Lecture 2b:

Nocturnal (Stable) Atmospheric Boundary Layers

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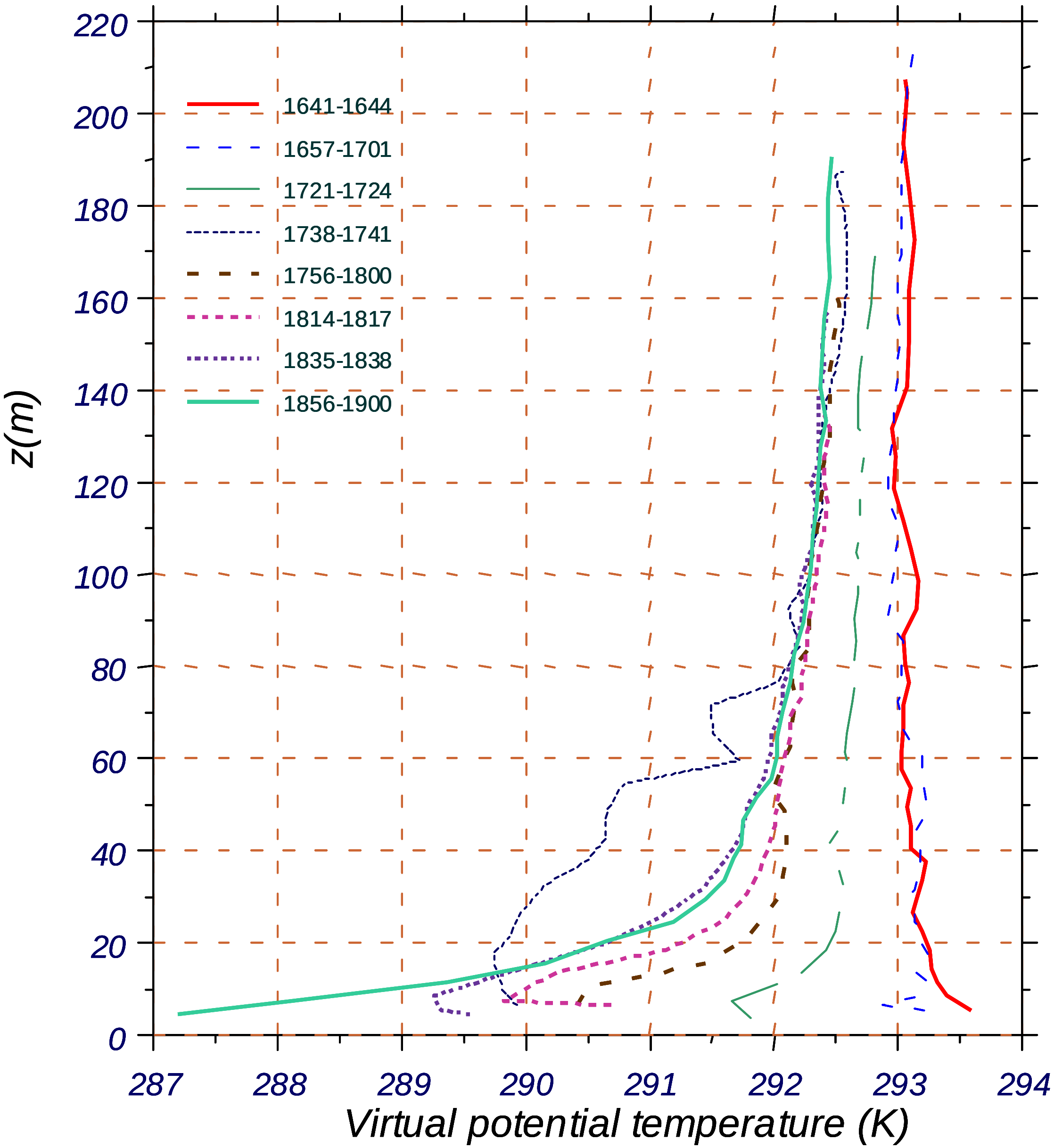
$$\frac{\partial \overline{\rho}}{\partial z} < 0; \qquad \frac{\partial \overline{T}}{\partial z} > 0; \qquad \overline{b' w'} < 0$$

Stable Boundary Layer

Nocturnal PBL



Critical Values for Instabilities

$$\frac{D(TKE)}{Dt} = -\overrightarrow{\overline{u_i u_j}} \frac{\partial \overline{U}_i}{\partial x_j} + \overline{b'w'} \ldots$$

< 0

$$Ri = \frac{N^2}{\left(\frac{d\overline{U}}{dz}\right)^2} = \frac{\left(\frac{\partial \overline{b}}{\partial z}\right)^2}{\left(\frac{\partial \overline{U}}{\partial z}\right)^2} < Ri_{cr}$$

Miles & Howard (1961, JFM) ; Ri > 0.25 – sufficient for stability; < 0.25 necessary for unstable

Ababarnel et al. (1984, PRS); Ri > 1 – Stable

Miles (1987, Phys. Fluids) -- same

Stable Stratification -- Characterization

$$Ri = \frac{N^2}{\left(\frac{\partial \overline{U}}{\partial z}\right)} = \frac{\left(\frac{\partial \overline{b}}{\partial z}\right)^2}{\left(\frac{\partial \overline{U}}{\partial z}\right)^2 + \left(\frac{\partial \overline{V}}{\partial z}\right)^2} = \frac{\frac{g}{\overline{\Theta}}\left(\frac{\partial \overline{\theta}}{\partial z}\right)}{\left(\frac{\partial \overline{U}}{\partial z}\right)^2 + \left(\frac{\partial \overline{V}}{\partial z}\right)^2}$$

Small Ri causes sporadic instabilities and turbulent patches

Scales

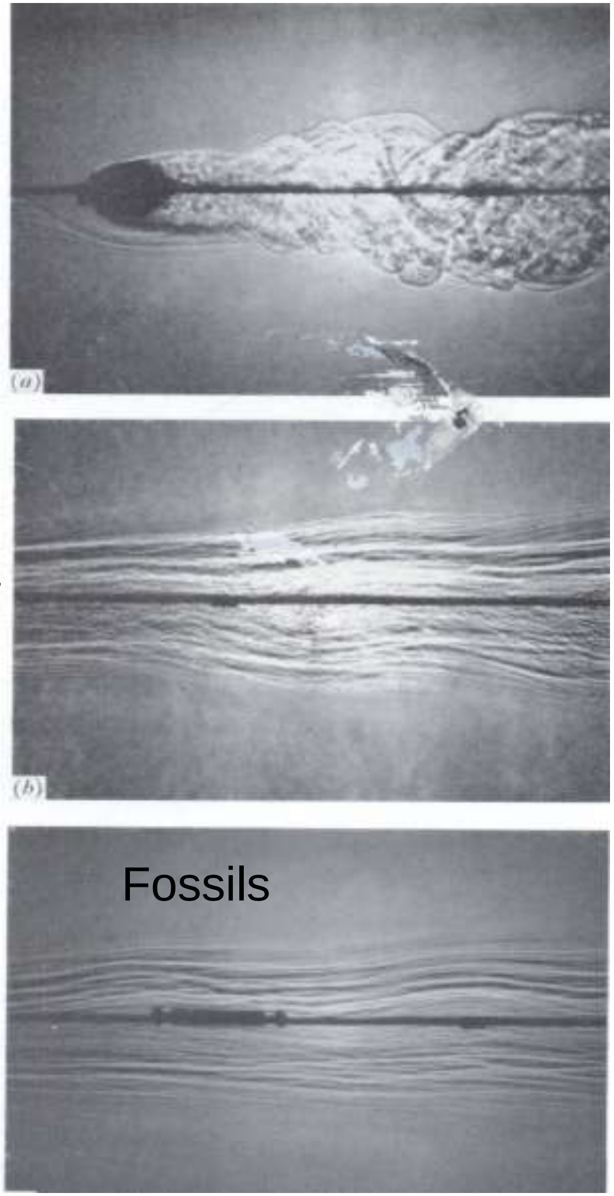
$$L_b = \frac{U_v}{N}$$

$$L_b \sim \left(\frac{\varepsilon}{N^3}\right)^{1/2} = L_R$$

Ozmidov

$$\varepsilon \sim \frac{U_v^{\,3}}{L_b}$$

$$L_R \sim L_b \sim L_K$$



H.P. Pao/Boeing

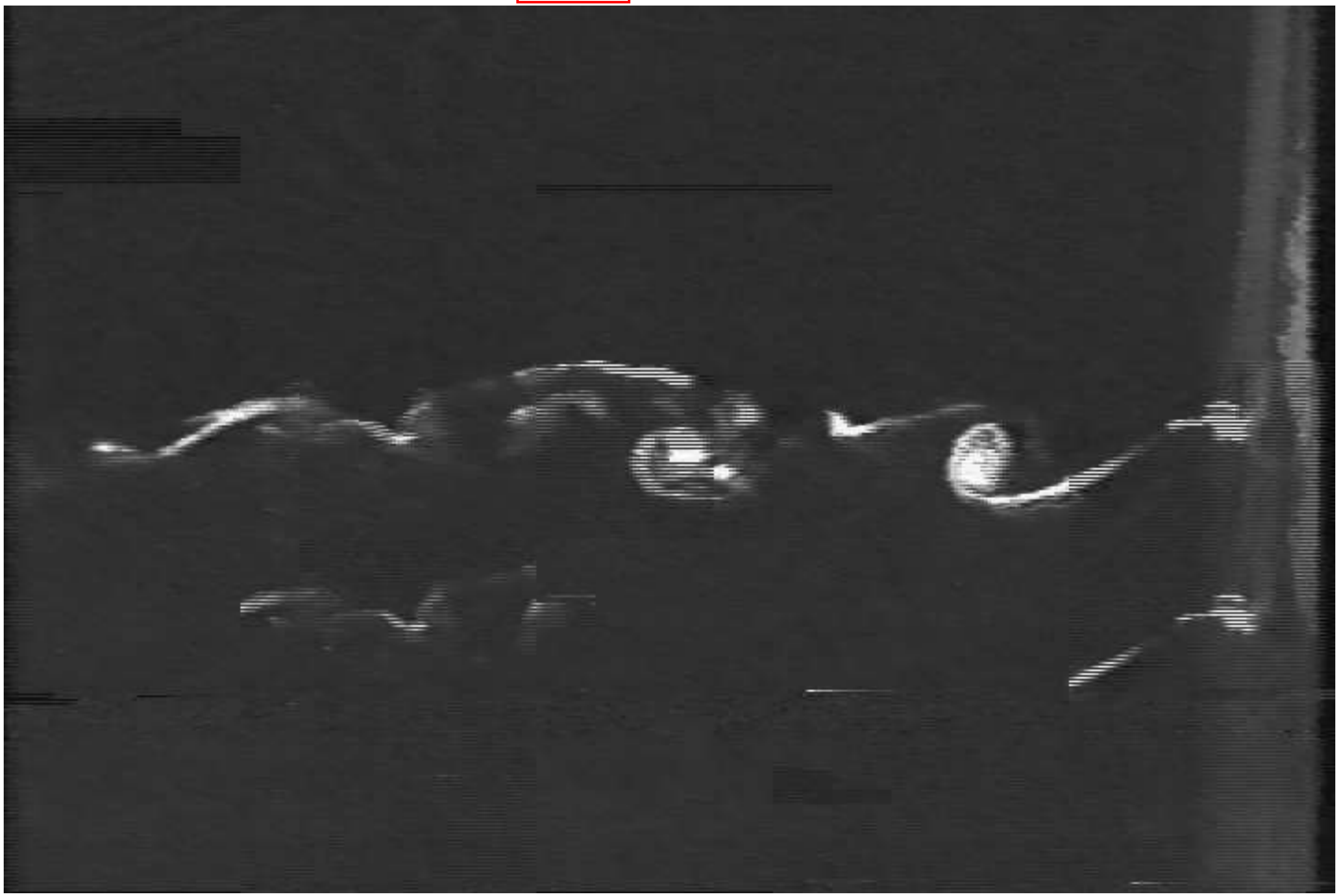
Gibson's Fossilization

$$\frac{\varepsilon}{\nu N^2} = 10 - 30 \approx 25$$

Plumes - Stable

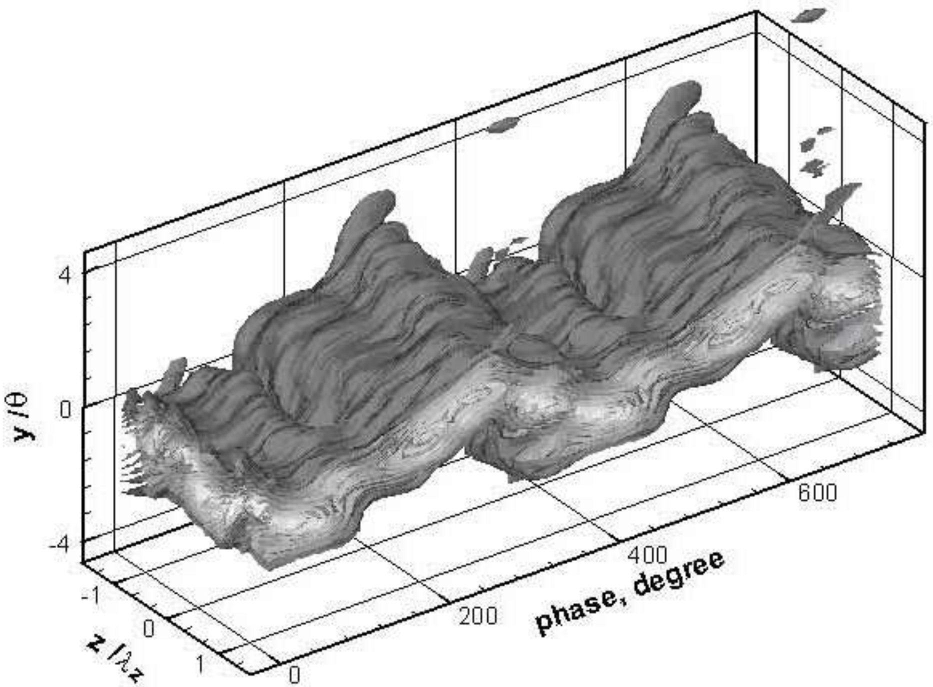
Instabilities in Stratified Shear Layers

Slow

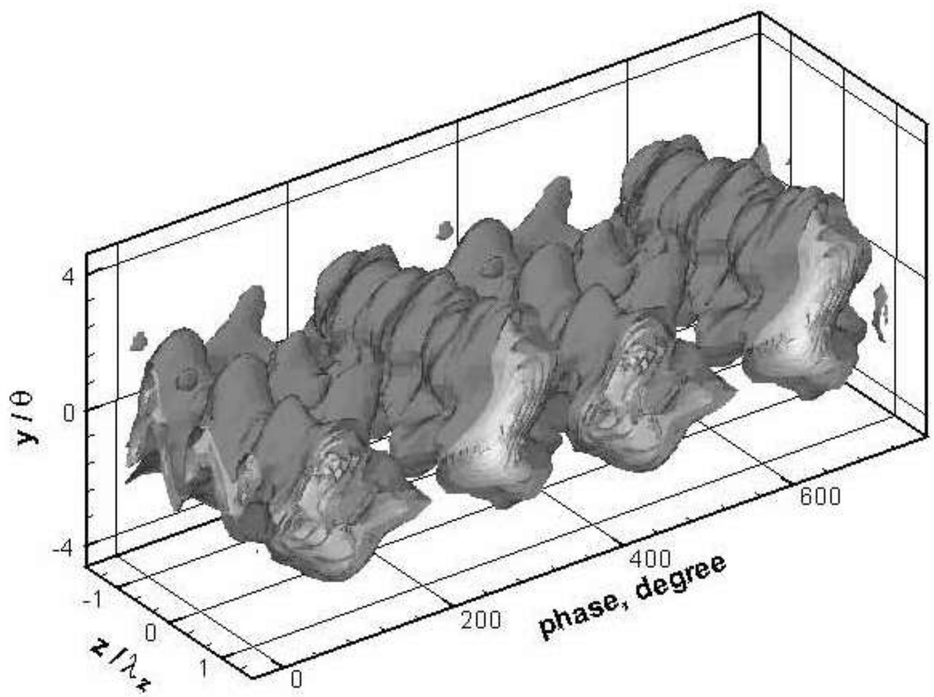


Faster

Rotter, Fernando & Kit, Physics of Fluids, 19, 2007.



y / θ
4
0
-4
-1
0
1
z / λz
0
200
400
600
phase, degree



y / θ
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z / λz
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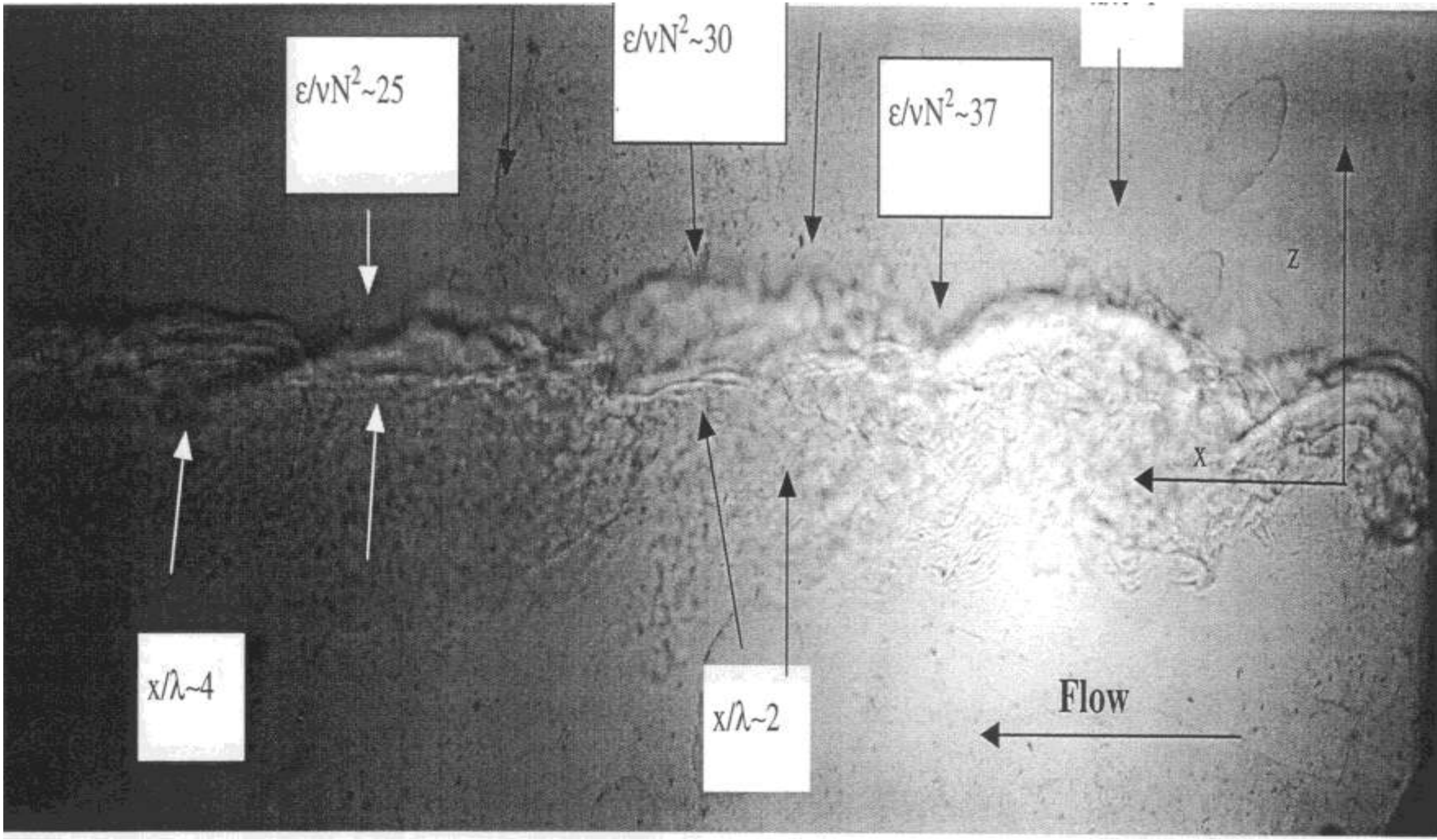
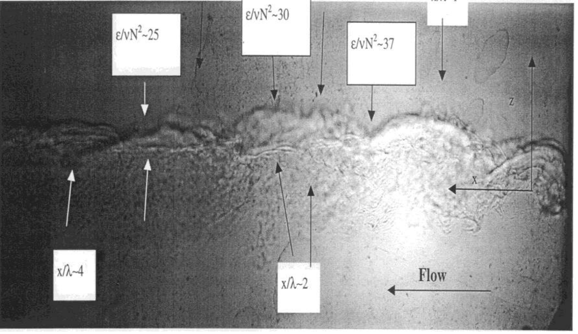
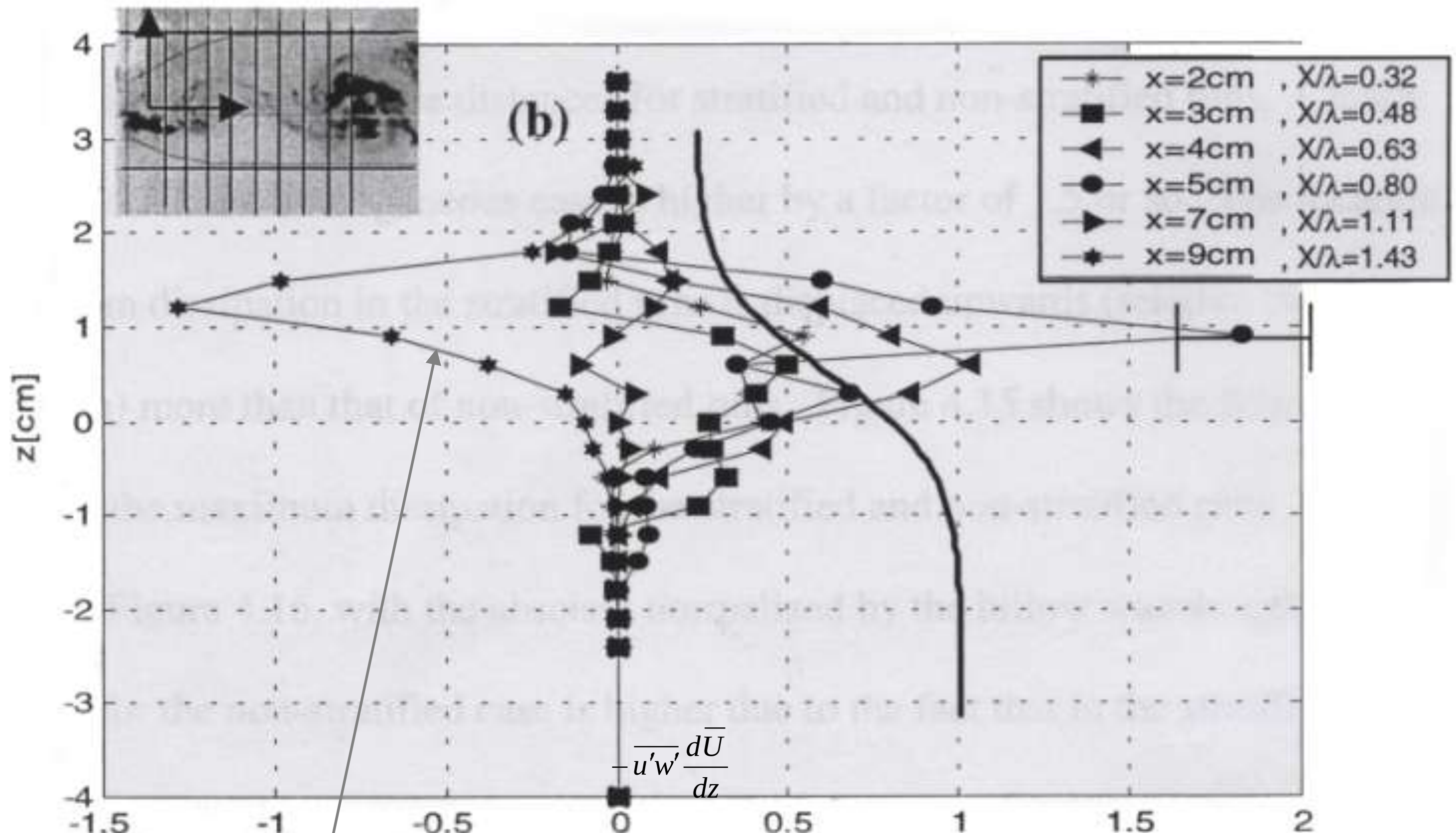
400
600
phase, degree



ε/νN²~25
ε/νN²~30
ε/νN²~37
z
x
x/λ~4
x/λ~2
Flow



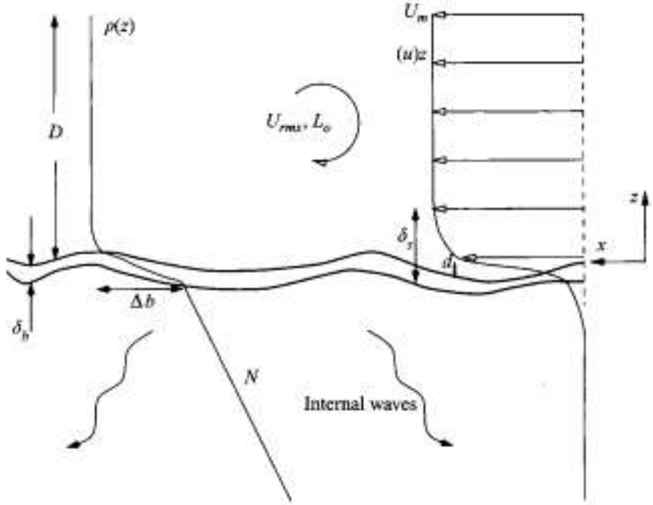


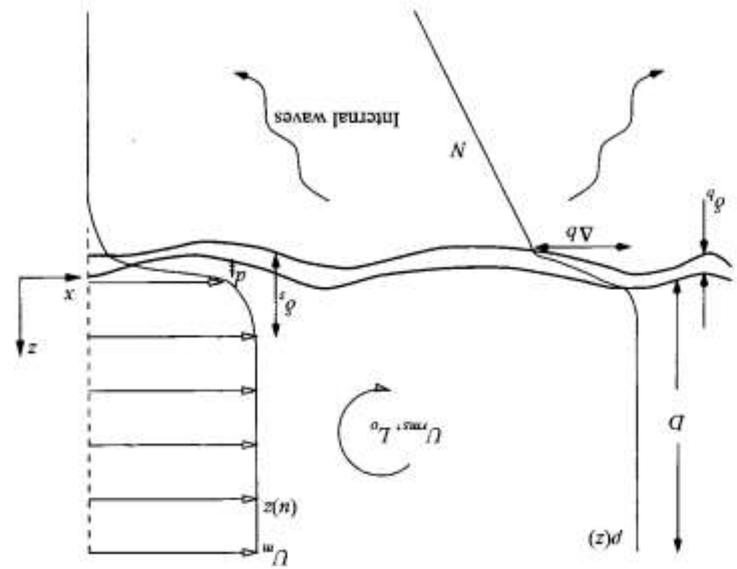
$$-u'w'\frac{dU}{dz} < 0$$

Theory/Laboratory Profiles

$$Ri = \frac{\Delta b D}{U^2}$$

$$\overline{Ri_g} = \frac{N^2}{\left(d\overline{U}/dz\right)^2}$$





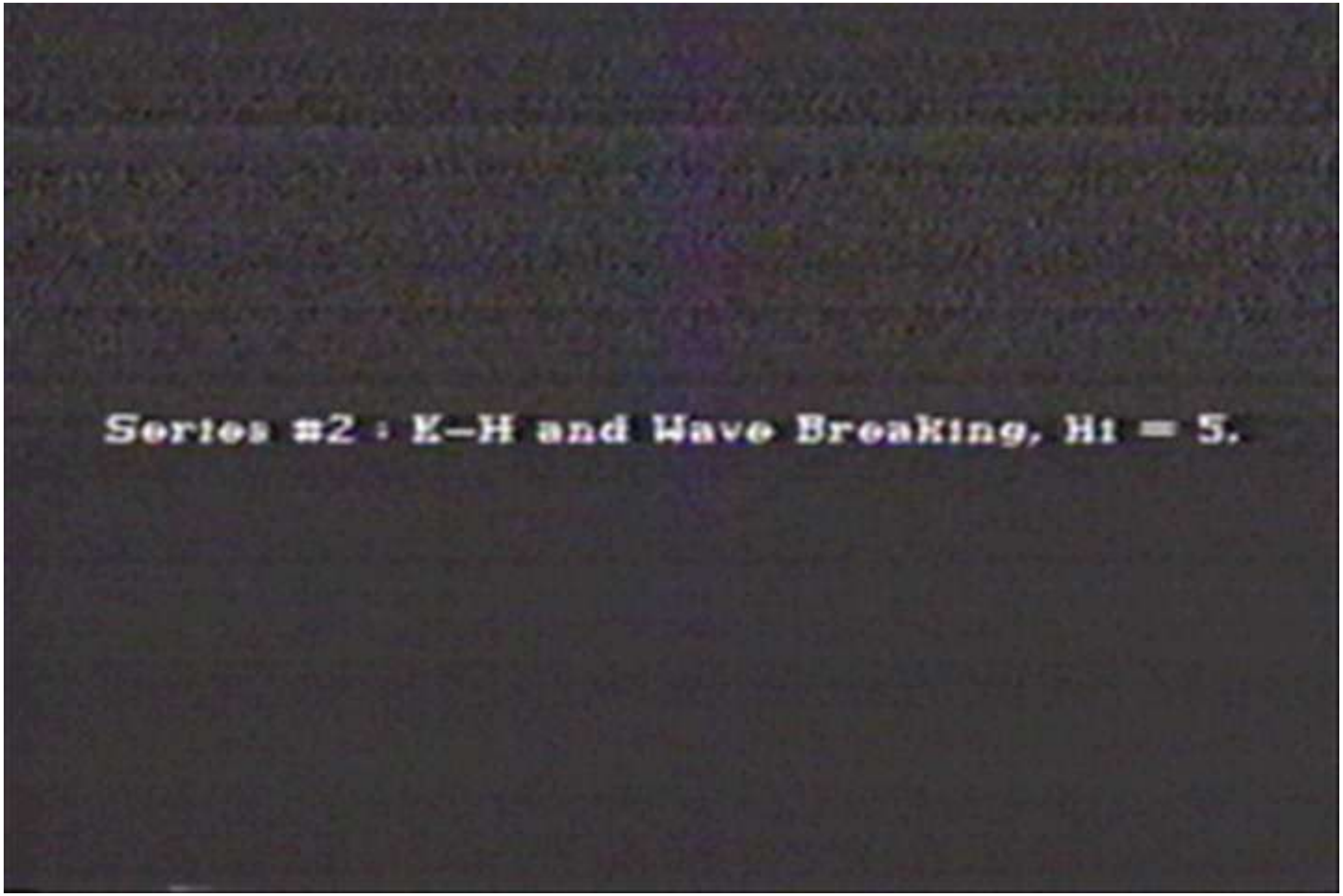
Stratified Shear Flow (Lab)

