frac{\partial}{\partial x}\langle C \rangle = 0$
- Tilting of vertical flux by vertical shear is important $f_1 \sim -f_3 \frac{\partial \overline{u}}{\partial z} T$
- No eddy viscosity model, including the "dynamic approach", can capture anisotropic production

SUBFILTER-SCALE PRESSURE DESTRUCTION

Momentum

$$-\frac{1}{\rho}\left[\overline{p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)} - \overline{p}\left(\frac{\partial \overline{u}_i}{\partial x_j}+\frac{\partial \overline{u}_j}{\partial x_i}\right)\right] = -\frac{\tau_{ij}\sqrt{e}}{C_m \triangle_f}$$

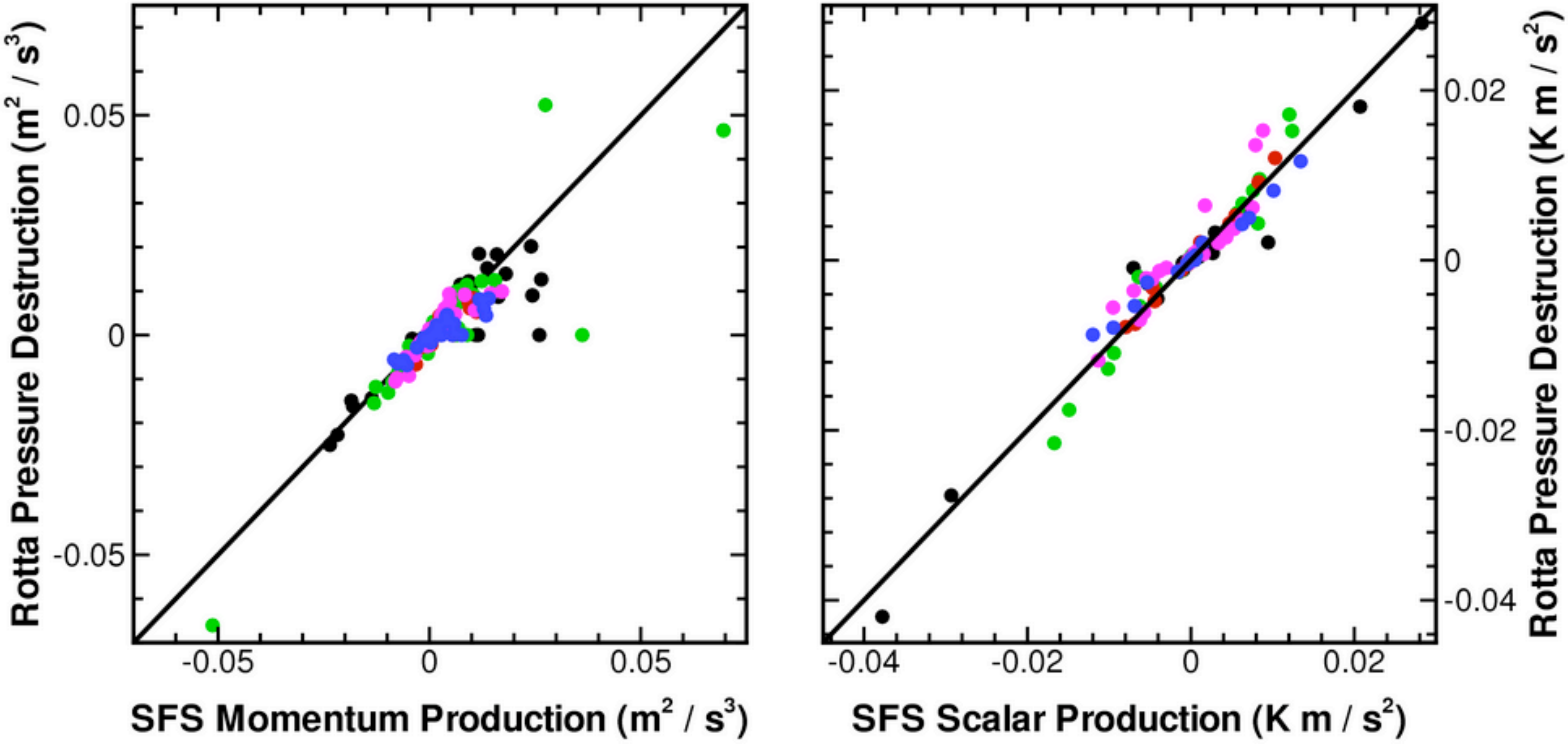
$$+\frac{1}{\rho}\left(\overline{p\frac{\partial c}{\partial x_i}} - \overline{p}\frac{\partial \overline{c}}{\partial x_i}\right) = -\frac{f_i\sqrt{e}}{C_s \triangle_f}$$

Scalar

CHATS PRESSURE SENSOR (Steven Oncley)

AHATS (2008) ``HORIZONTAL ARRAY” OF PRESSURE SENSORS

VALIDATION OF ROTTA MODEL FOR MOMENTUM AND SCALARS



$$Production \approx Destruction$$

SUMMARY

- LES is being applied to a richer set of boundary layer flows because of advances in parallel computing
- Subgrid-scale parameterizations in LES need to be validated/improved for geophysical applications
- Multi-point measurements from the HATS field campaigns compliment our ability to compute
  - Evaluation of subgrid scale models with high Re data
  - Rate equations provide insight into SGS dynamics
  - Importance of anisotropic production for stress and scalar especially for $\Lambda_w/\Delta_f \sim \mathcal{O}(1)$ or less
  - Data highlights the shortcomings of an eddy viscosity approach