$Ri_g < 1$

(Strang & Fernando, Journal of Fluid Mechanics, 2001)

Stratified Shear Flow #2

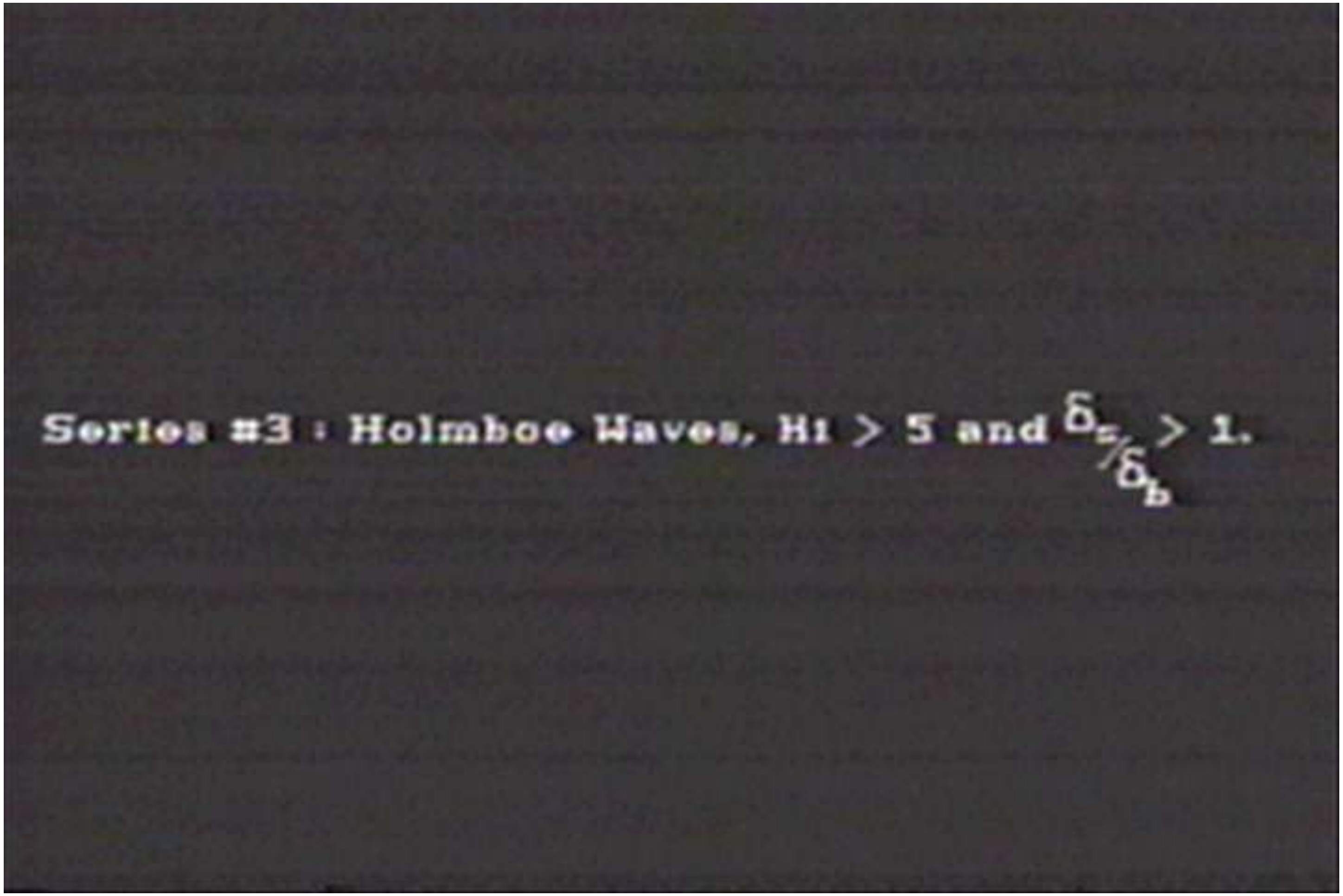
$Ri_g \approx 1$



K-H and Resonant waves

Stratified Shear Flow #3

$$Ri_g > 1$$



Holmboe Instabilities

Mechanisms of Entrainment

(a) $\overline{Rig} = 0.12$

(b) $\overline{Rig} = 0.36$

(c) $\overline{Rig} = 0.83$

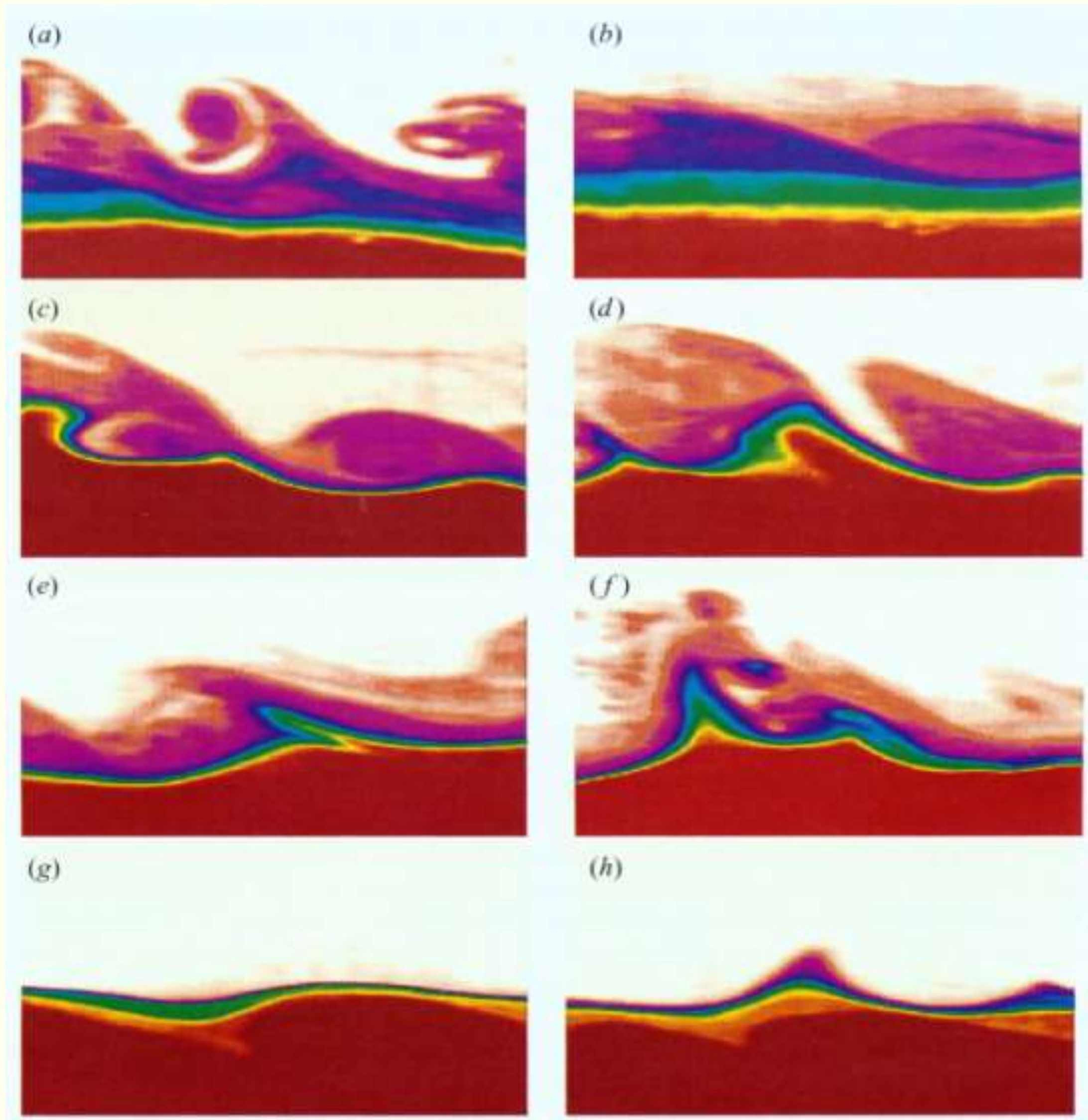
(d) $\overline{Rig} = 1.21$

(e) $\overline{Rig} = 1.21$

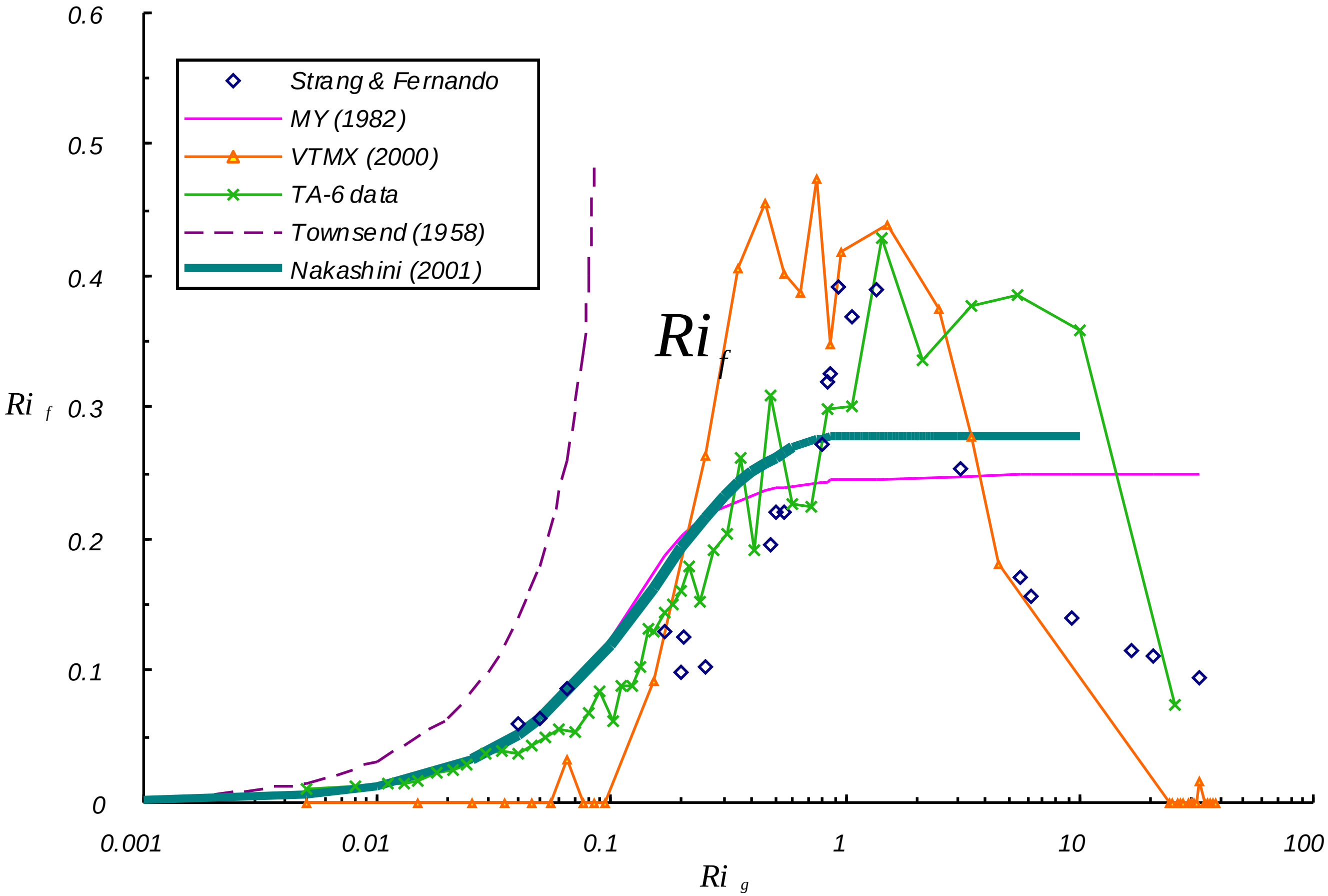
(f) $\overline{Rig} = 1.78$

(g) $\overline{Rig} = 2.82$

(h) $\overline{Rig} = 2.82$

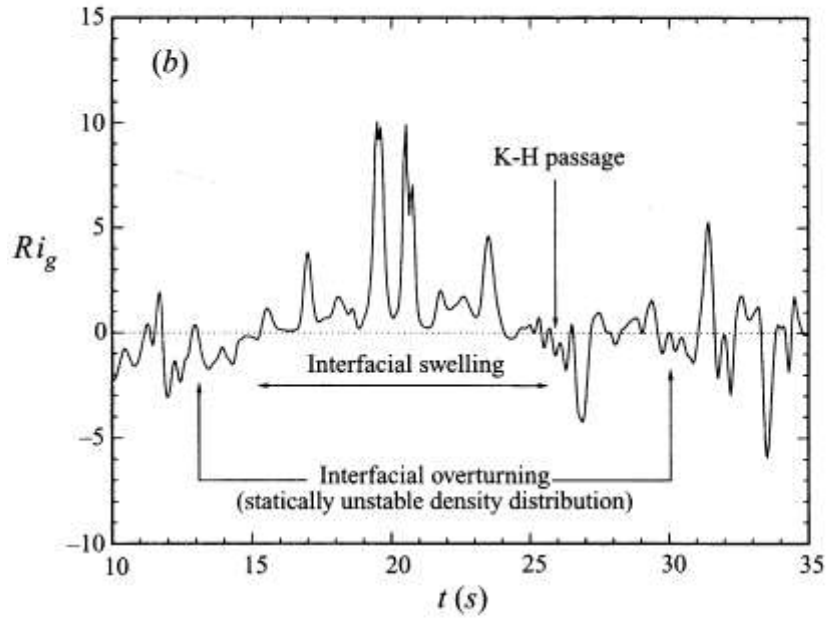


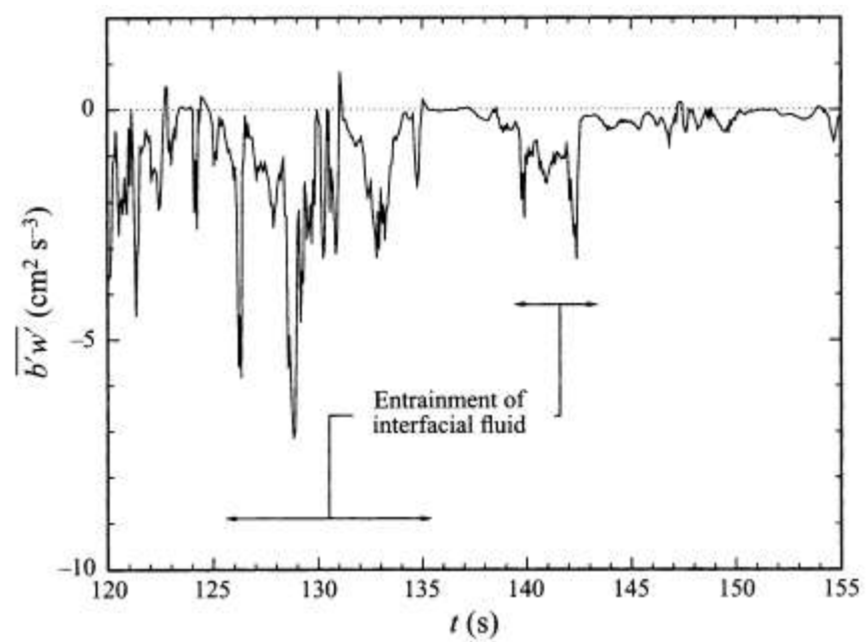
Flux versus Gradient Richardson Numbers



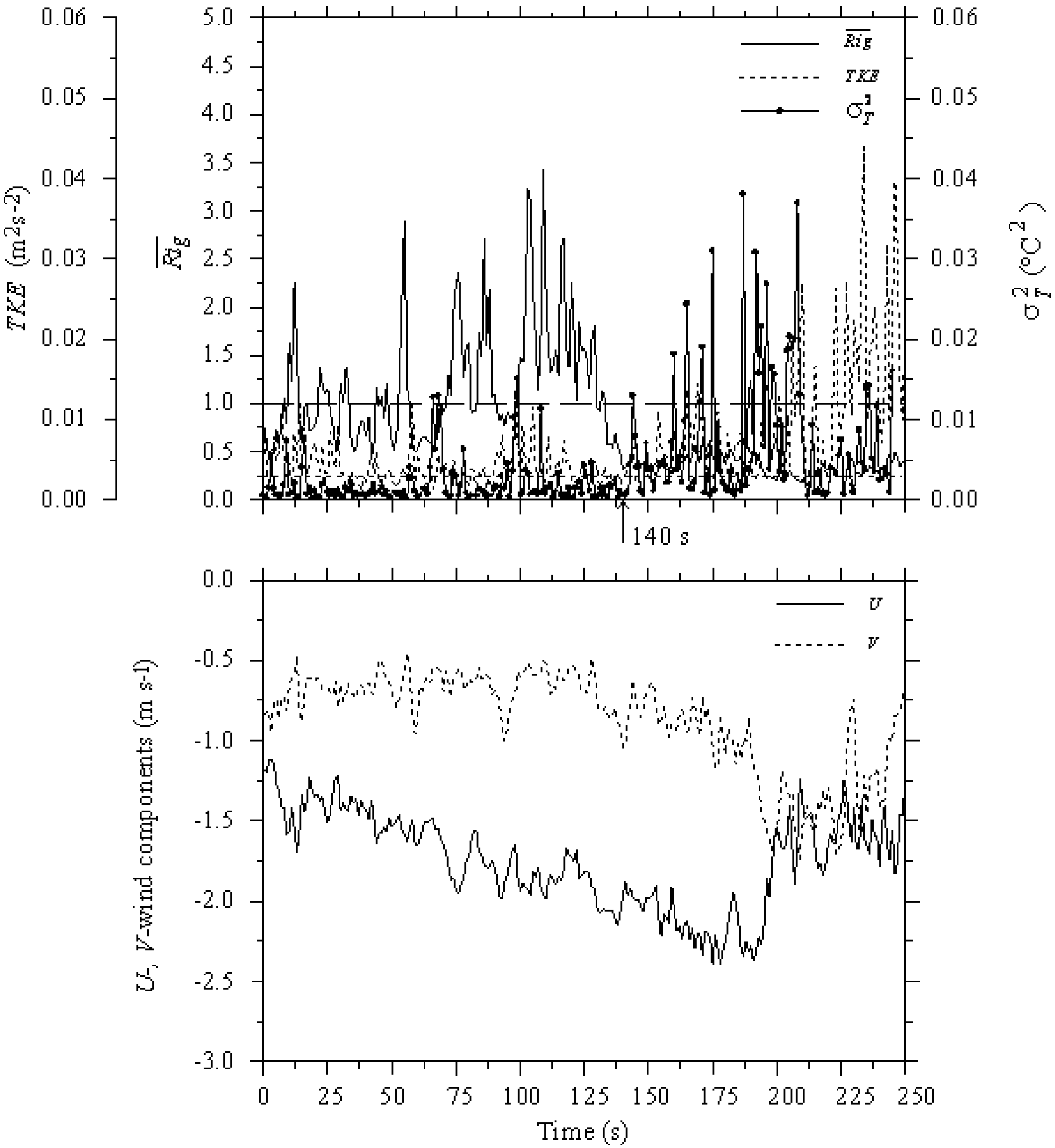
J. Fluid Mech. 2002

Interfacial Measurements



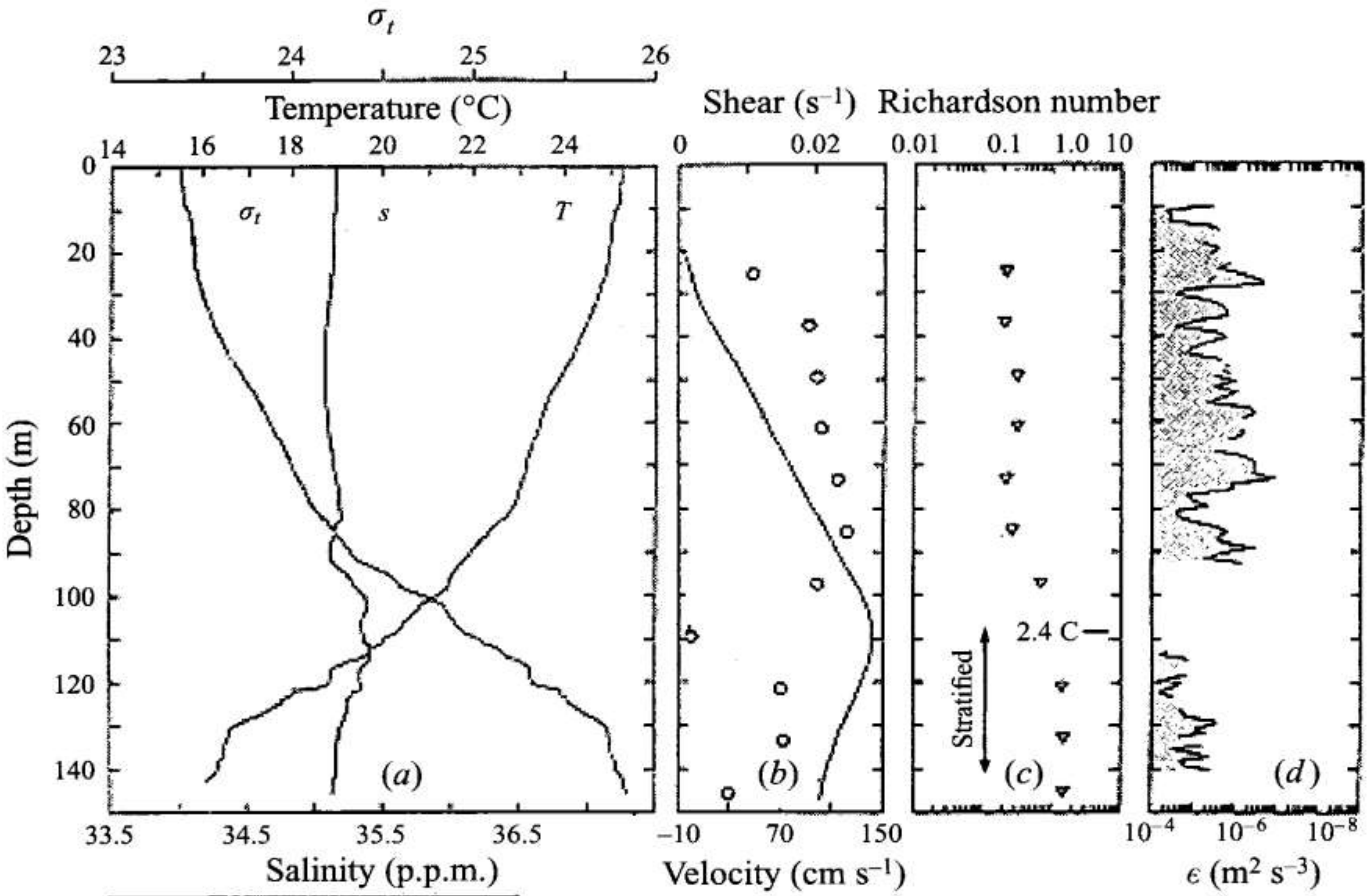


Turbulence in ABL

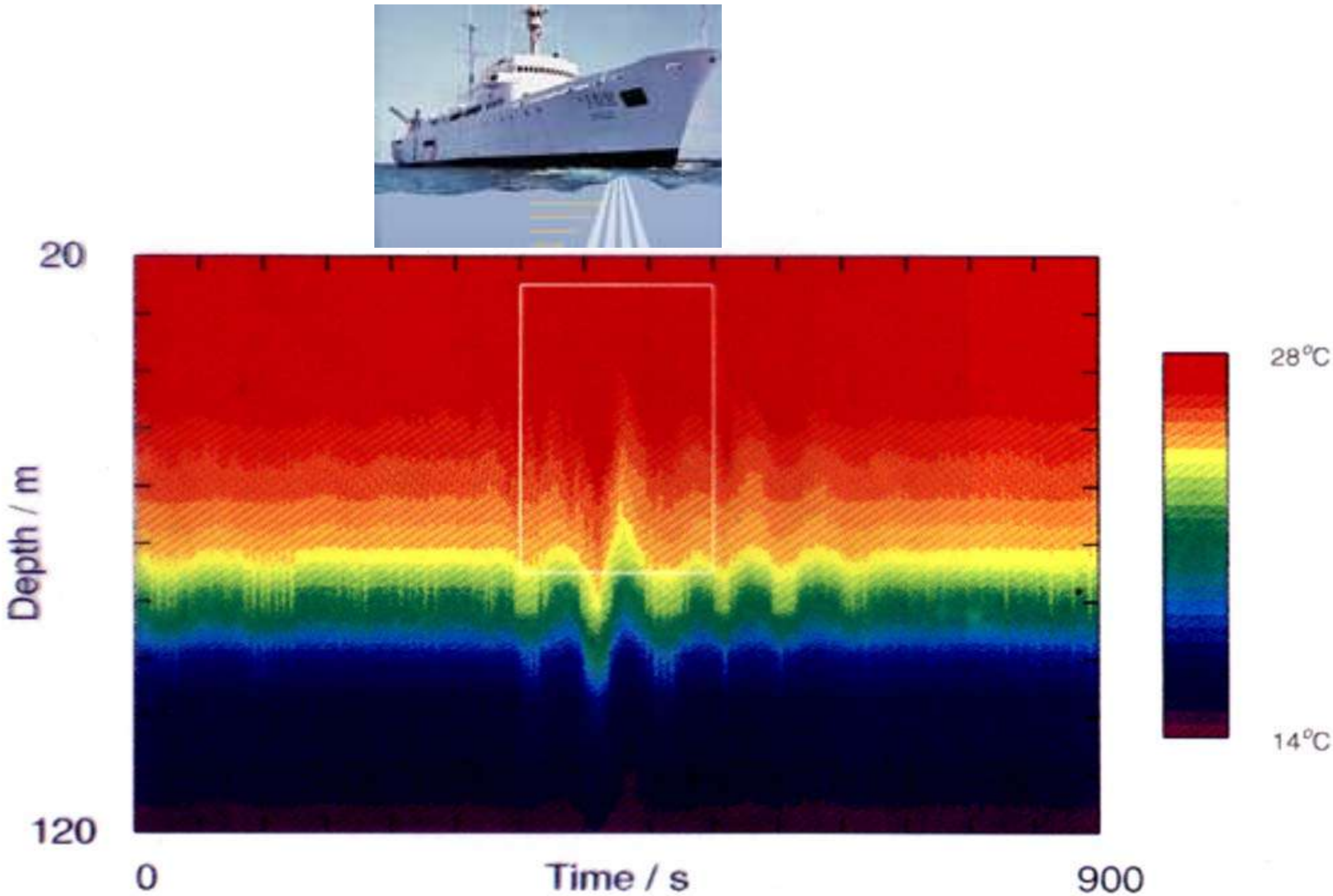


Global Intermittency

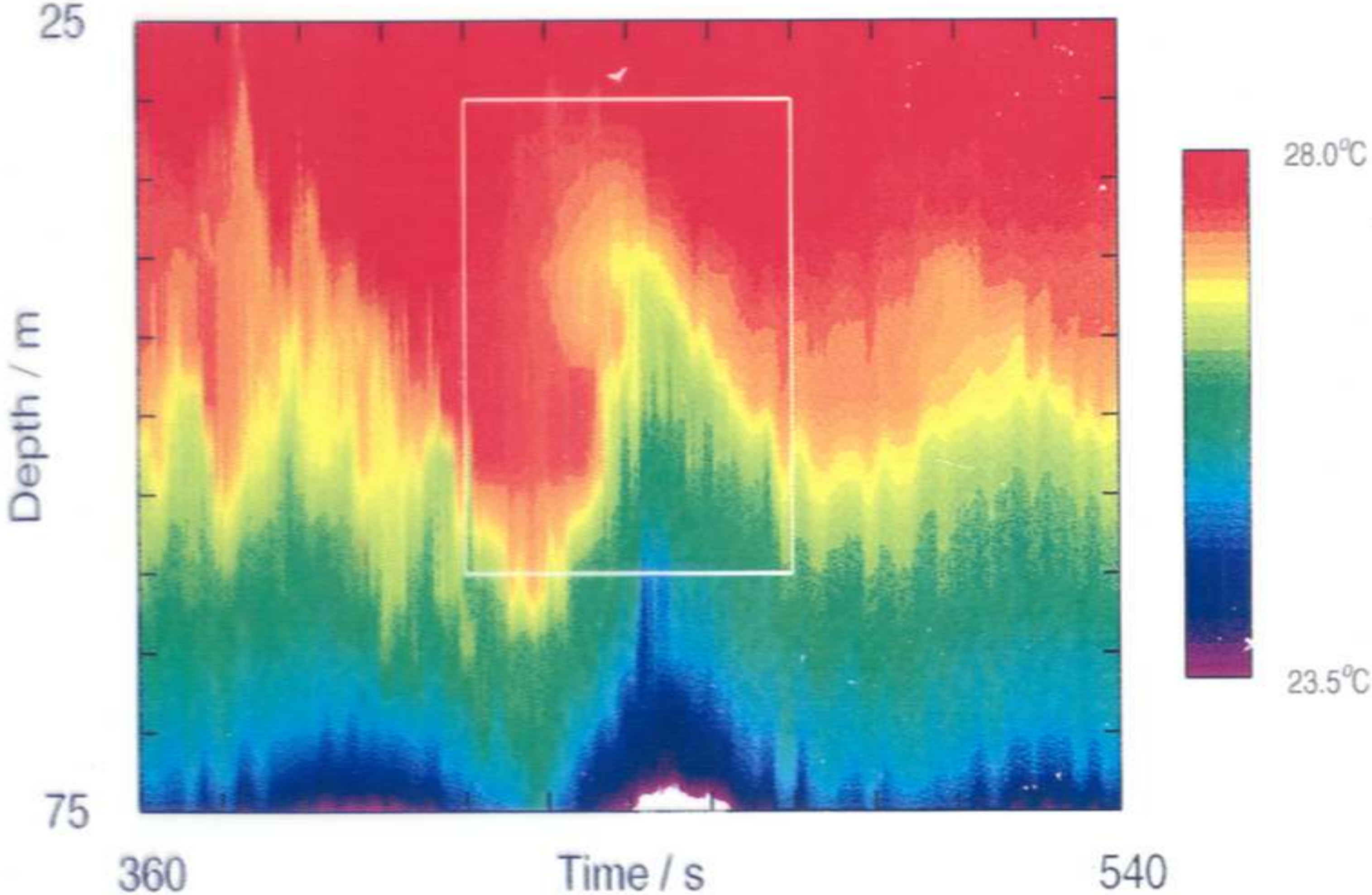
Oceans: Daytime Vertical Profile - Equatorial Undercurrent



Moum et al. 1990

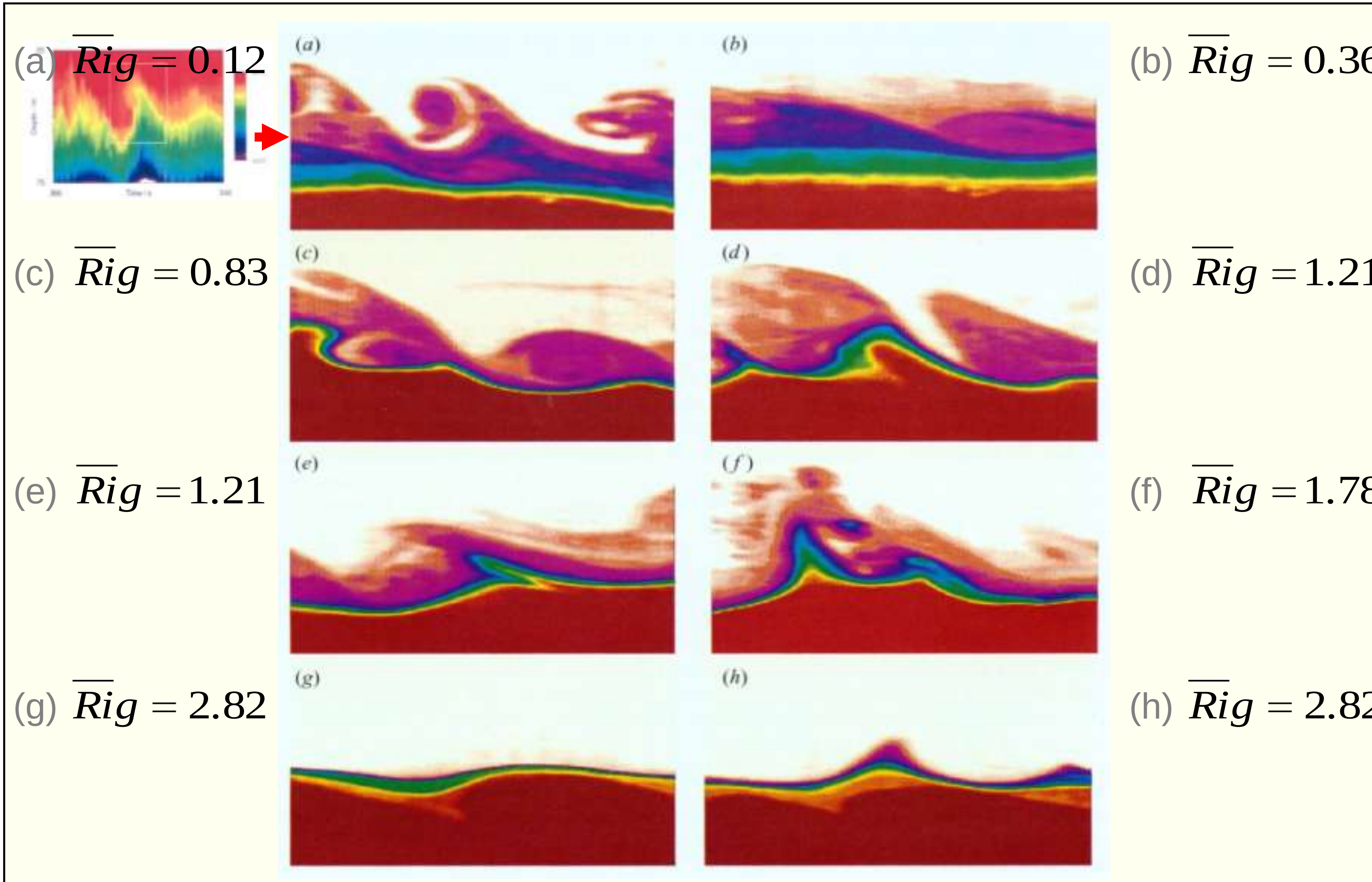


DeSilva, Fernando, Hebert & Eaton,
Earth Planetary Sci. Lett. , 1996

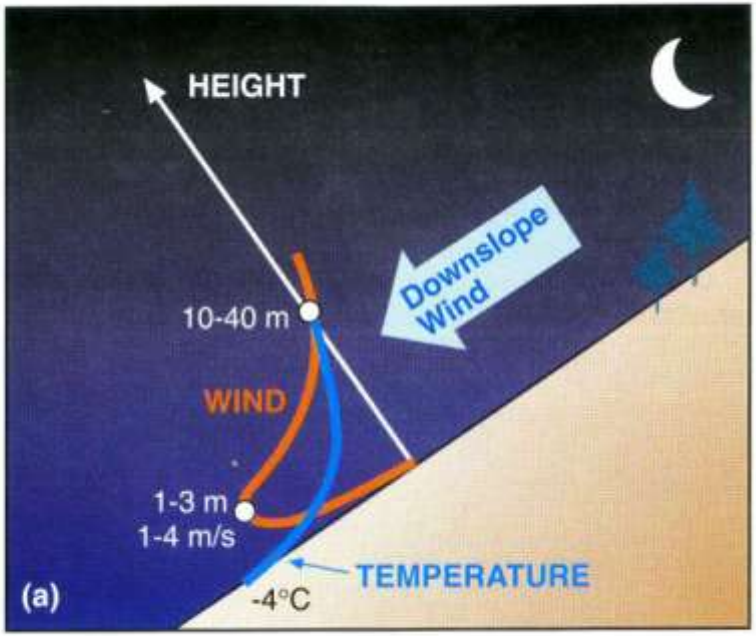


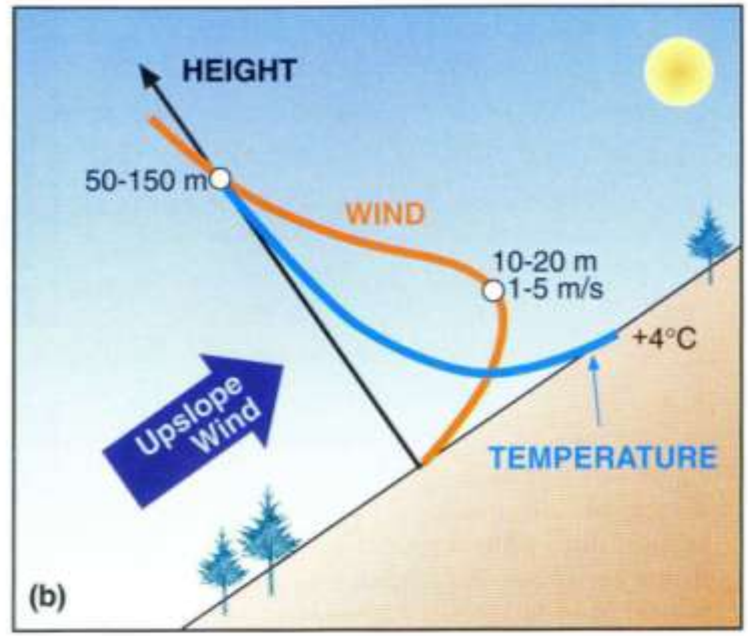
25
75
Depth / m
360
Time / s
540
28.0℃
23.5℃

Mechanisms of Entrainment



Complex Terrain SBL





Reproduced from *Mountain Meteorology* (2000). Courtesy of Dr. Whiteman, PNNL.

VTMX Campaign, Salt Lake City October, 2000

Desert Research Institute

Argonne National Laboratory

University of Utah

Pacific Northwest National Laboratory

NOAA/Environmental Technology Laboratory

University of Massachusetts

Oregon State University

Arizona State University

National Center for Atmospheric Research

Stanford University

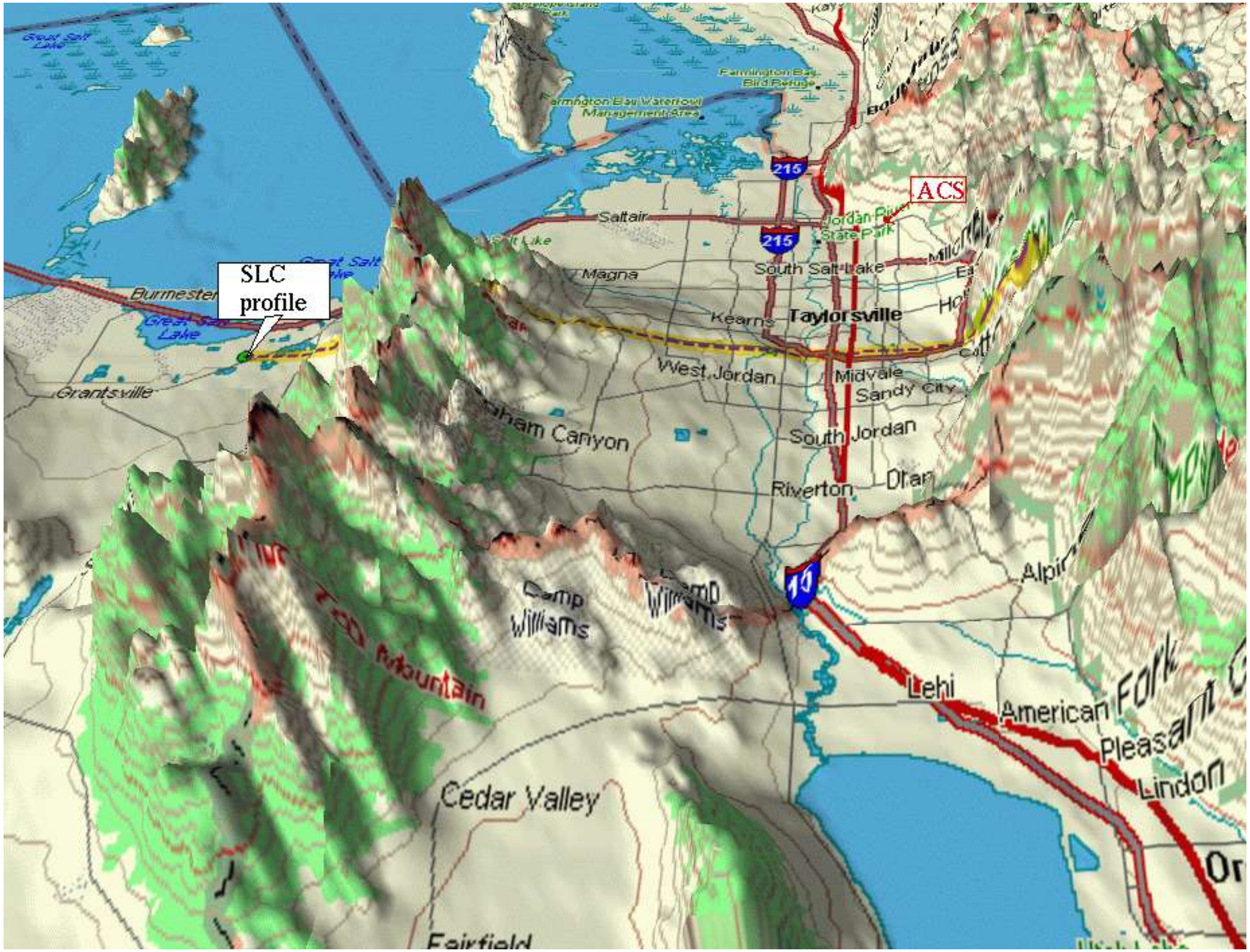
Brookhaven National Laboratory

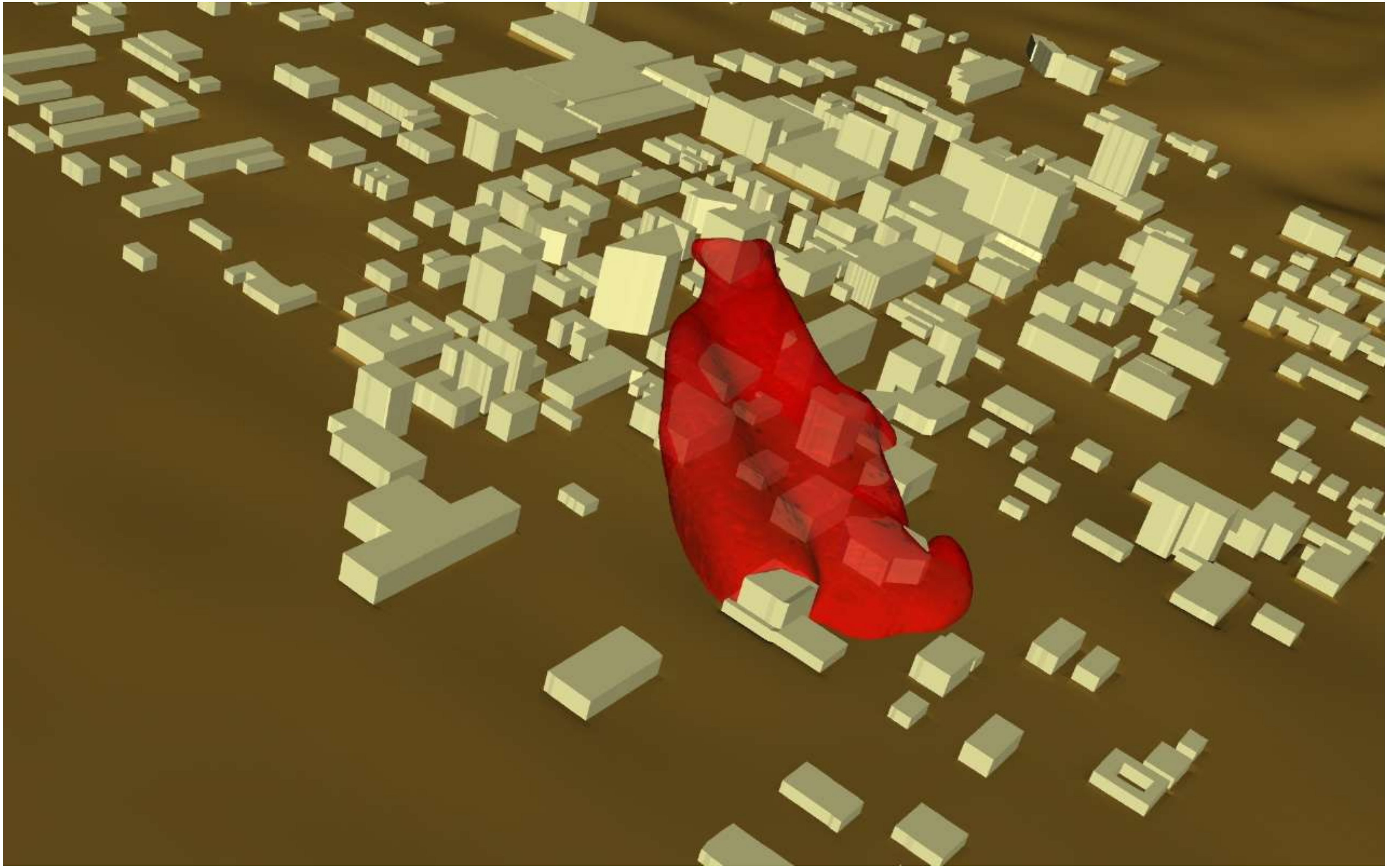
Colorado Research Associates (CoRA)

Los Alamos National Laboratory

National Oceanic and Atmospheric Administration

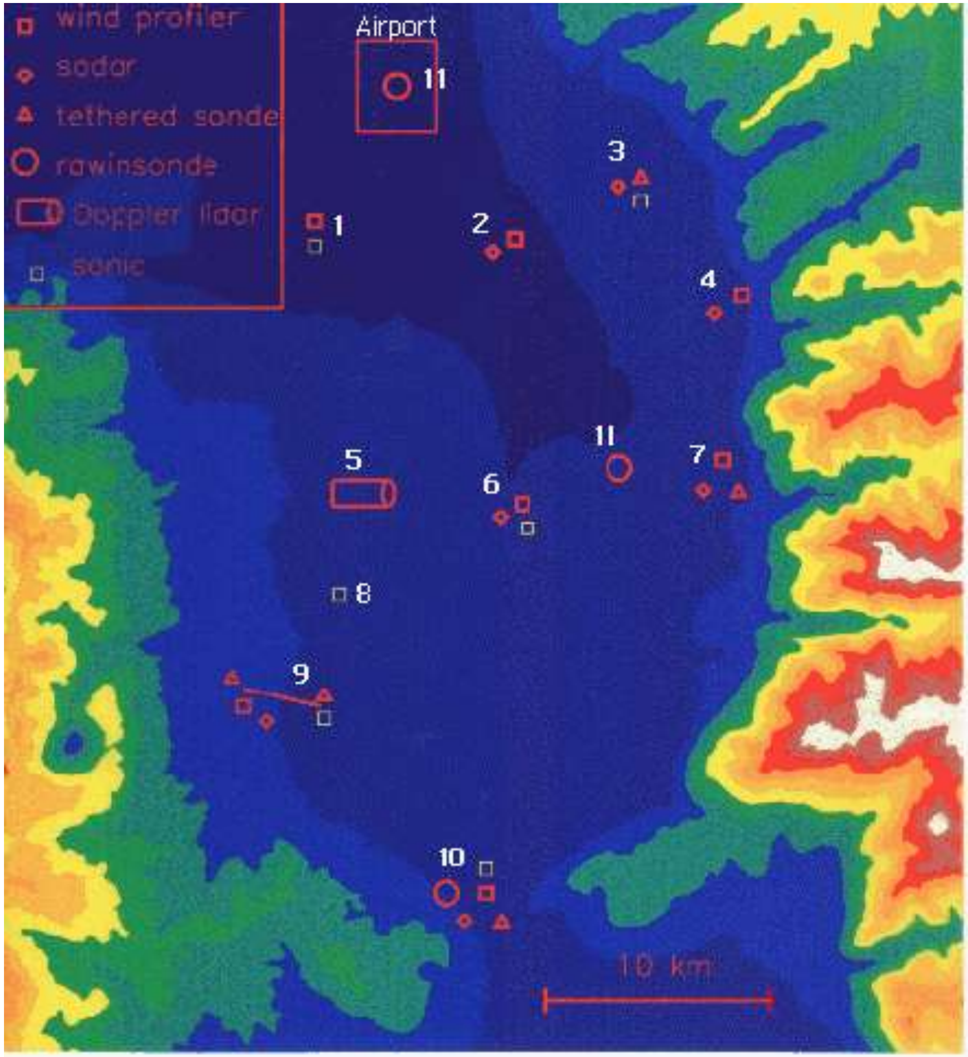
Salt Lake City





Simulated gas release in Salt Lake City. Gas plume in Red. Computer model shared with ASU by Lawrence Livermore National Laboratory.

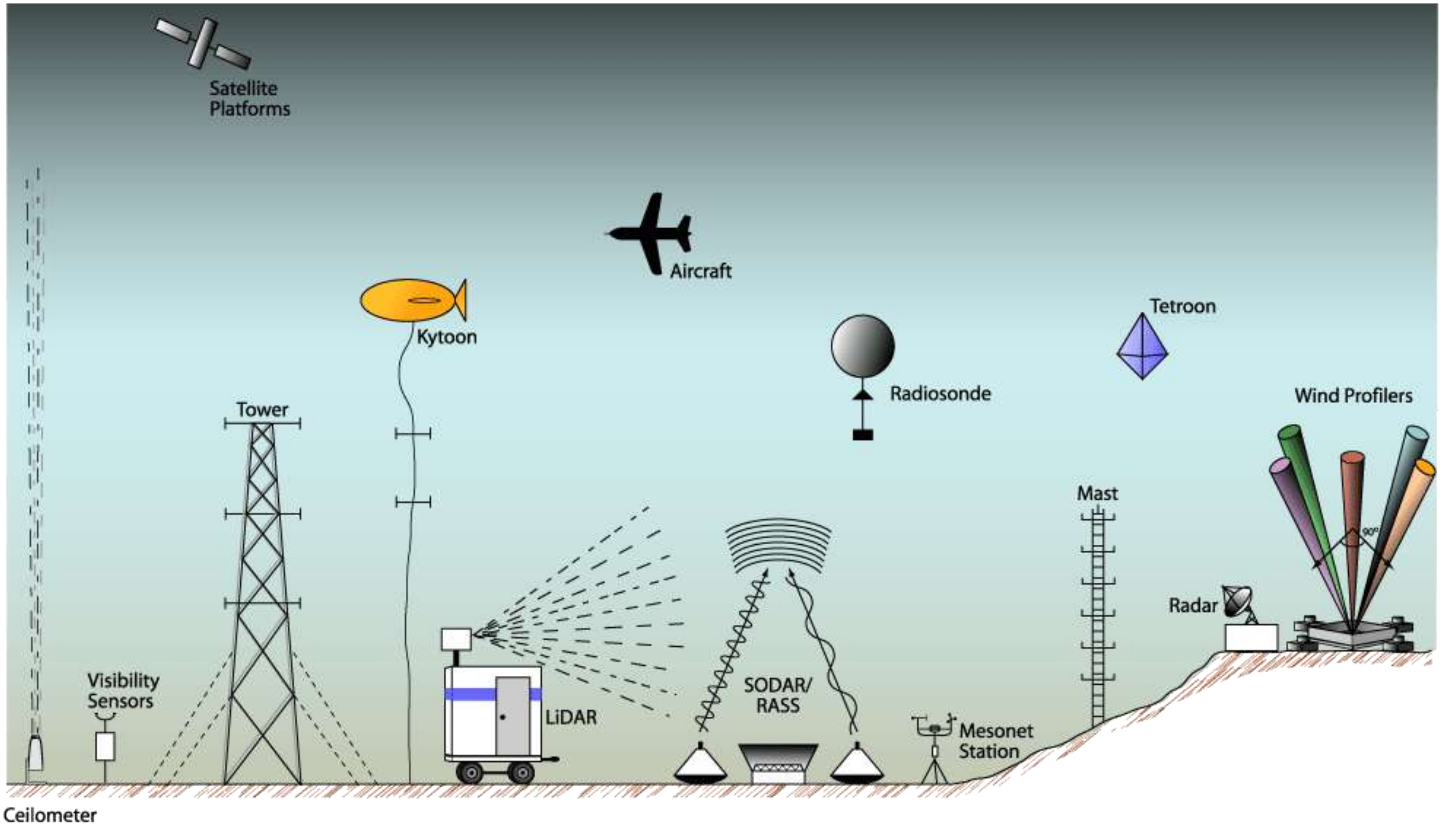
Participants and their sites



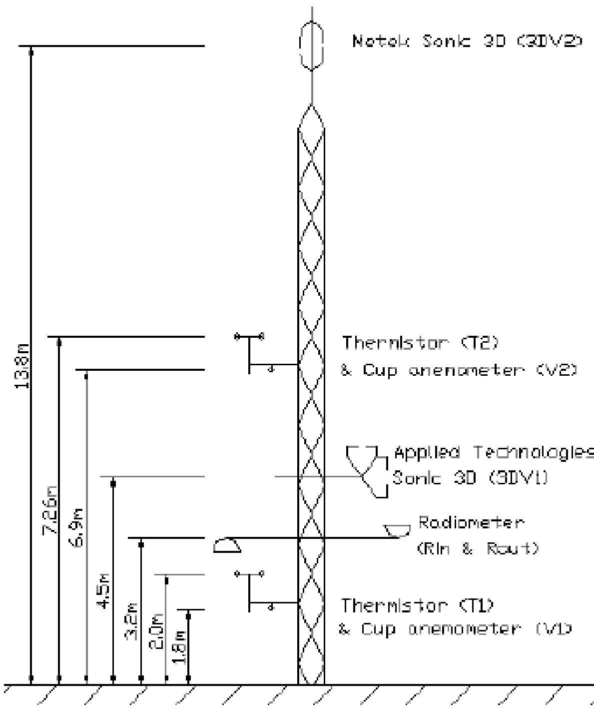
1- UMass
2- NOAA (Idaho Falls)
3- **ASU**, LLNL
4- LANL
5- NOAA (ETLab)
6- PNNL (Will Shaw)
7- ARGON Nlab
8- NOAA (ATDD Oak Ridge)
9- PNNL (Dave Whiteman)
10- NCAR
11- NWS

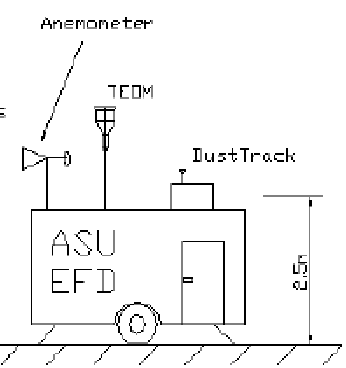
from: http://www.met.utah.edu/vtmx/

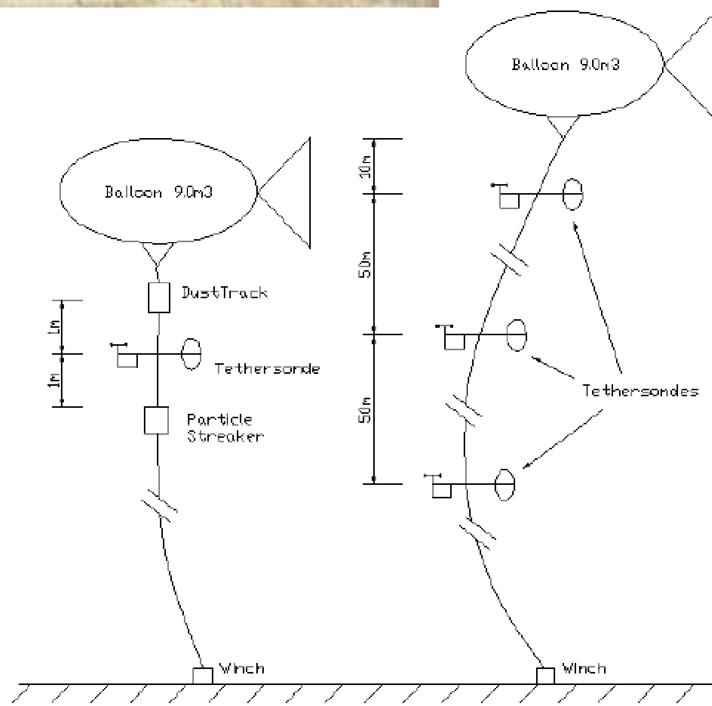
A Typical Field Experiment



VTMX ASU Equipment





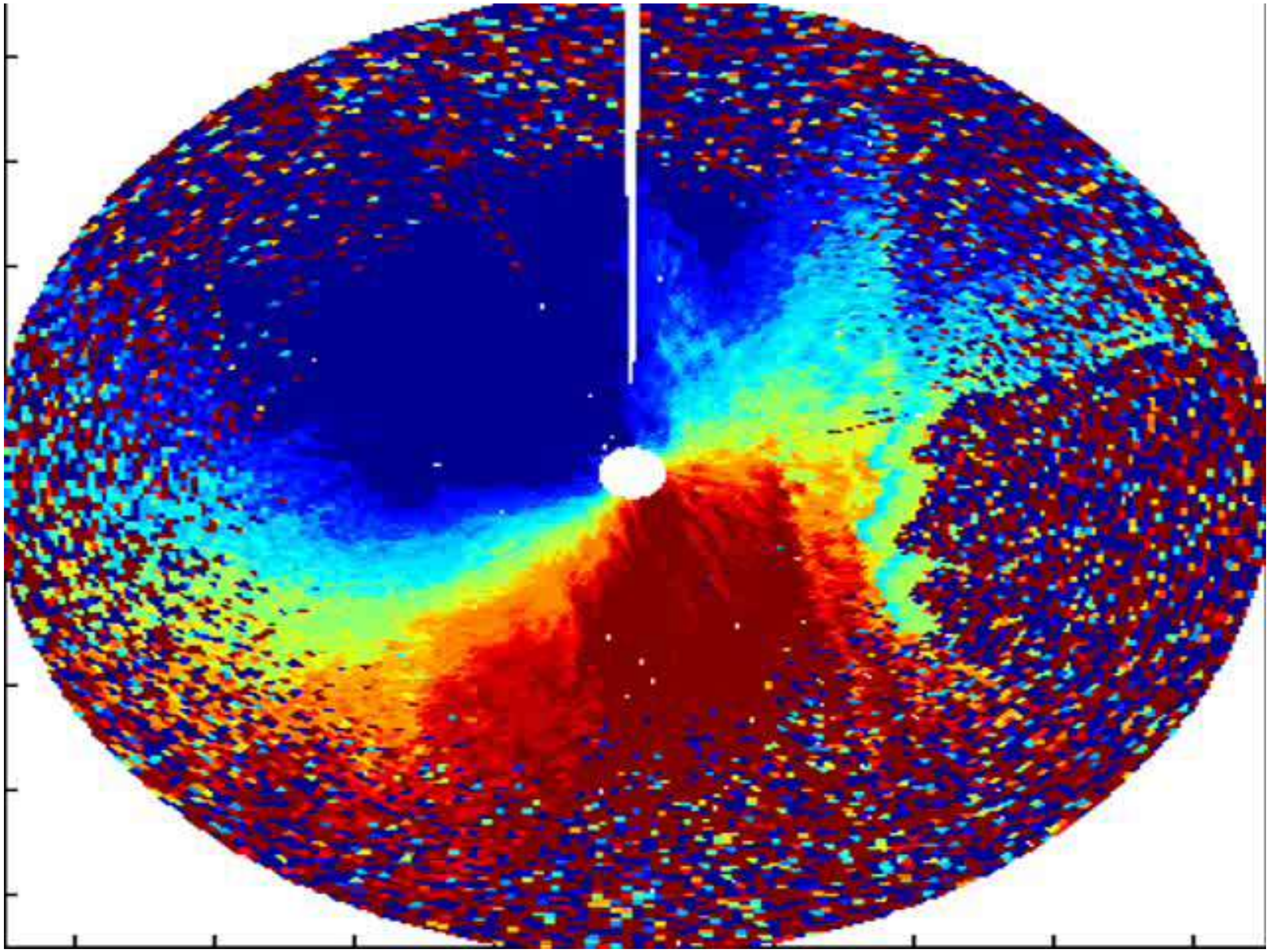


Network

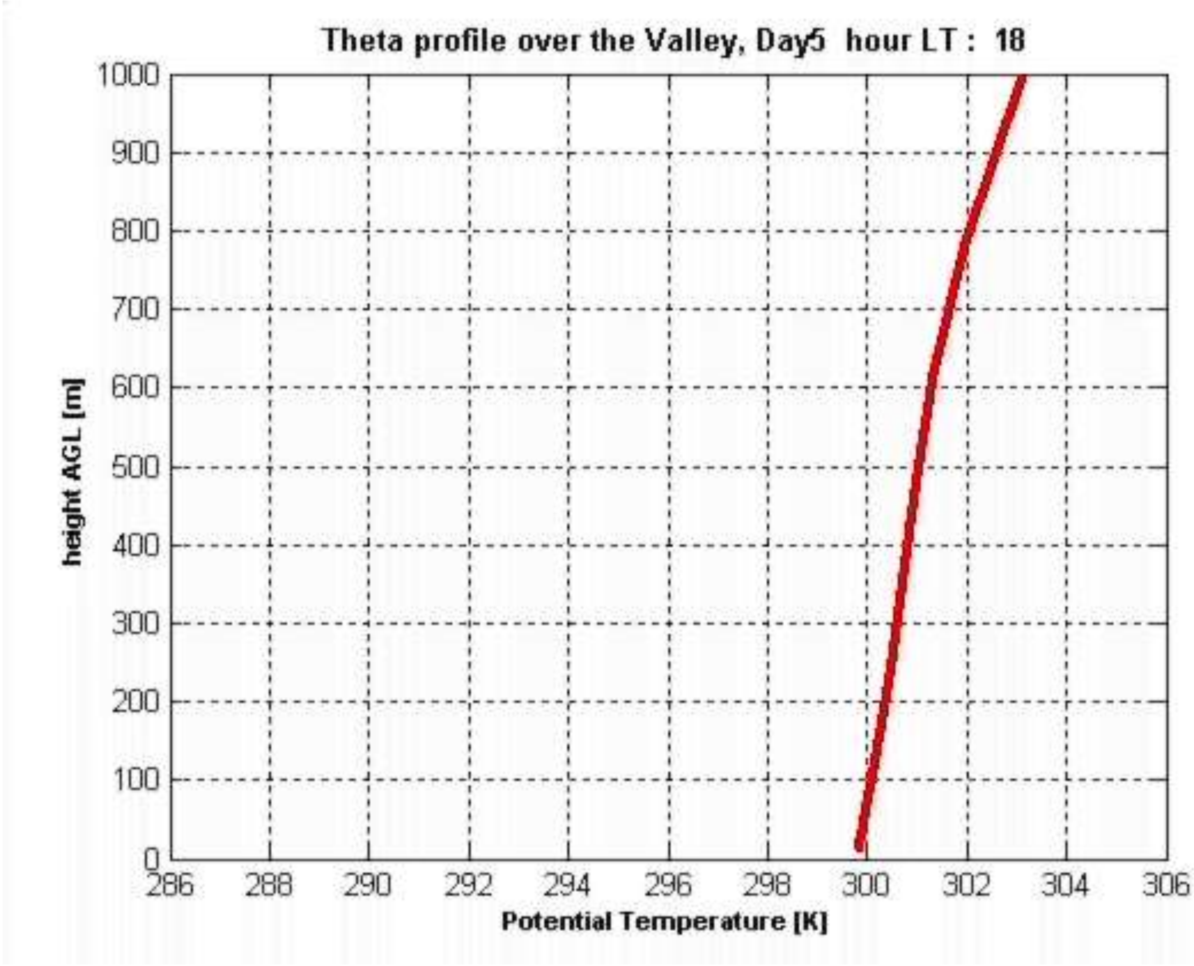
sodar ceilometer radar

ASU Doppler Lidar

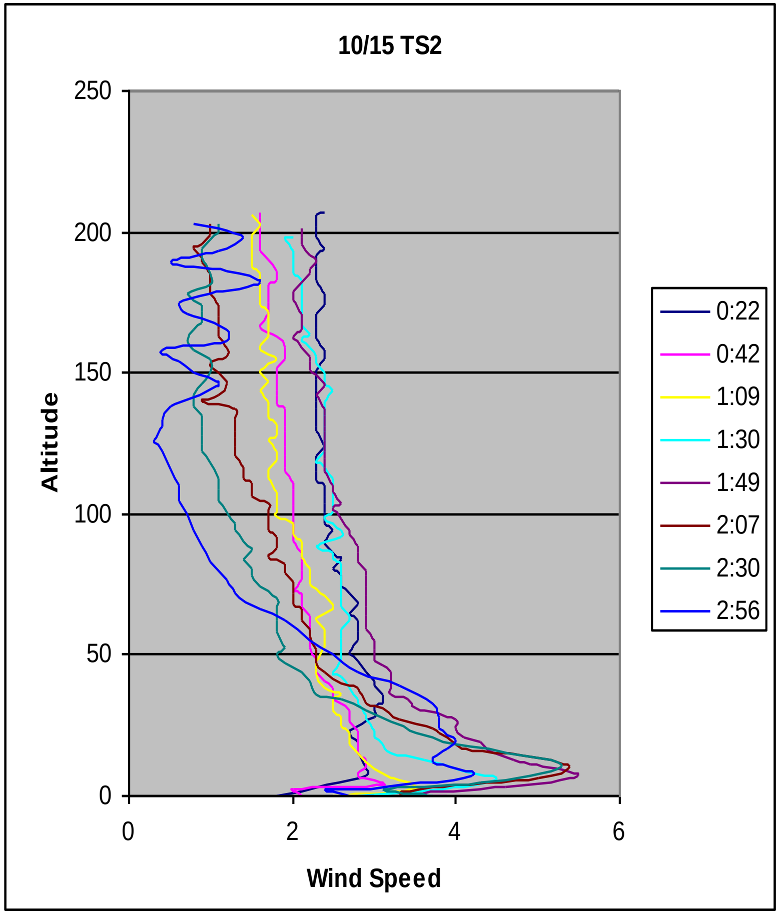
Wind Fields

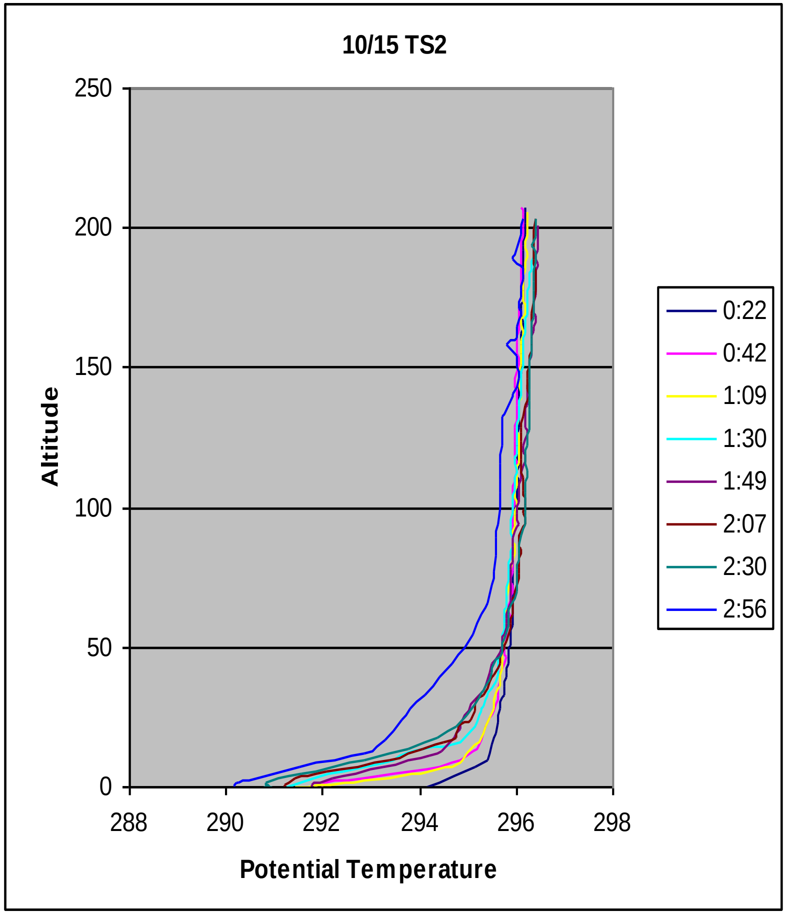


Theta profile in the valley

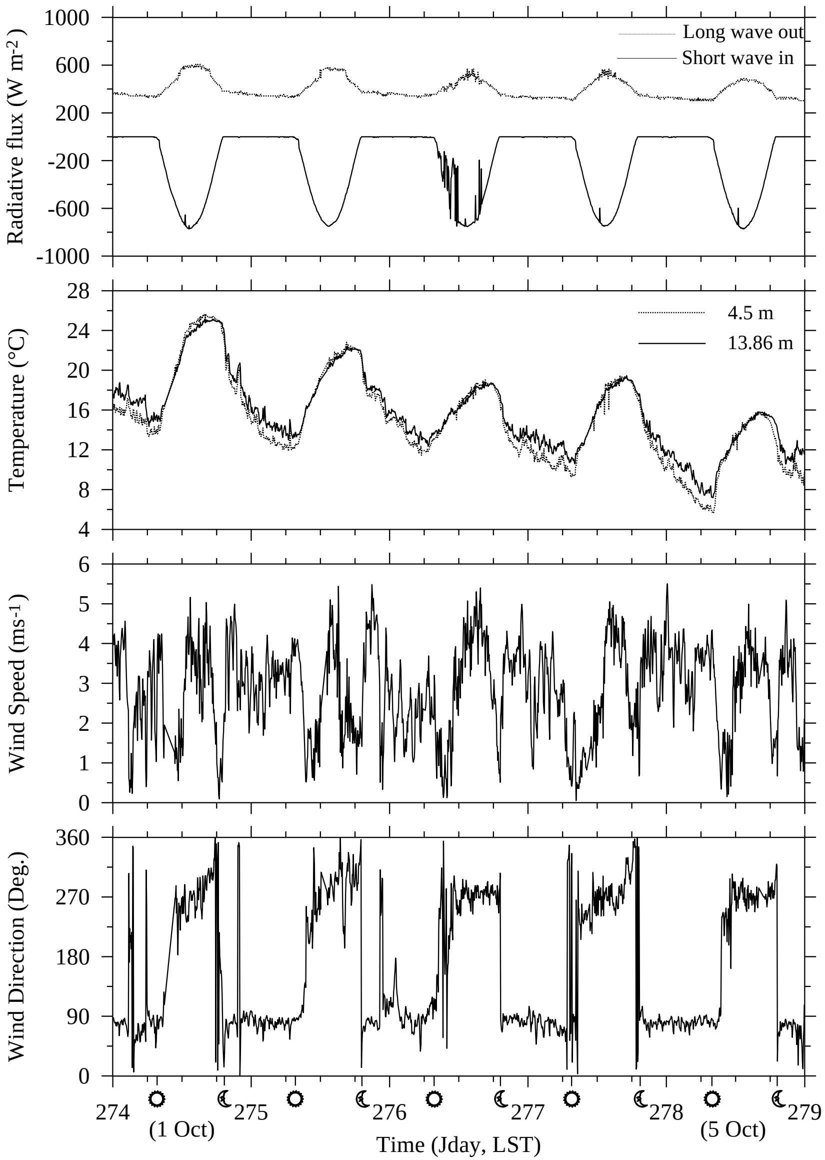


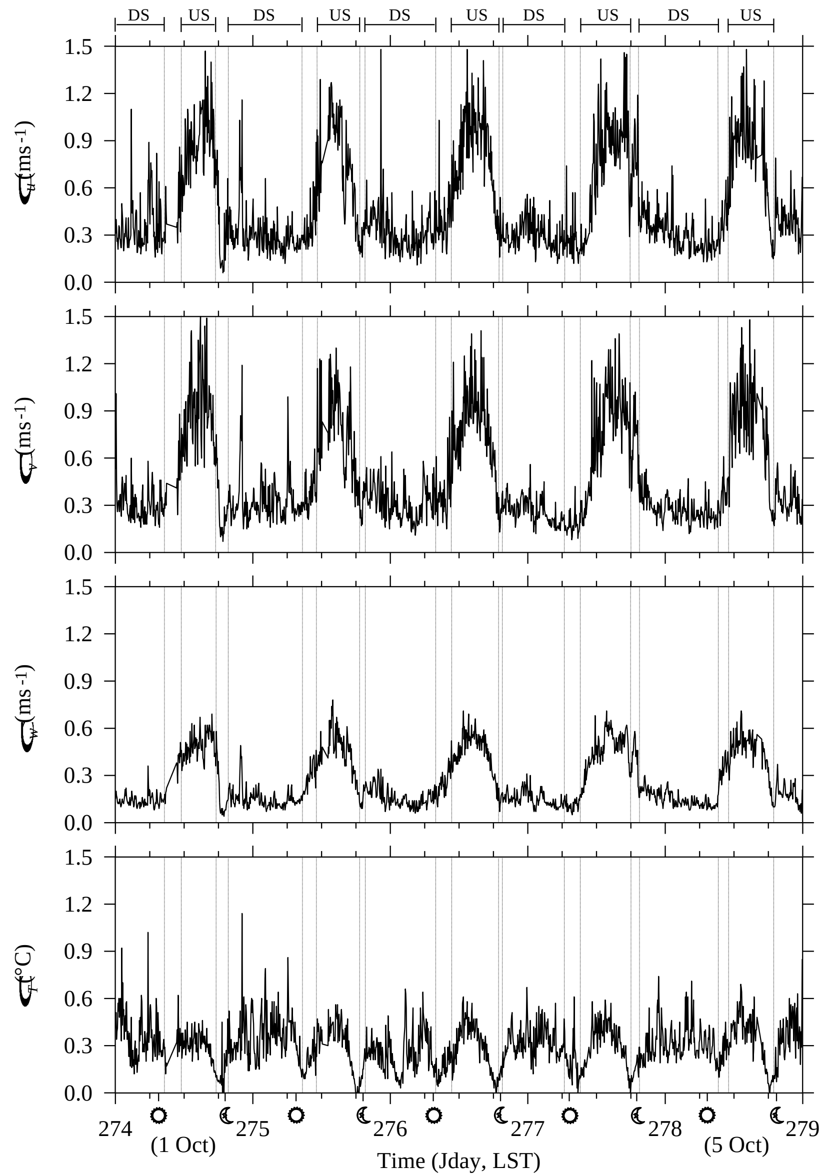
Downslope – Field Data





VTMX Measurements





Flow Analysis

Idealized Slope Flow Analysis

$$\frac{\partial u}{\partial s} + \frac{\partial w}{\partial n} = 0$$

$$Uh \qquad -EU$$

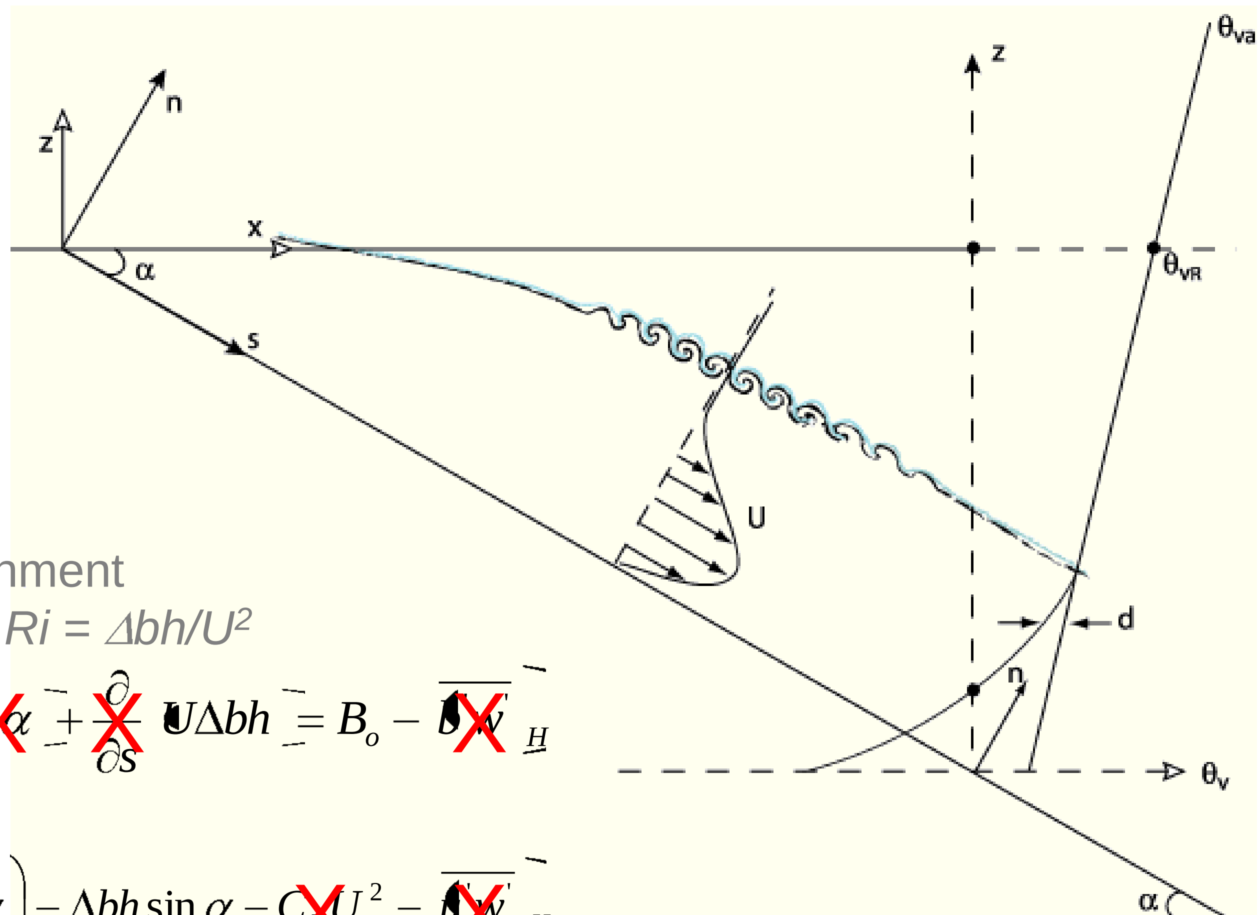
$$\frac{\partial}{\partial s}\int_0^H u\,dn + w_H = 0$$

$$\frac{\partial}{\partial s}(Uh) = EU$$

Entrainment Coefficient, *Ri = Δbh/U*²

$$\frac{\partial}{\partial t}(\Delta b h) + UhN^2(\sin\alpha - E\cos\alpha) + \frac{\partial}{\partial s}(U\Delta b h) = B_o - \left(\overline{b'w'}\right)_H$$

$$\frac{\partial Uh}{\partial t} + \frac{\partial U^2 h}{\partial s} = \frac{\partial}{\partial s}\left(\frac{1}{2}\Delta b h^2\cos\alpha\right) - \Delta b h\sin\alpha - C_D U^2 - \left(\overline{u'w'}\right)_H$$



Downslope flow - Pulsation

Linearized governing equations with neglected flux divergence and the entrainment-rate,

$$\frac{\partial U}{\partial t} = -a\Delta b$$

$$\frac{\partial \Delta b}{\partial t} = a^2 N_E^2 U$$

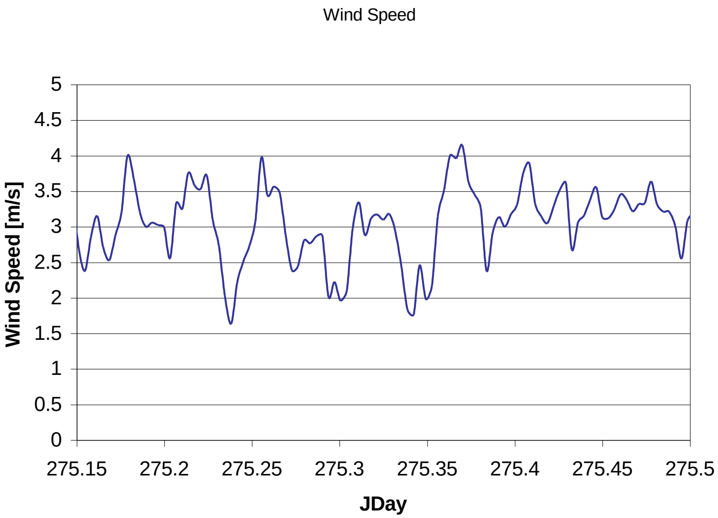
$$\frac{\partial^2 U}{\partial t^2} + a^2 N_E^2 U = 0$$

have oscillatory solution with the frequency

$$\omega = N_E \sin \alpha$$

or period

$$T \sim \frac{2\pi}{N_E \sin \alpha}$$



Journal of the Atmospheric Sciences, **65 (2)**, 627-643, 2008

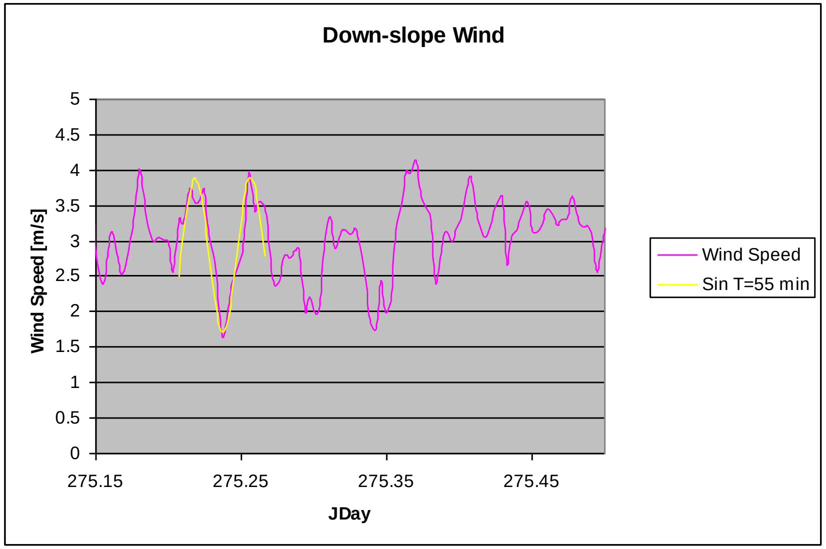
Downslope flow - Pulsation

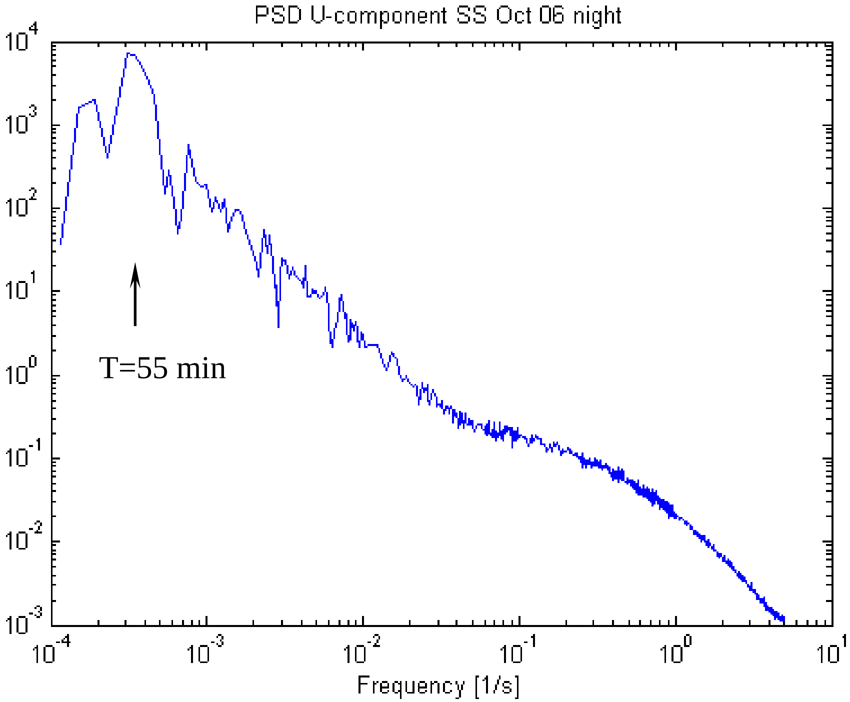
$$T \sim \frac{2\pi}{N_E \sin\alpha}$$

$$\omega = N_E \sin\alpha$$

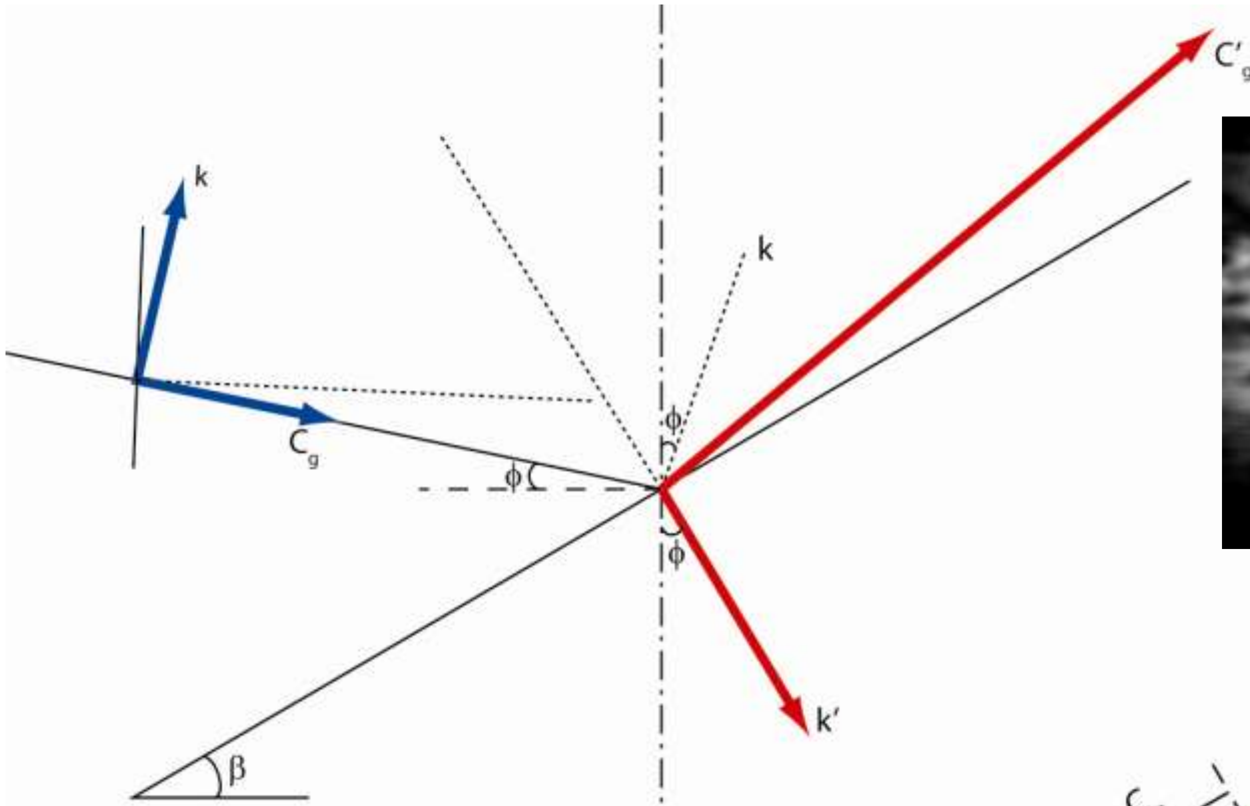
ACS $\alpha = 4.7$ deg: T=20 – 50 min

SS $\alpha = 1.8$ deg: T=50 – 130 min



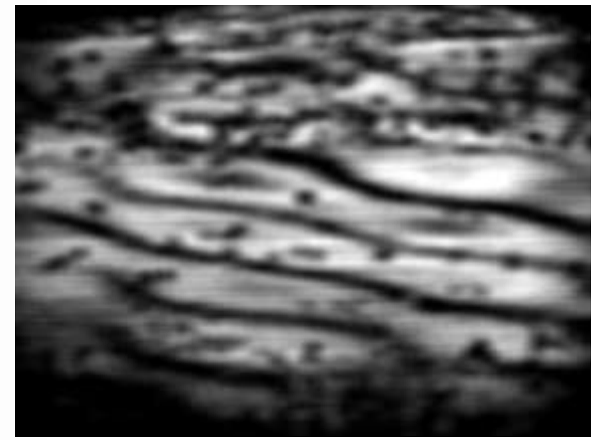


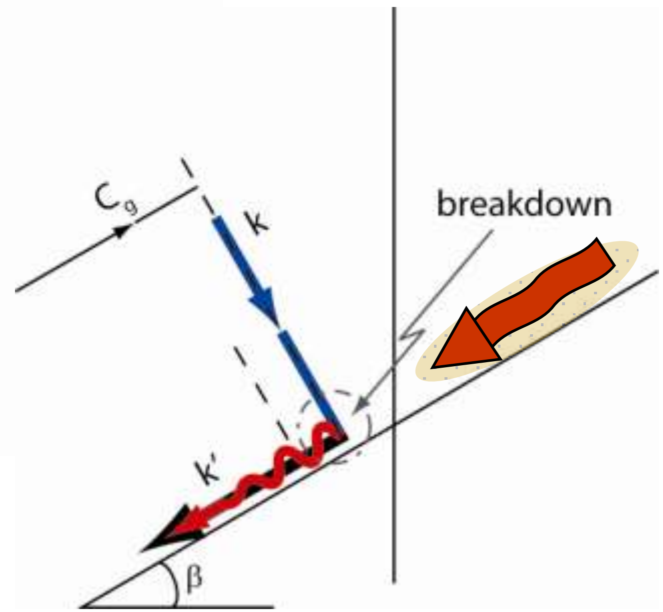
$\omega = aN_E = N_E \sin\beta$ *Critical slopes*



$$k \sin(\phi+\beta) = k' \sin(\phi-\beta)$$

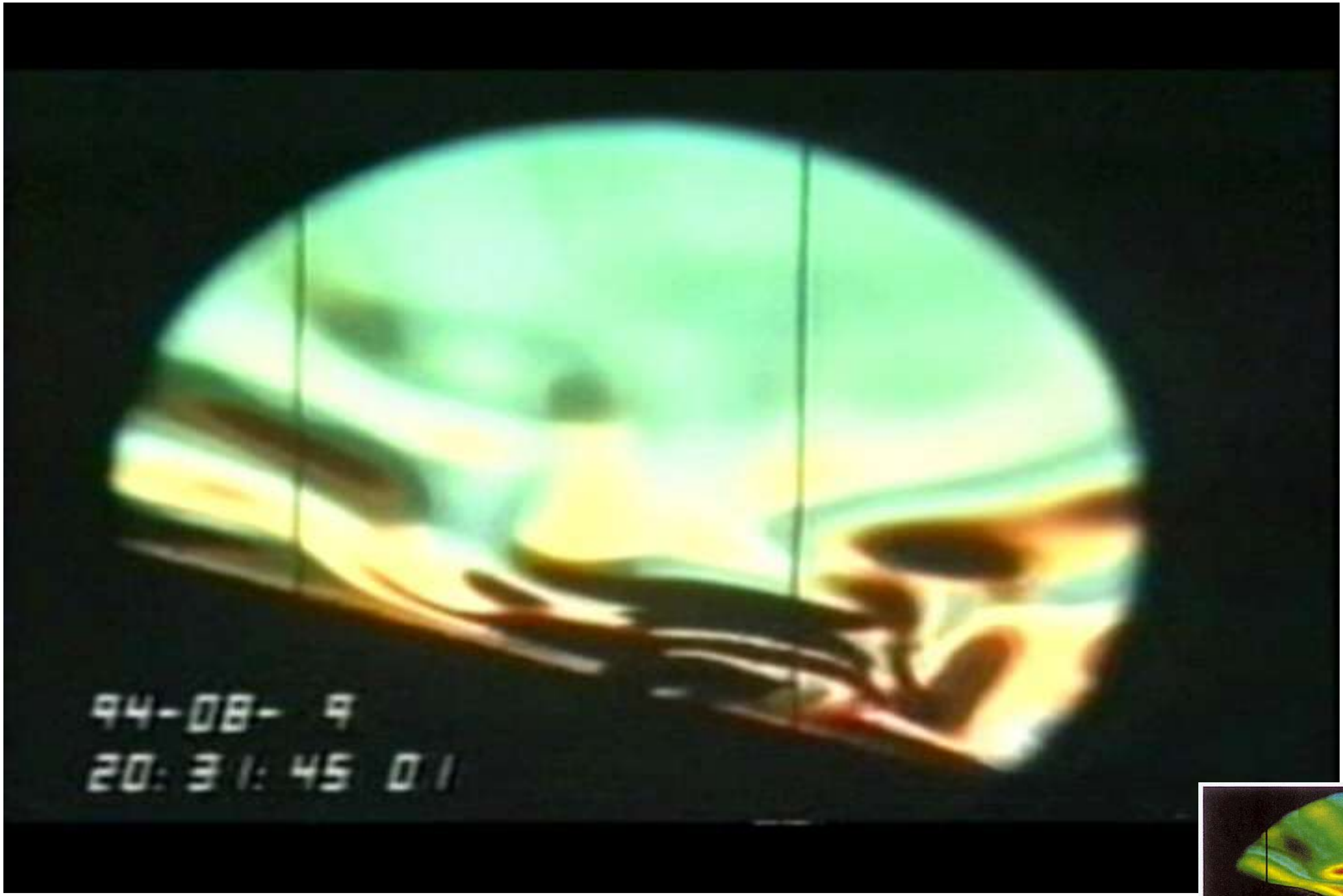
$$k' \to \infty \qquad \phi = \beta$$

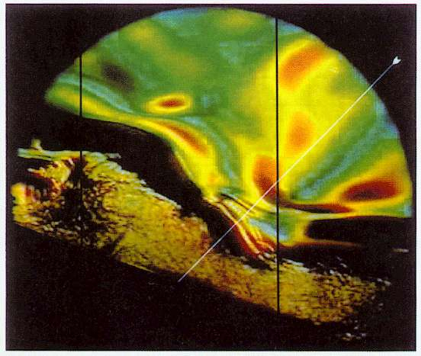




$\omega = N \cos\theta$

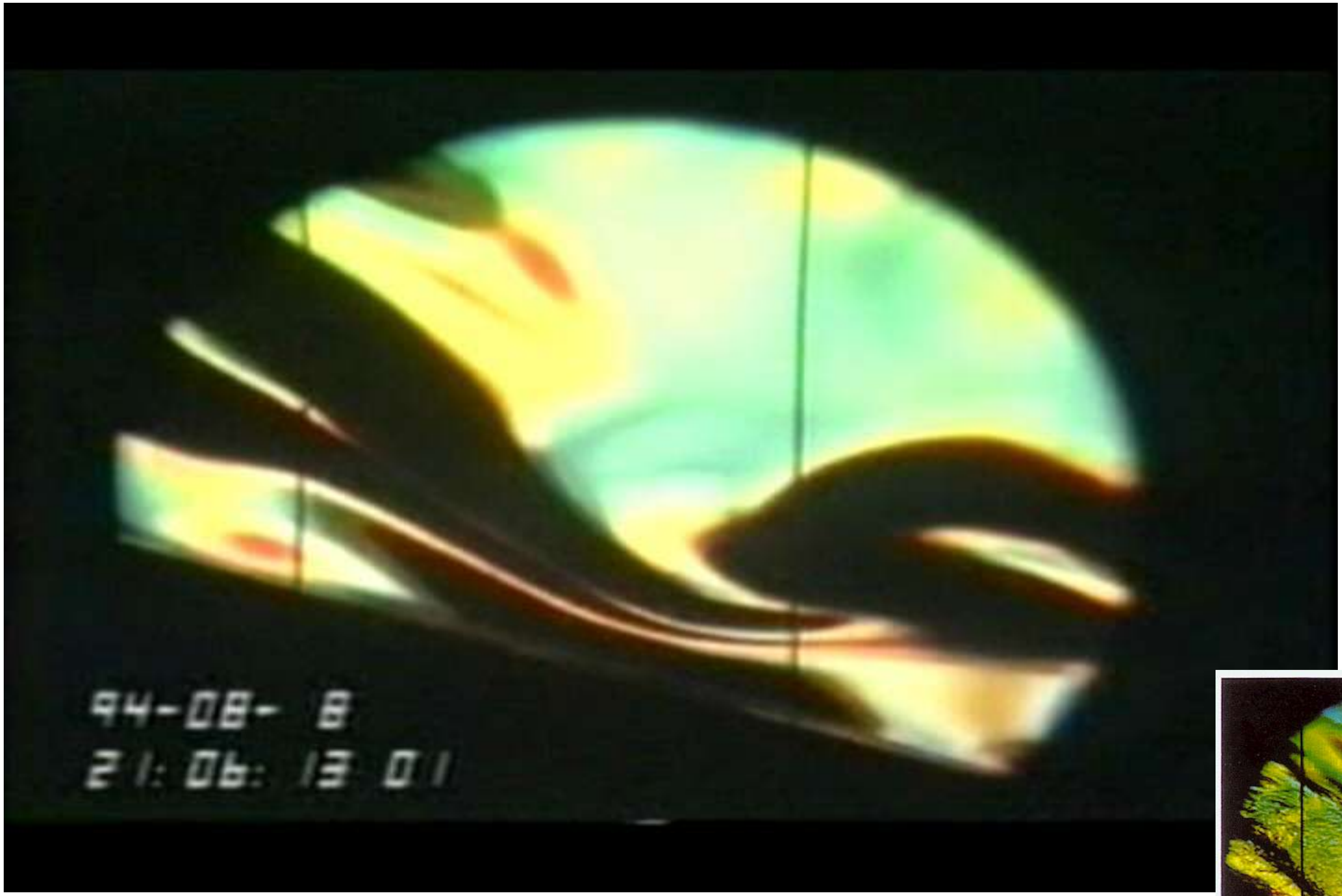
Subcritical angles

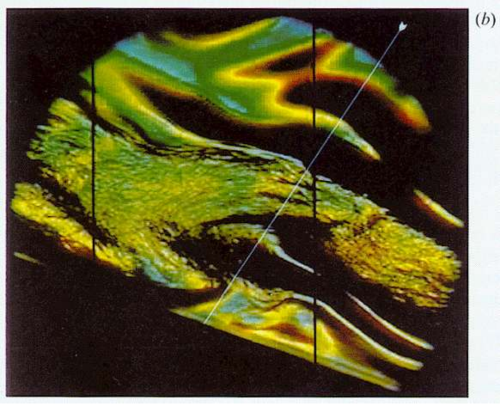




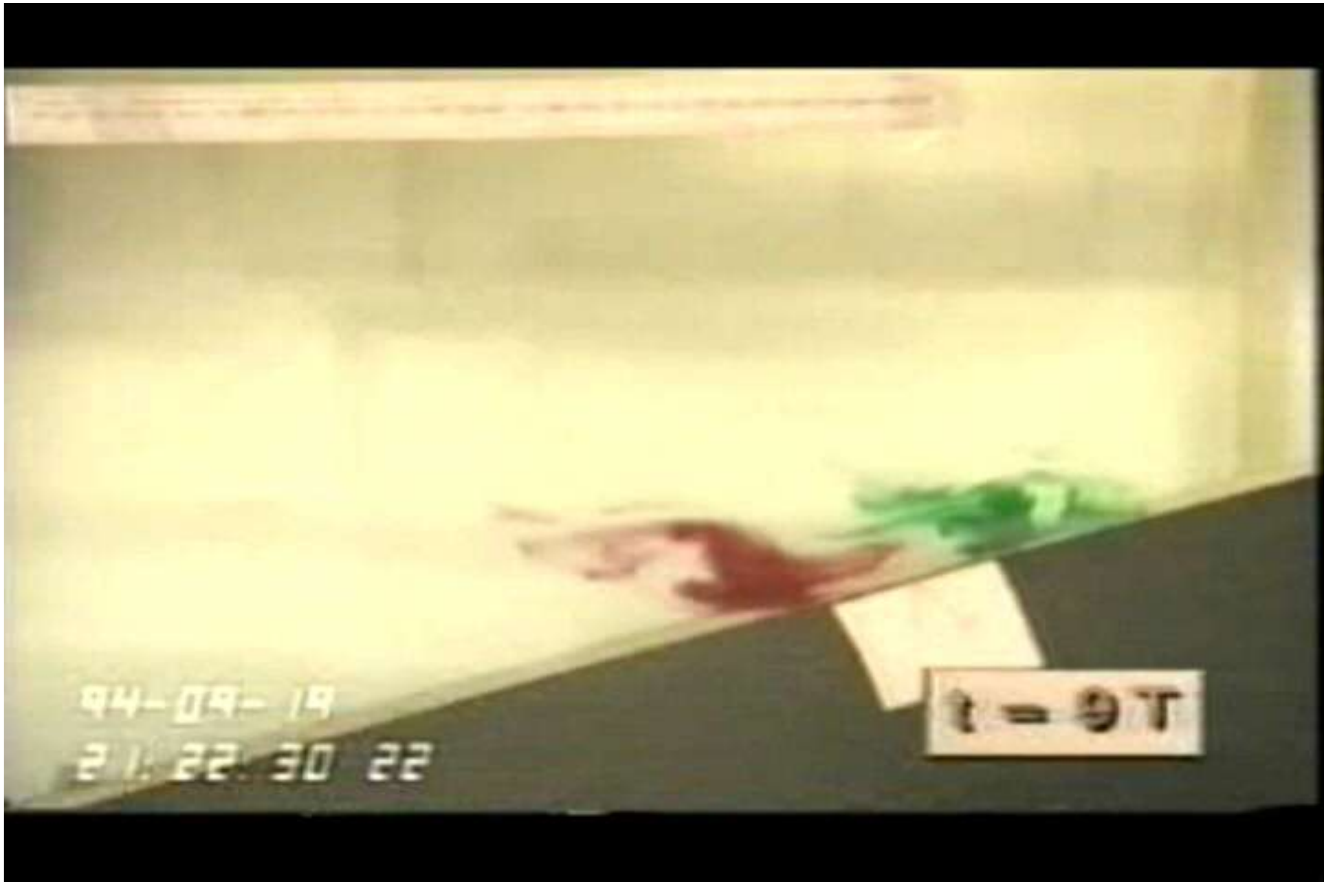
(a)

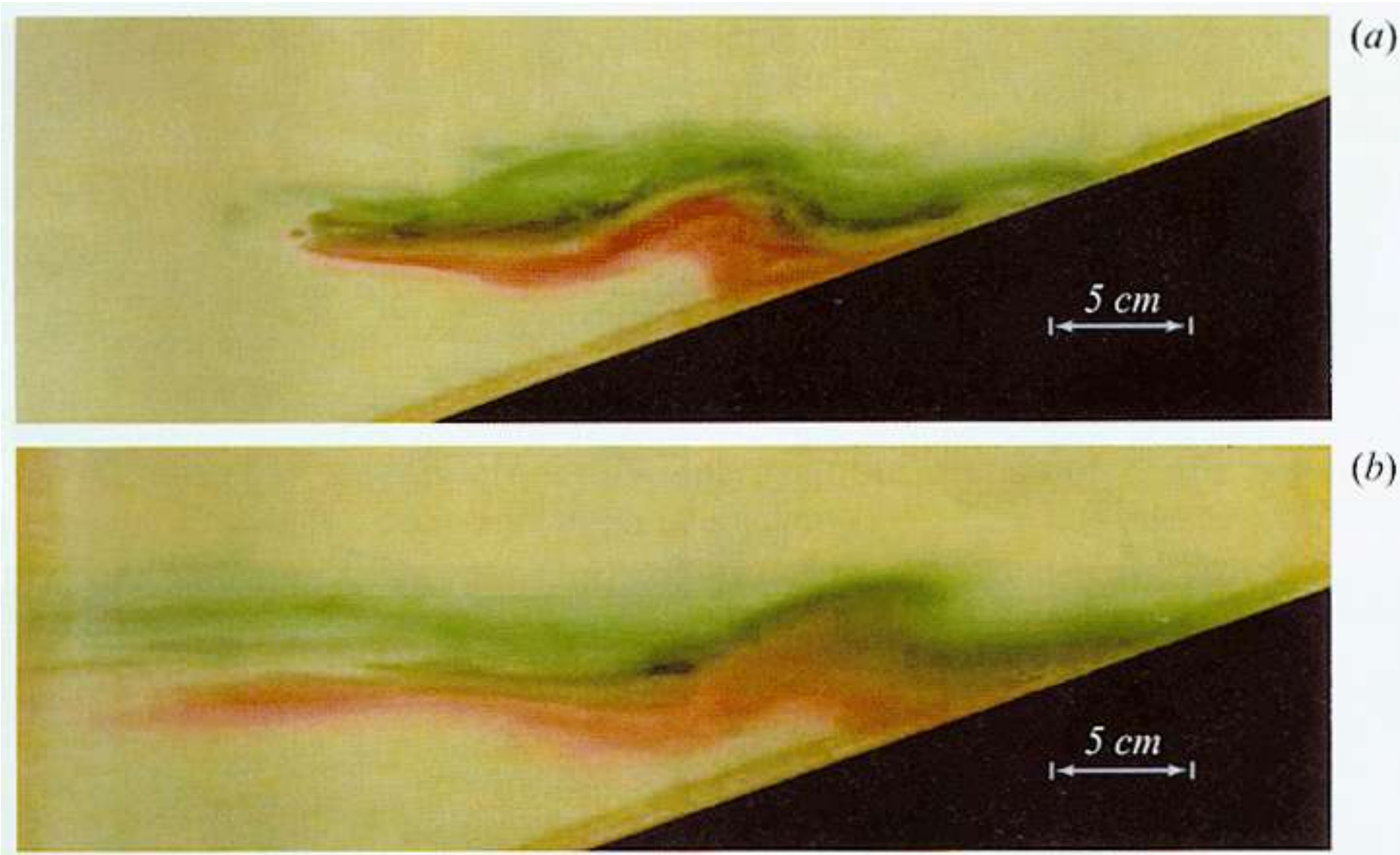
Close to critical angle



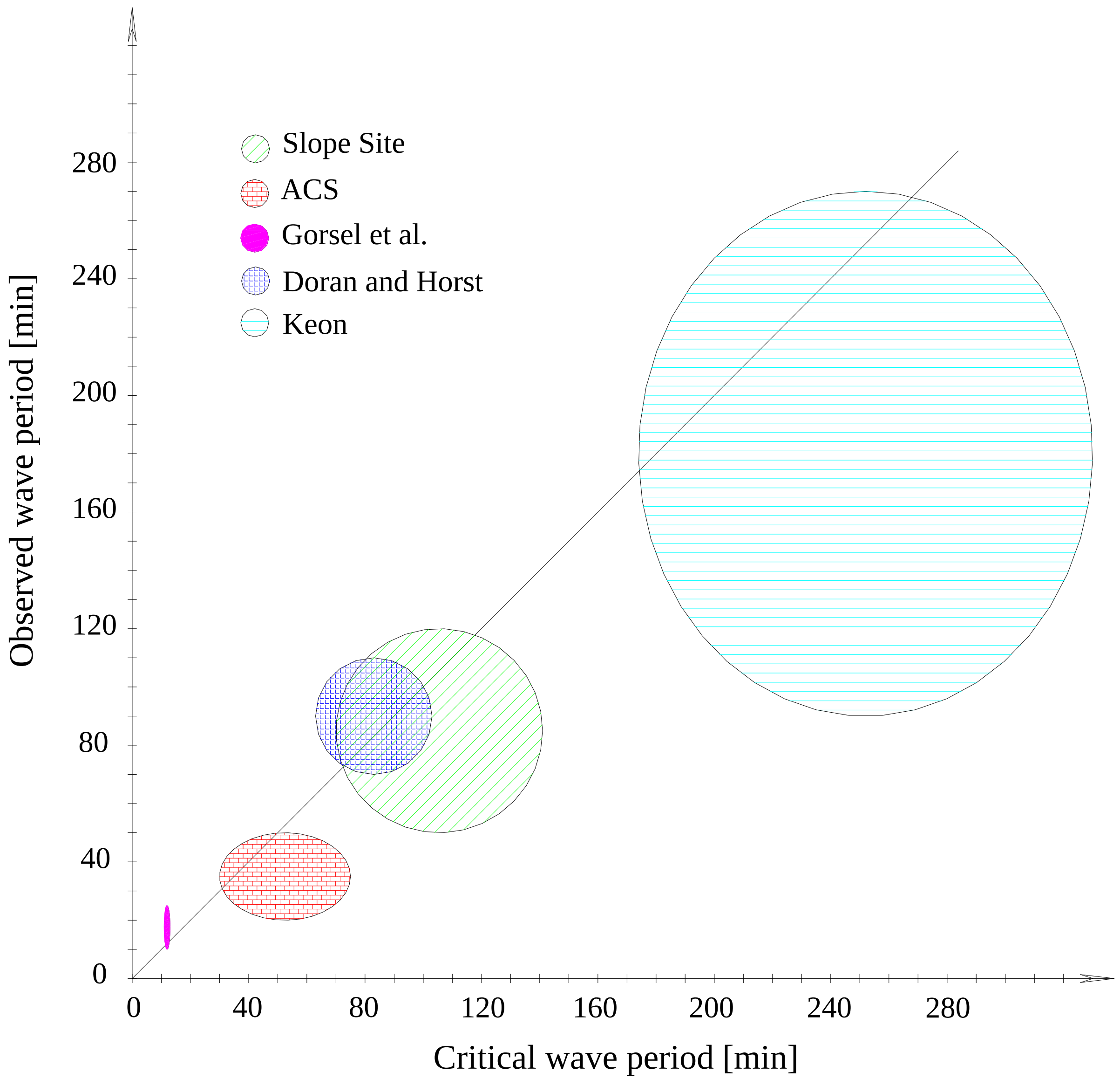


Intrusions keep BL turbulent




(a)
5 cm
(b)
5 cm

Other Observations



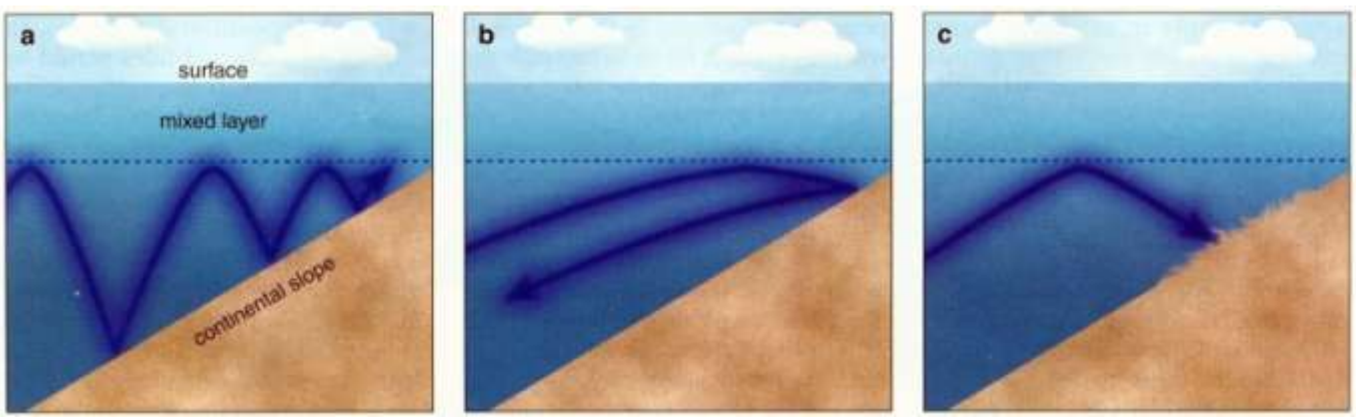
- the Riviera valley (Gorsel et al., ICAM/MAP proceedings, 2003)
- Cobb Mountain (Doran and Horst, JAM, 20(4), 361-364, 1981)
- Phoenix valley (Keon, Master Thesis, ASU, 1982)
- Slope and ACS sites of the VTMX campaign in Salt Lake City (Doran et al., BAMS, 83(4), 537-554).

American Scientist 2004

David A Cacchione & Lincoln F. Pratson (Am. Sci., 2004)

Hypothesis --

Tidal Internal waves coursing beneath the surface of the sea may shape the margins of the world's landmasses



$$\omega = N \sin \alpha$$

Flow Velocity

$$\frac{\partial Uh}{\partial t} + \frac{\partial U^2 h}{\partial s} = \frac{\partial}{\partial s}\left(\frac{1}{2}\Delta b h^2 \cos\alpha\right) - \Delta b h \sin\alpha - C_D U^2 - \left(\overline{u'w'}\right)_H$$

$$Uh\frac{\partial U}{\partial s} + U\frac{\partial Uh}{\partial s} \Rightarrow EU^2 \qquad \frac{\partial}{\partial s}\left(Uh\right) = EU$$

High Ri $\longrightarrow$ Entrainment is Unimportant

$$U \approx \lambda_u \left(\Delta b L_H \sin\alpha\right)^{1/2}$$

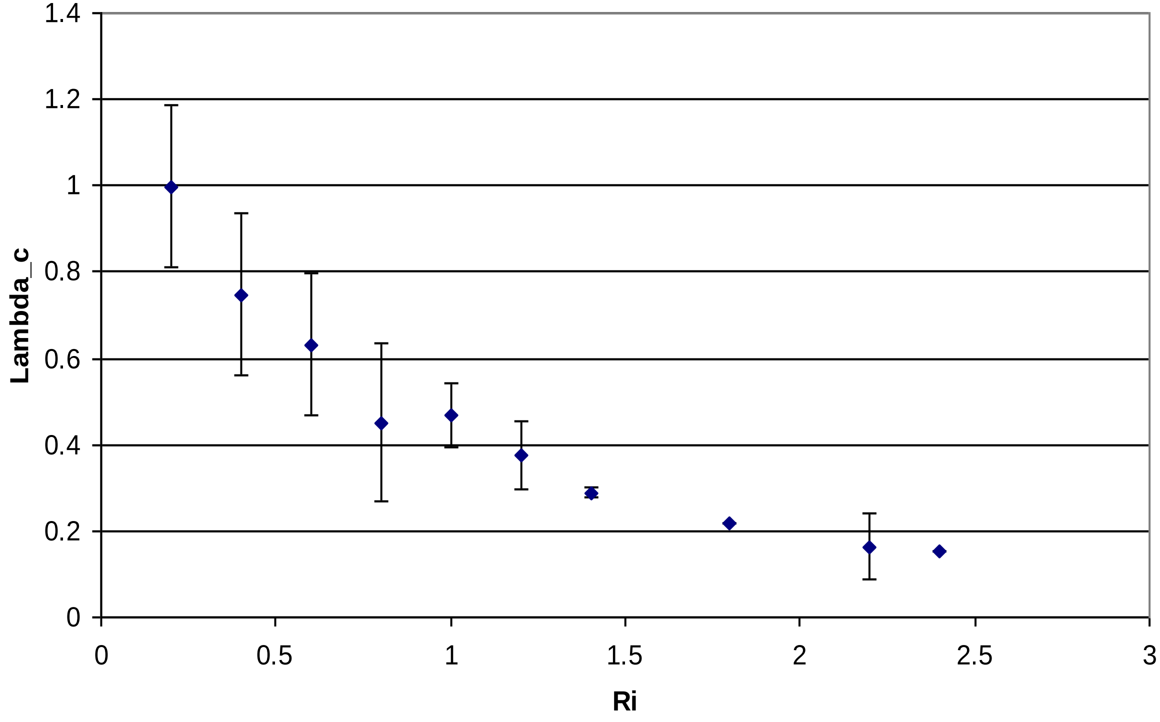
(Businger & Rao 1966)

Low Ri $\longrightarrow$ Entrainment is dominant

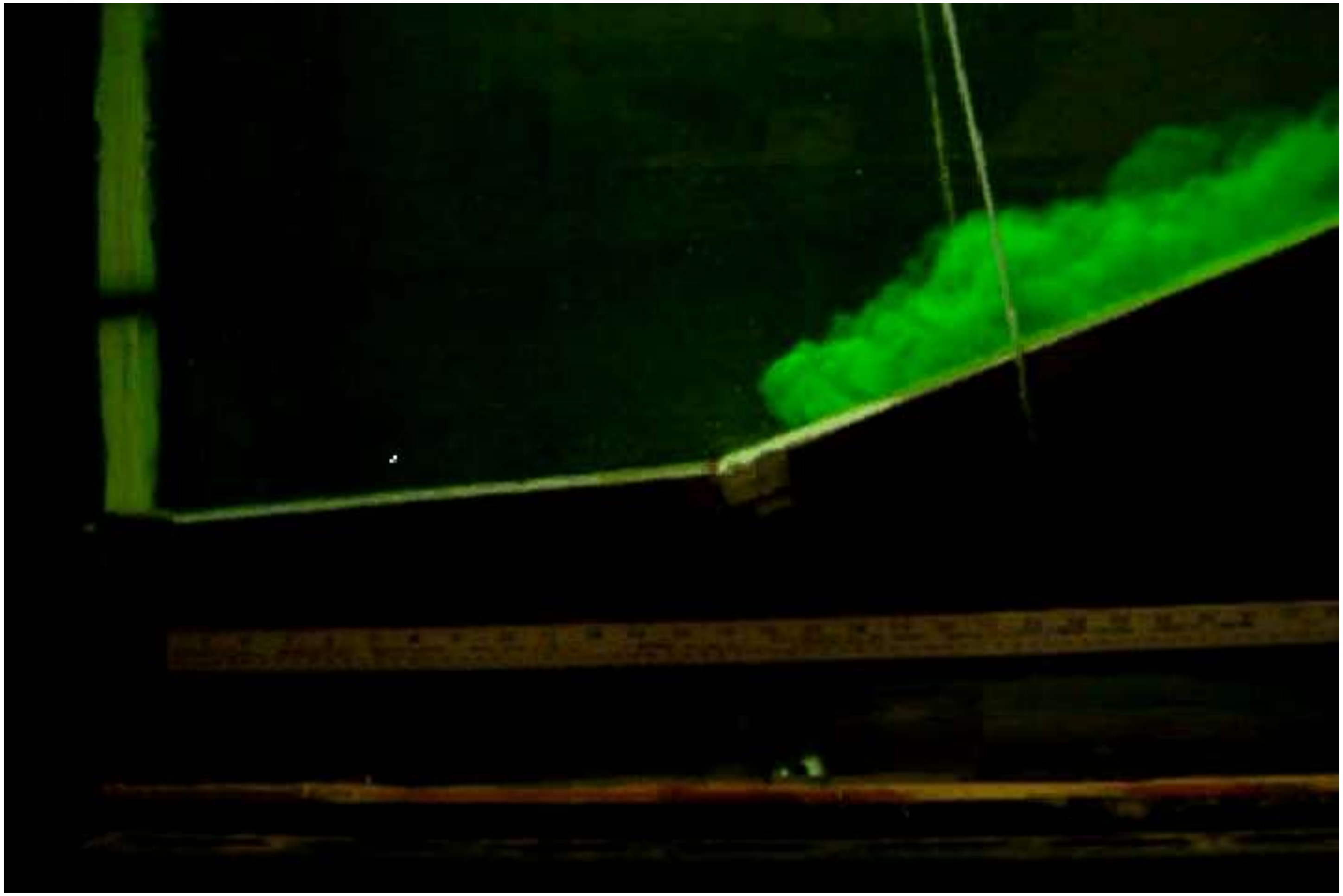
$$U \approx \lambda_{u_1}\left(\frac{\Delta b h \sin\alpha}{E}\right)^{1/2}$$

High Ri   Entrainment is Unimportant

$$U \approx \lambda_u \left(\Delta b L_H \sin\alpha\right)^{1/2}$$

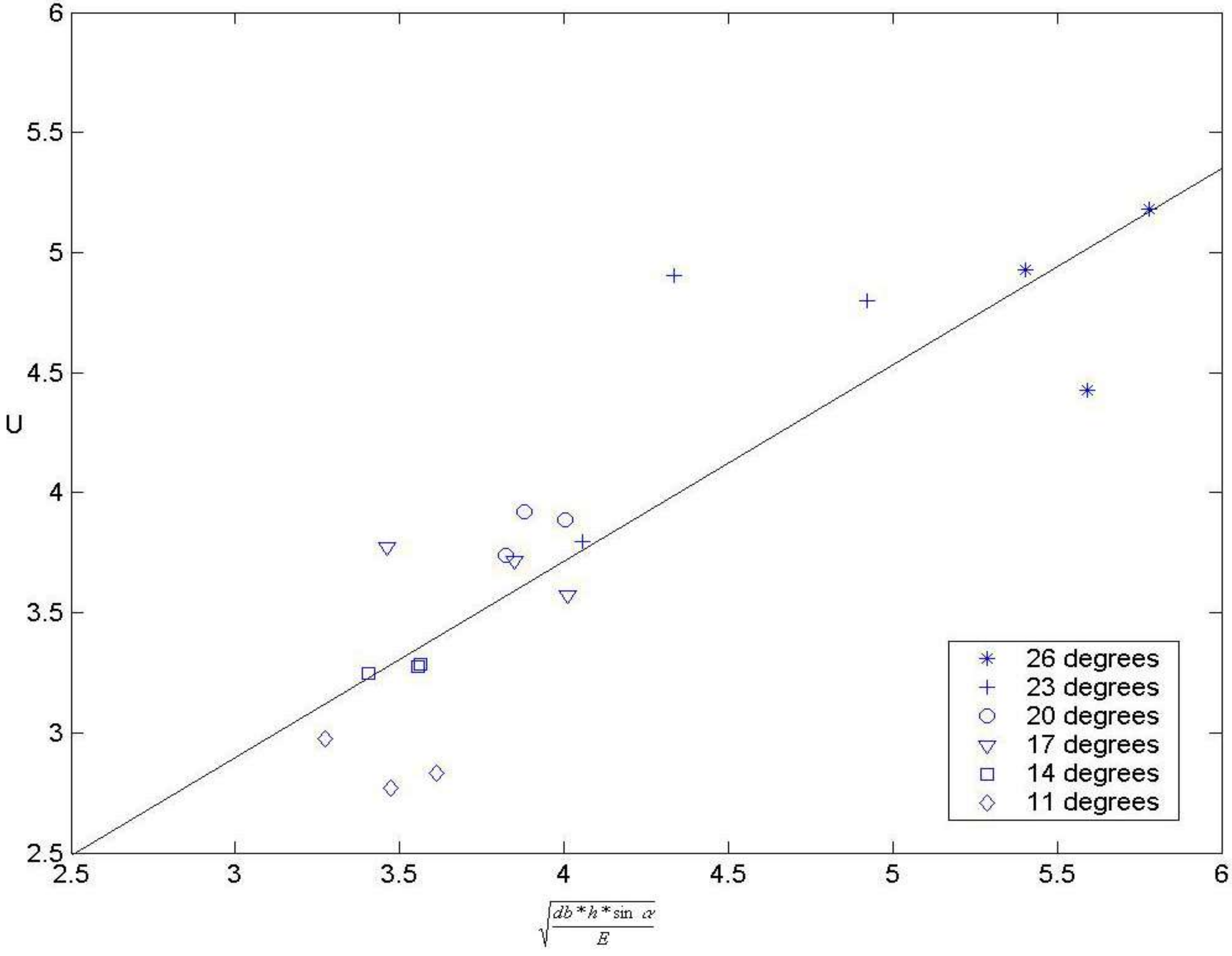


$$U \approx \lambda_{u1}\left(\frac{\Delta bh\sin\alpha}{E}\right)^{1/2}$$



$$U \approx \lambda_{u_1} \left(\frac{\Delta b h \sin \alpha}{E} \right)^{1/2}$$

Low Ri   Entrainment is dominant



Entrainment Velocity

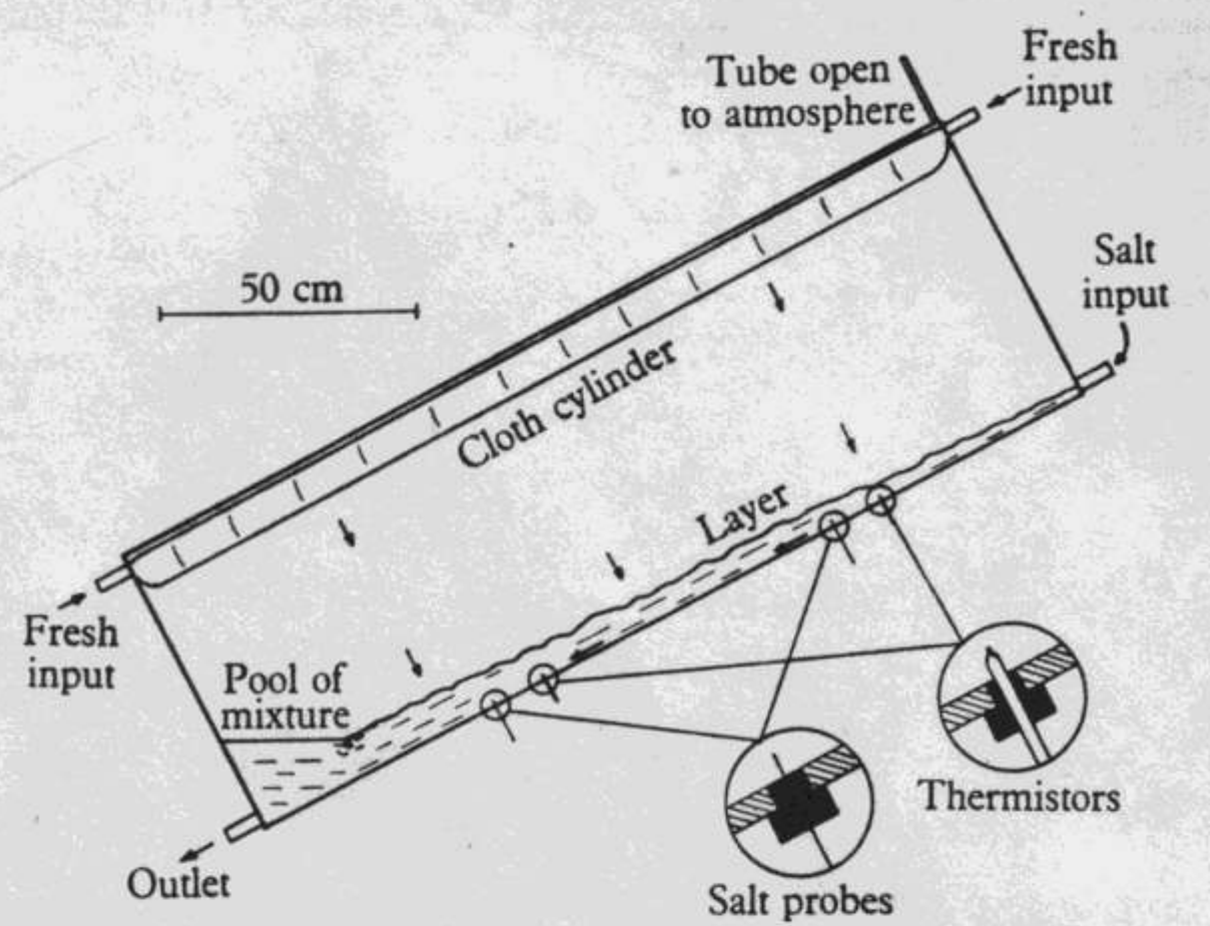
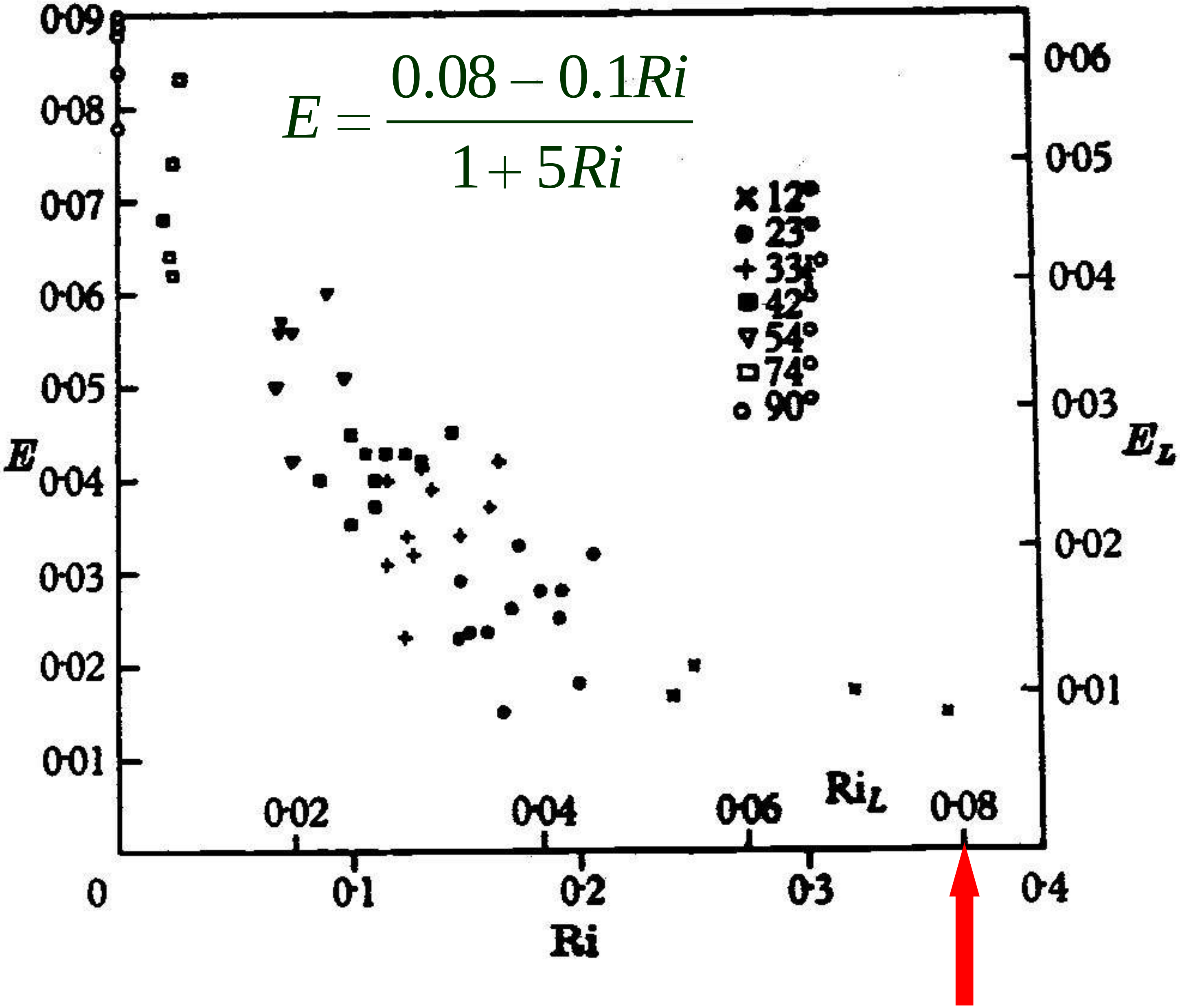


FIGURE 3. Sketch of the channel used to investigate inclined plumes.

Ellison and Turner (1959, JFM)

Results

$$E = \frac{0.08 - 0.1Ri}{1 + 5Ri}$$



A30 SUNDAY, APRIL 9, 2006 DV NATI

Mining company in Utah building new megasuburb

By Paul Foy
Associated Press

WEST JORDAN, Utah — It's a plan for development that will take more than 50 years from start to finish, on the largest piece of privately owned land next to a U.S. metropolis for an expected half-million residents.

This megasuburb, twice the size of San Francisco, will be the work of a mining company, Kennecott Utah Copper Corp., which has no experience in real-estate development.

The Utah company is a subsidiary of London-based Rio Tinto, a mining multinational and avowed convert to environmentalism, which decided to make a showcase out of its surplus Utah lands instead of just selling them off for cookie-cutter subdivisions.

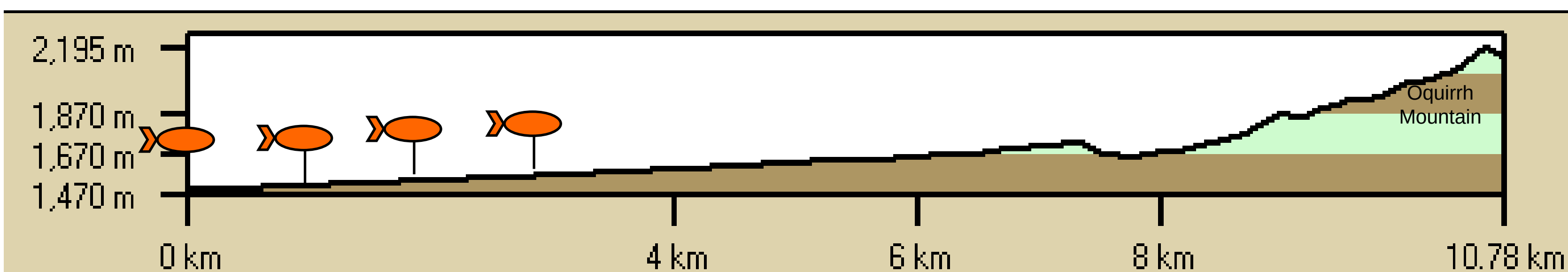
DOUGLAS C. PIZAC/ASSOCIATED PRESS

Craig and Cathy Douglass walk home through the Daybreak subdivision with the Kennecott mine in the background in South Jordan, Utah.

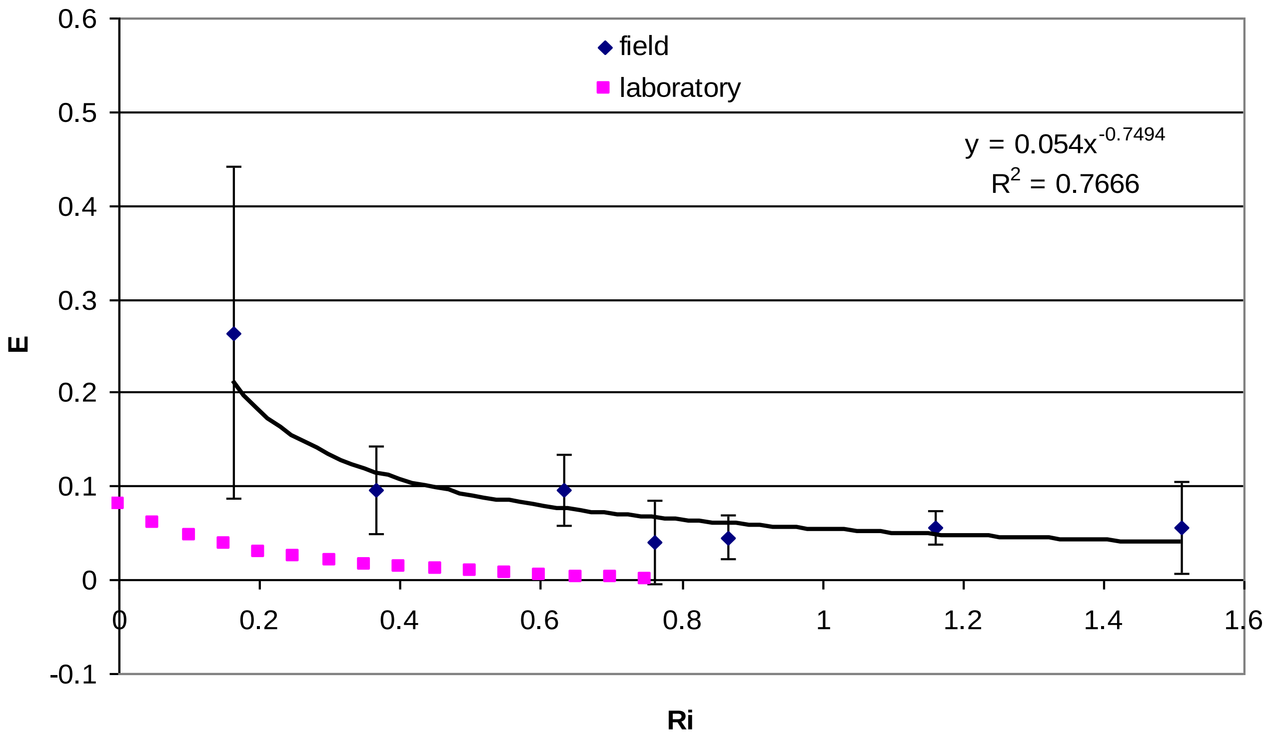
square miles of land, which ranks as the largest piece of land anywhere in the United States that's under the control of a single, private owner and

vide ground-source heating and cooling for a new elementary school and community center and contributed $400,000 to kick-start an envi-

$$\frac{\partial}{\partial s}\left(Uh\right) = EU$$

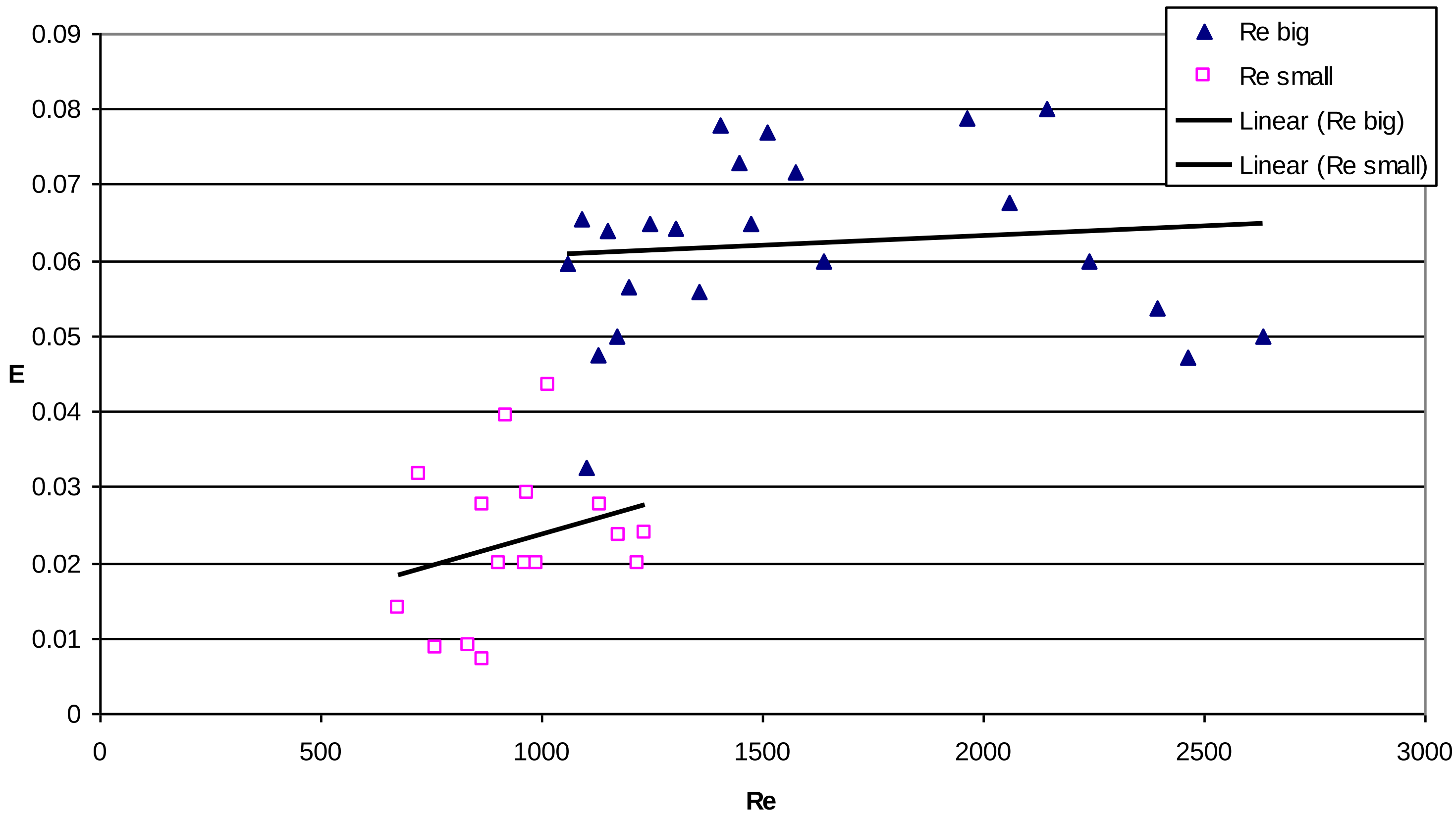


Entrainment Coefficient



Princevac, Fernando and Whiteman, J*. Fluid Mech*., 2005

Re vs. E



E
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
500
1000
1500
2000
2500
3000
Re
Re big
Re small
Linear (Re big)
Linear (Re small)

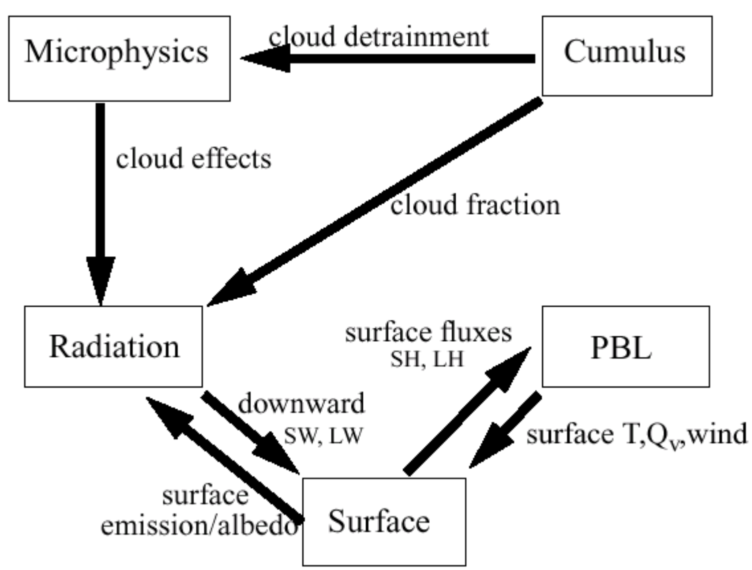
Applications

Flow Prediction

The Fifth-generation Penn State / NCAR Mesoscale Model (MM5)

Terrain following σ - coordinate

- Non-hydrostatic dynamics
- Four-dimensional data assimilation
- Multiple nest capability
- Physics



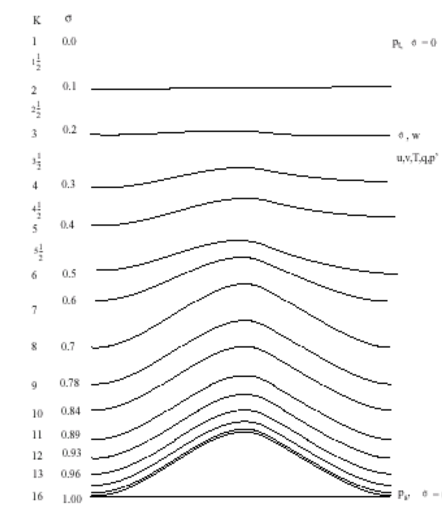


Figure 1.2 Schematic representation of the vertical structure of the model. The example is for 15 vertical layers. Dashed lines denote half-sigma levels, solid lines denote full-sigma levels.

MRF (Medium Range Forecast model)

MRF is a vertical diffusion non-local scheme in which is taken into account that the transport of mass, momentum and heat is mostly accomplished by large eddies (Troen and Mahrt 1986, Hong and Pan 1996).

The turbulence diffusion equation for prognostic variable is:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z}\left[K_c\left(\frac{\partial C}{\partial z} - \gamma_c\right)\right]$$

Below the PBL:

$$K_{zm} = k w_s z \left(1 - \frac{z}{h}\right)^p \qquad \frac{K_{zm}}{K_{zh}} = Pr$$

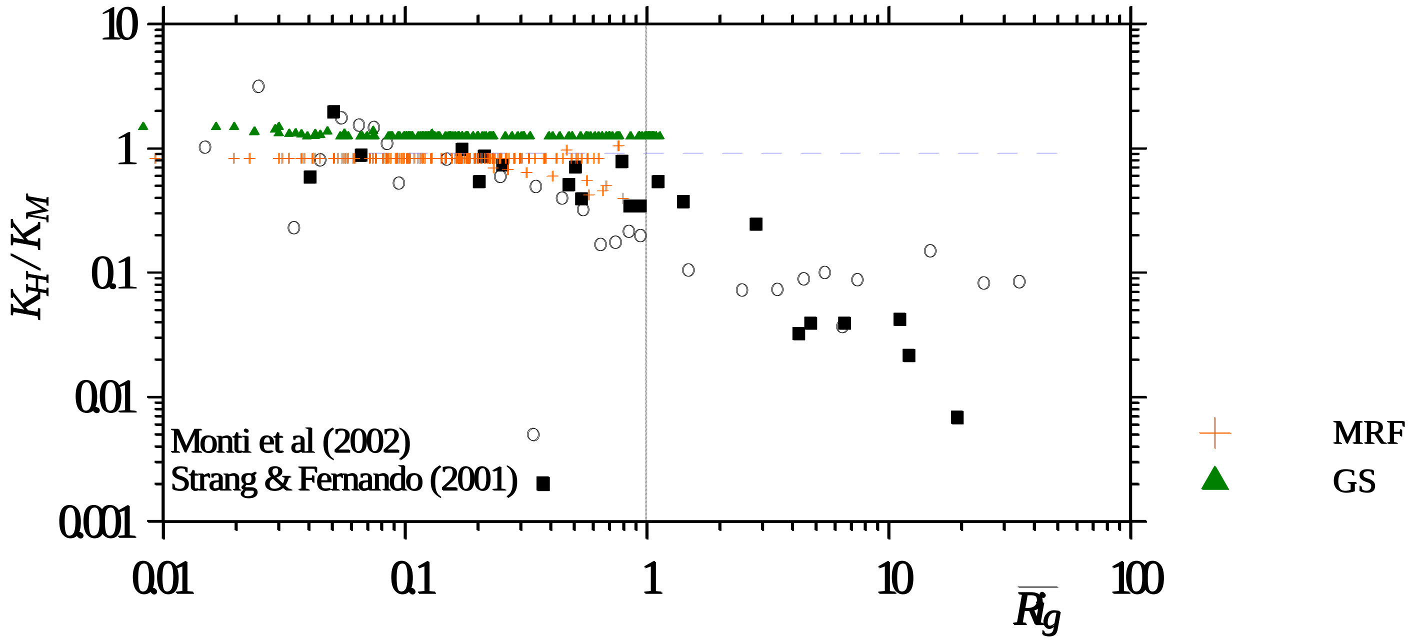
-Regime 1 ==========➔ **Nighttime Stable conditions** **(BR>0.2)**

-Regime 2 ==========➔ **Dumped mechanical turbulence** **(0<BR<0.2)**

-Regime 3 ==========➔ **Forced Convection conditions** **(BR=0)**

-Regime 4 ==========➔ **Free Convection conditions** **(BR<0)**

Eddy Diffusivity Ratio



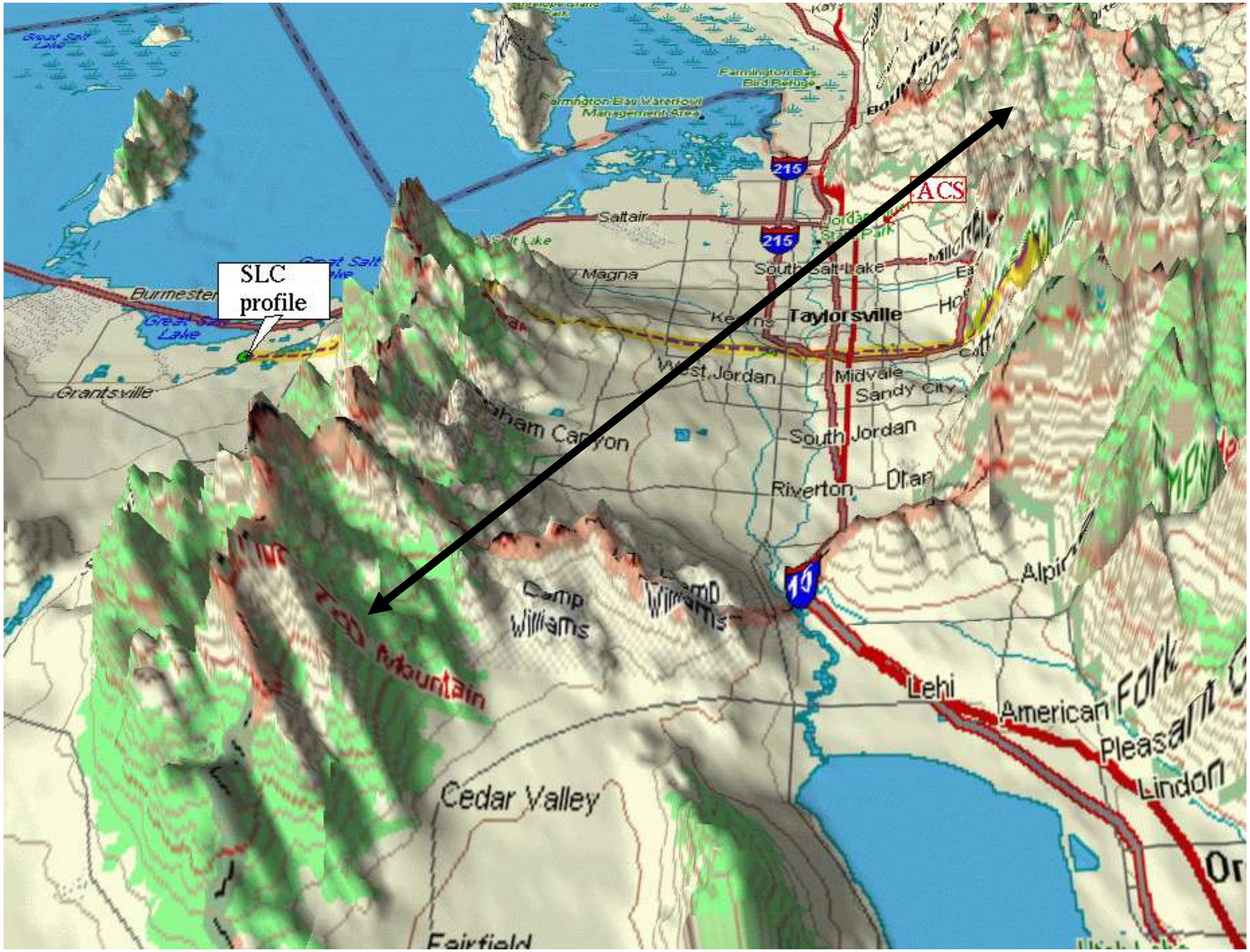
Inverse Prandtl Number

- J. Physical Oceanography 2001
- Boundary layer Meteor. 2005

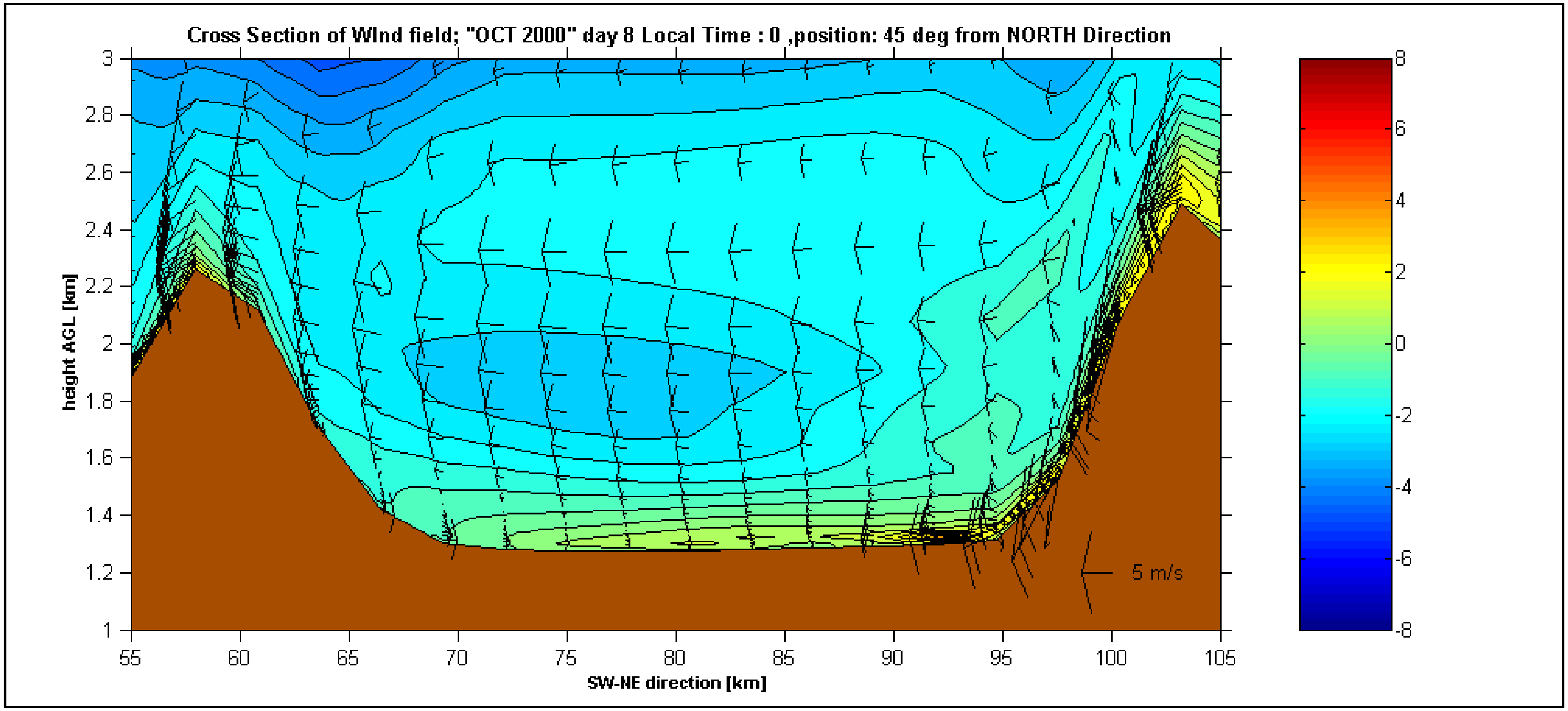
New Eddy Diffusivities

$$\begin{cases} \dfrac{K_m}{\sigma_w^2 / \left| d\tilde{V} / dz \right|} = (0.34)\overline{Ri_g}^{-0.02} \approx 0.34 \\ \dfrac{K_h}{\sigma_w^2 / \left| d\tilde{V} / dz \right|} = (0.08)\overline{Ri_g}^{-0.49} \approx (0.08)\overline{Ri_g}^{-0.5} \end{cases}$$

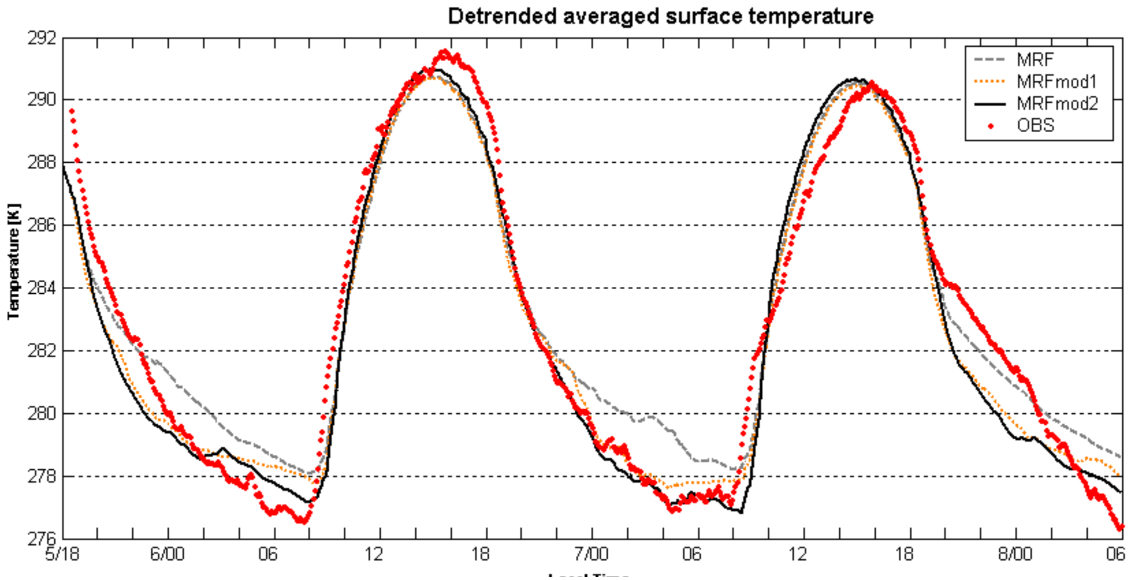
Salt Lake City

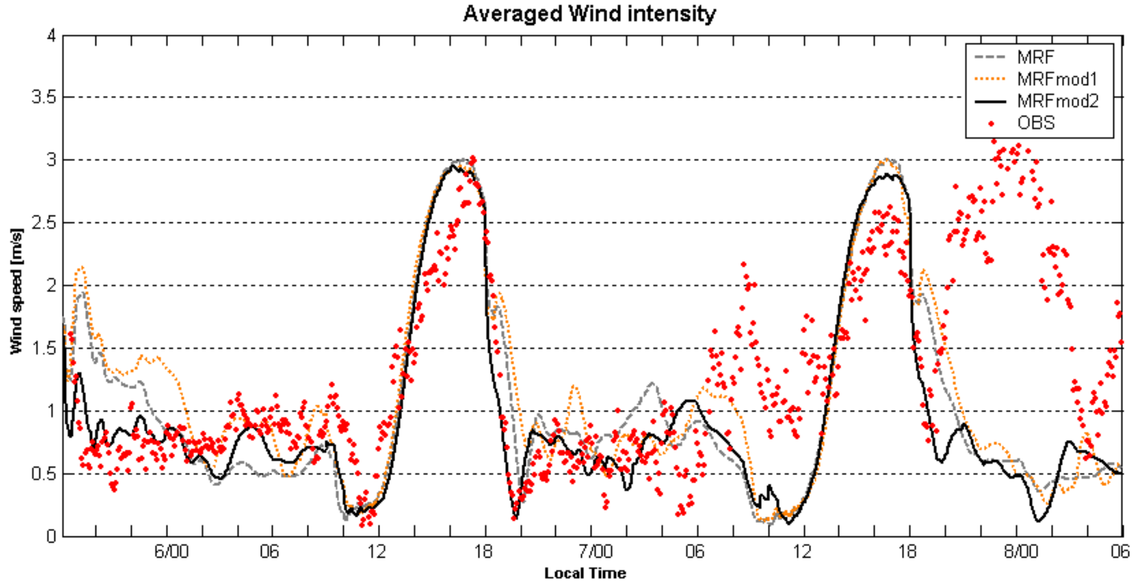


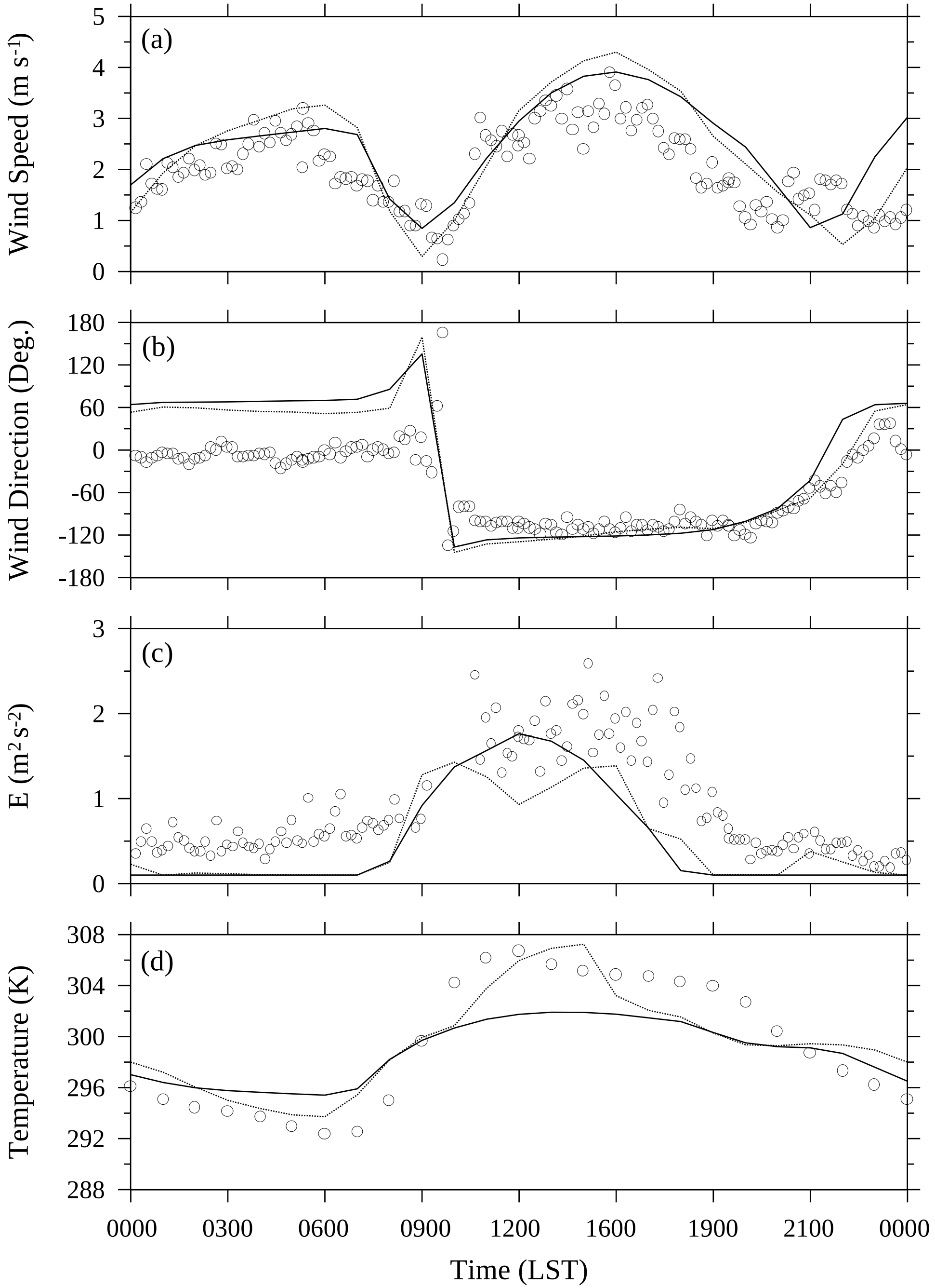
CROSS SECTION SW-NE 45 deg.



Temperature & Wind comparison



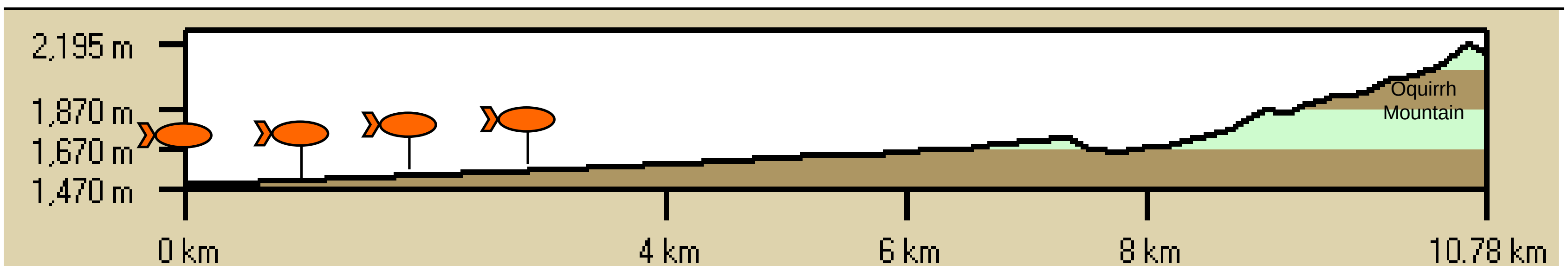




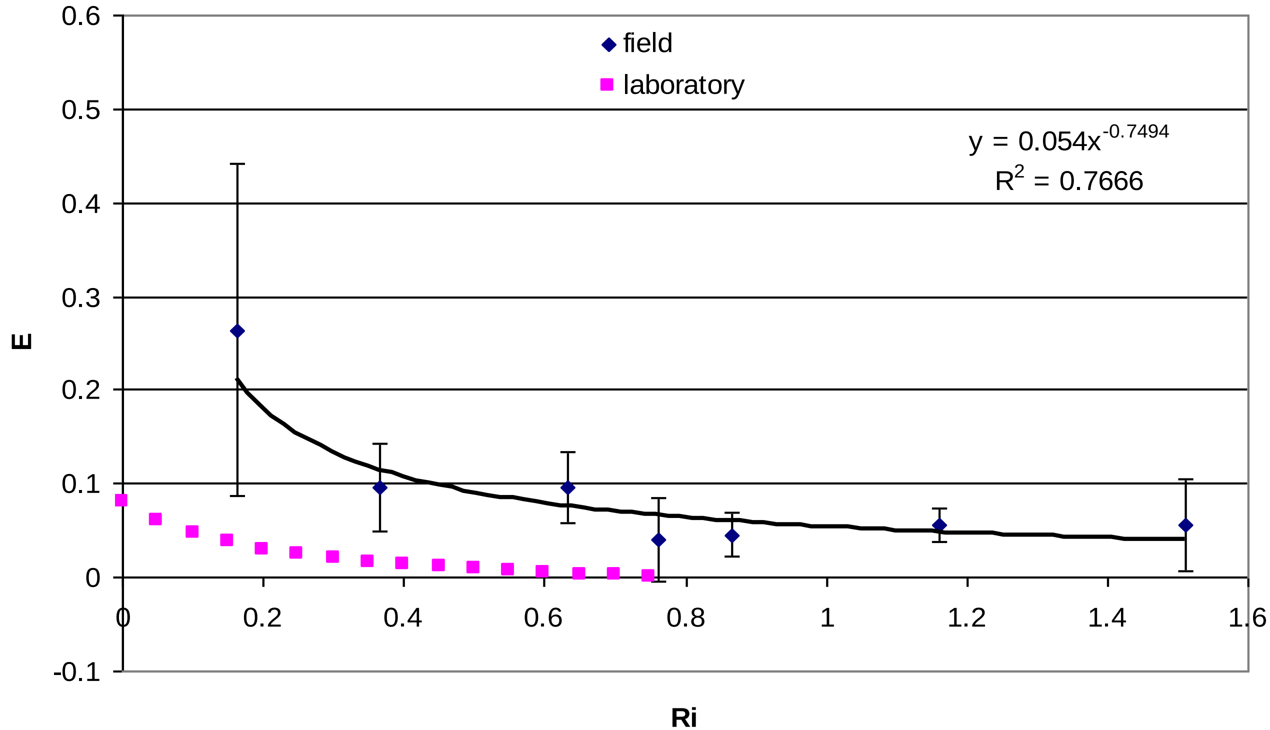
(averaged over 1-h, at 10 km inland versus simulations) RAMS uses Therry and Lacarrere's (1983) parameterization (200x200 km domain, including Rome)

Thank You !

$$\frac{\partial}{\partial s}\left(Uh\right) = EU$$

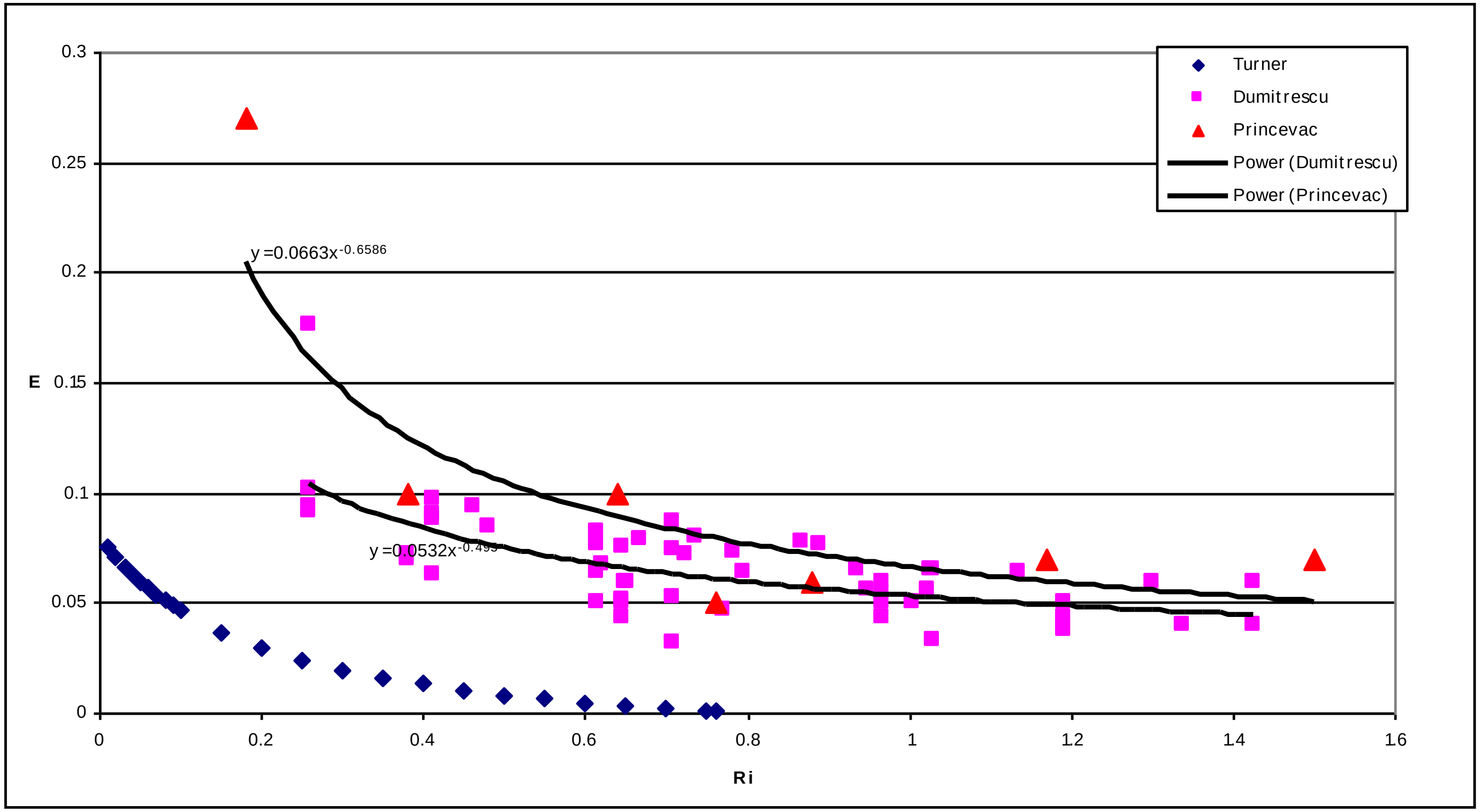


Entrainment Coefficient



Mixing Transition -- above a certain critical Reynolds number, entrainment increases

J. Fluid Mech. 2005,



Turner
Dumitrescu
Princevac
Power (Dumitrescu)
Power (Princevac)
y = 0.0663x^-0.6586
y = 0.0532x^-0.49
E
Ri
0
0.05
0.1
0.15
0.2
0.25
0.3
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6

Hydraulic Adjustment

$$\frac{\partial Uh}{\partial t}+\frac{\partial U^2 h}{\partial s}=\frac{\partial}{\partial s}\left(\frac{1}{2}\Delta b h^2\cos\alpha\right)-\Delta b h\sin\alpha - C_D U^2 - \left(\overline{u'w'}\right)_H$$

Steady state, small angle

$$\frac{d}{ds}\left(U^2 h\right)=\Delta b h\sin\alpha-\Delta b h\frac{dh}{ds}-C_D U^2$$

$$\frac{dh}{ds}\left(1-F^2\right)=\left(\alpha-C_D F^2\right)$$

$$F=\frac{U}{\sqrt{\Delta b h}}$$

Hydraulic Equation

$$Critical\ \alpha = C_D, when\ F=1$$

$$C_D \sim 10^{-3}$$

$$\alpha \approx 0.05^\circ$$

$Ri < 1$

$$U=\lambda_u^*\left(\frac{\Delta b h \sin\alpha}{E}\right)^{1/2}$$

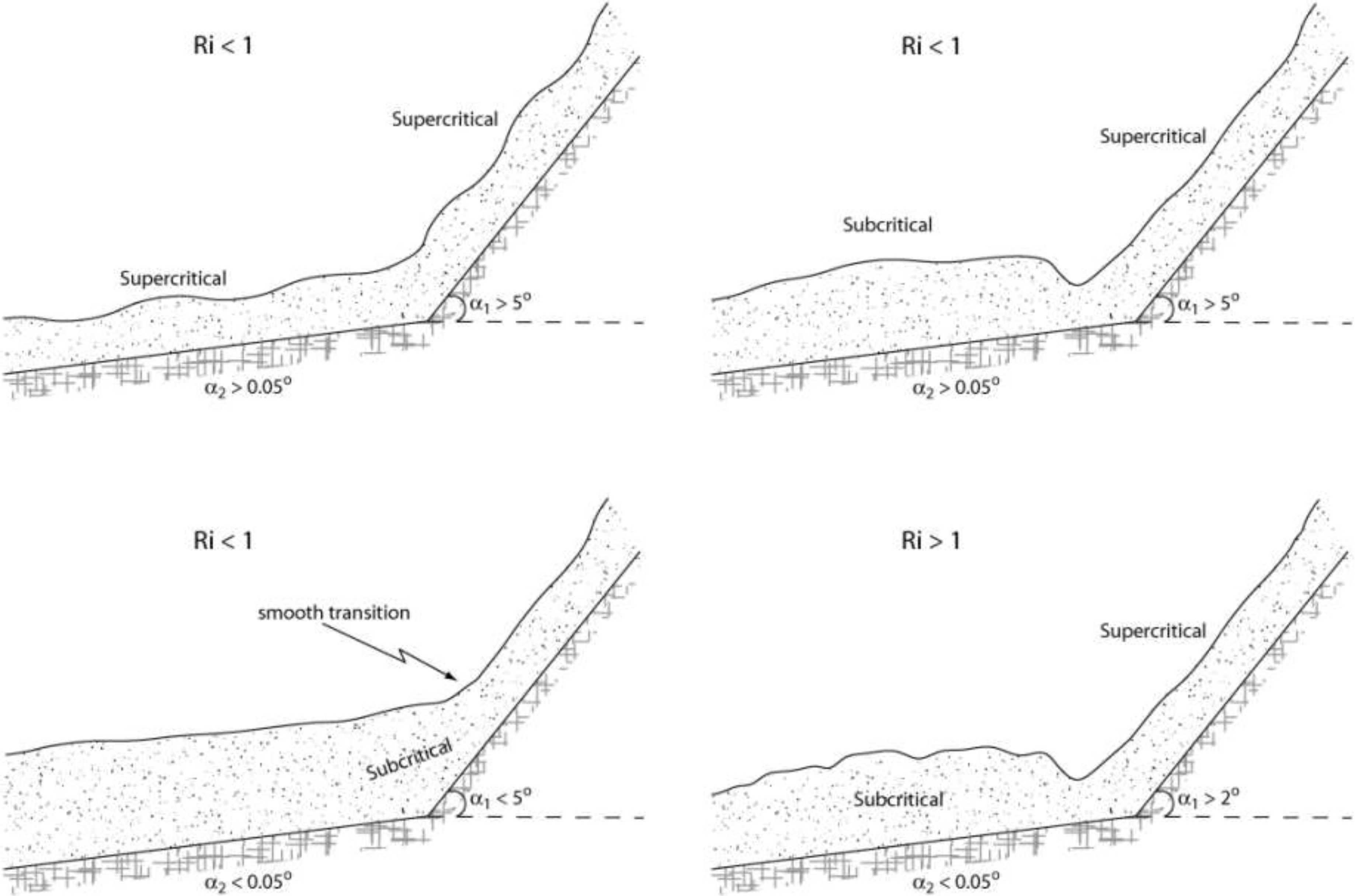
$$F=\frac{U}{\sqrt{\Delta b h}}=\lambda_u^*\left(\frac{\sin\alpha}{E}\right)^{1/2}$$

$$\alpha > 5\ is\ supercritical$$

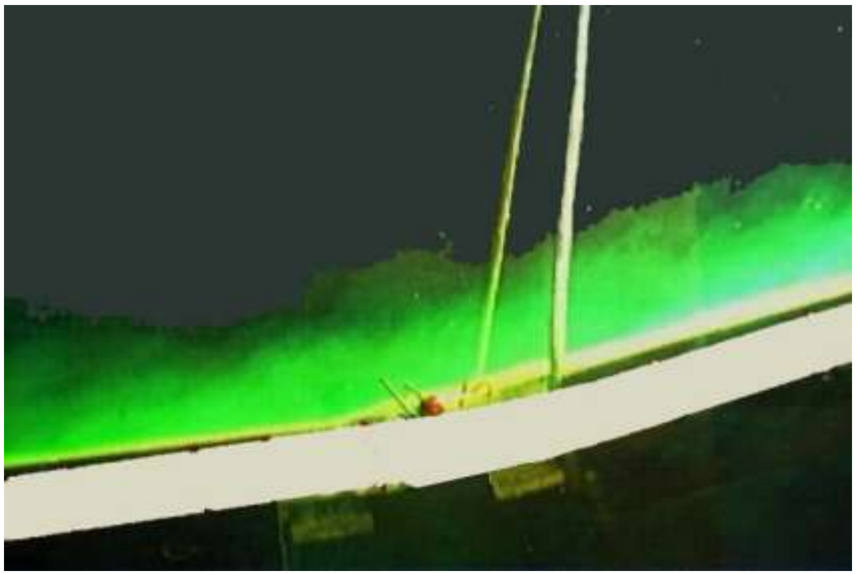
$Ri > 1$

$$F=\frac{U}{\sqrt{\Delta b h}}=\lambda_u^*\left(\frac{\sin\alpha}{h/L_H}\right)^{1/2}$$

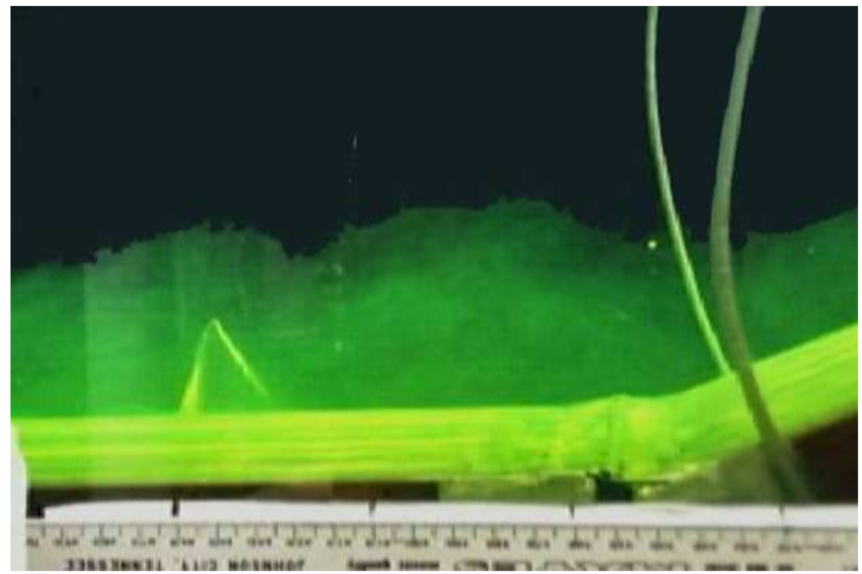
$$\alpha > 2^\circ\ \ is\ \ supercritical$$



Ri < 1
Supercritical
Supercritical
α1 > 5°
α2 > 0.05°
Ri < 1
Supercritical
Subcritical
α1 > 5°
α2 > 0.05°
Ri < 1
smooth transition
Subcritical
α1 < 5°
α2 < 0.05°
Ri > 1
Supercritical
Subcritical
α1 > 2°
α2 < 0.05°



a) α= *(10˚, 20˚)*

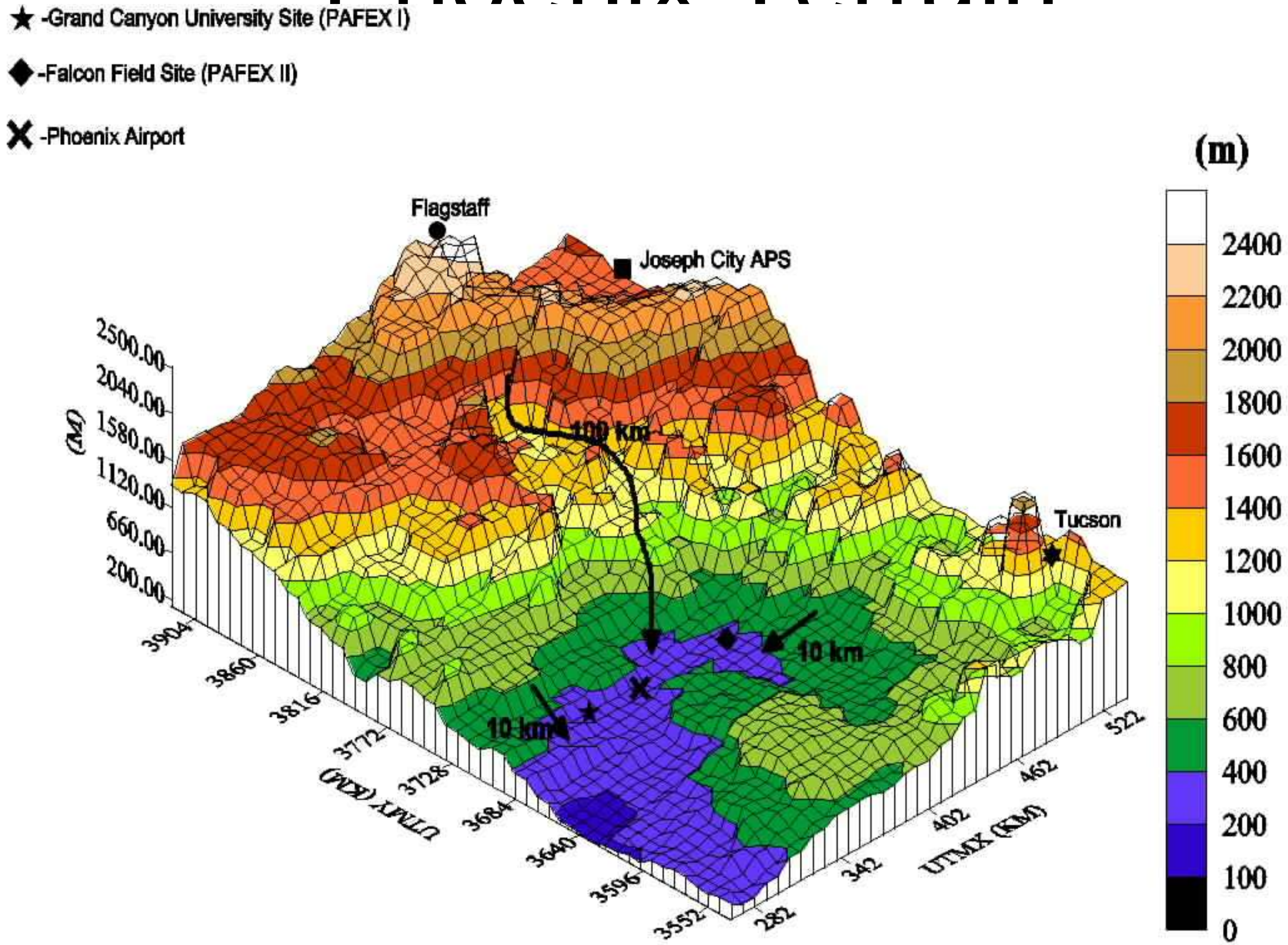


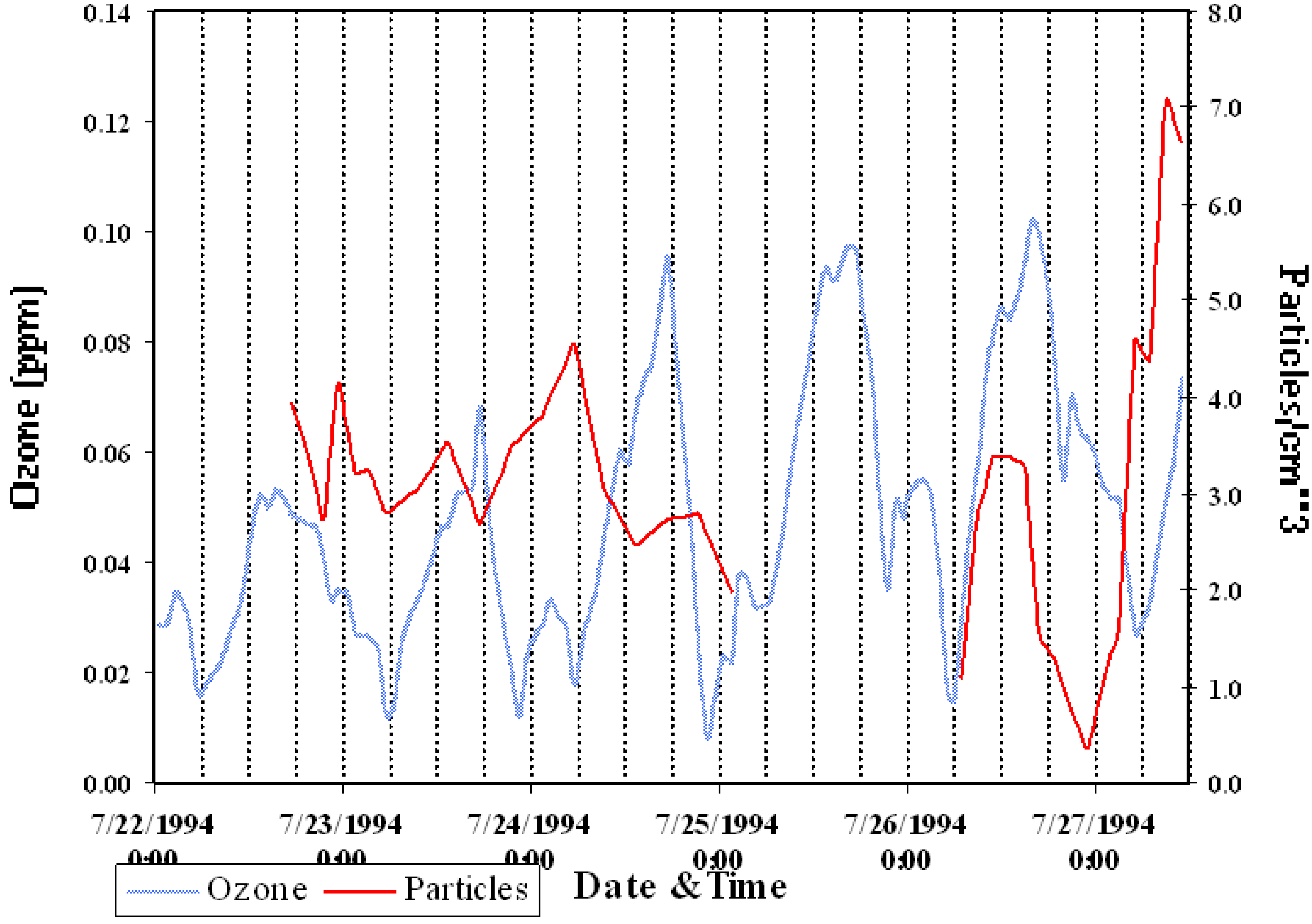
b) α= *(0˚, 26˚)*

Applications

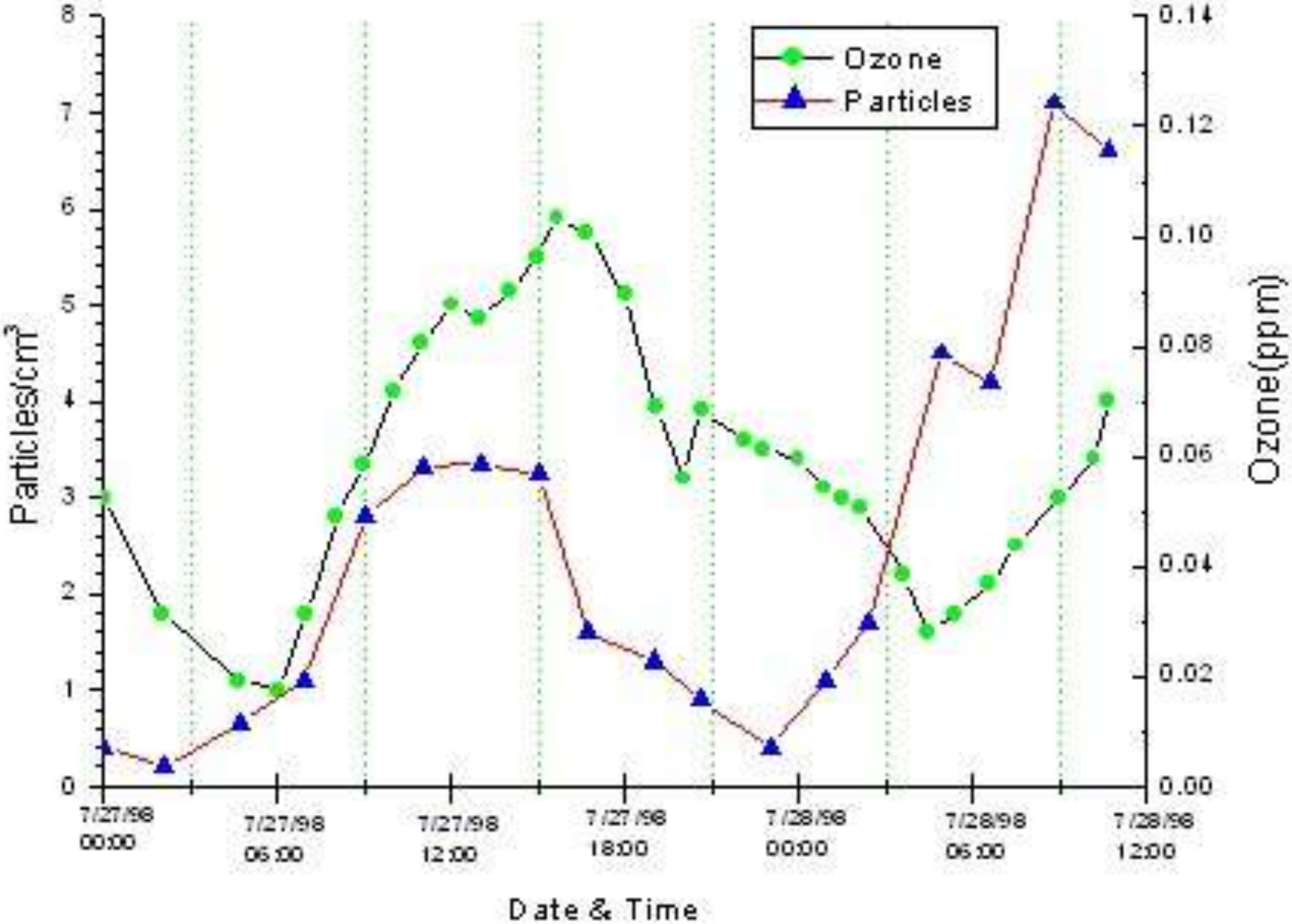
Power plant emissions

Phoenix Terrain

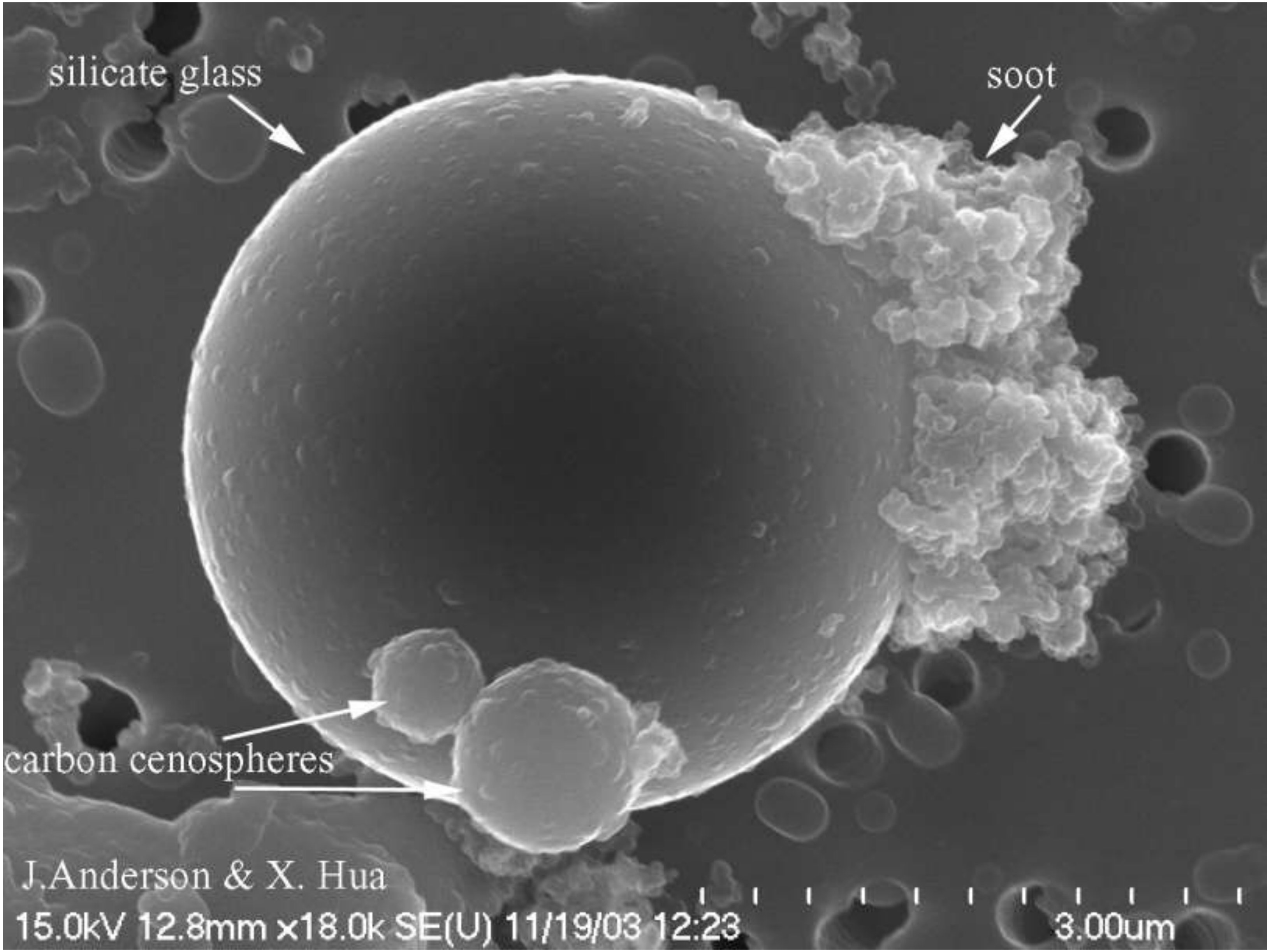




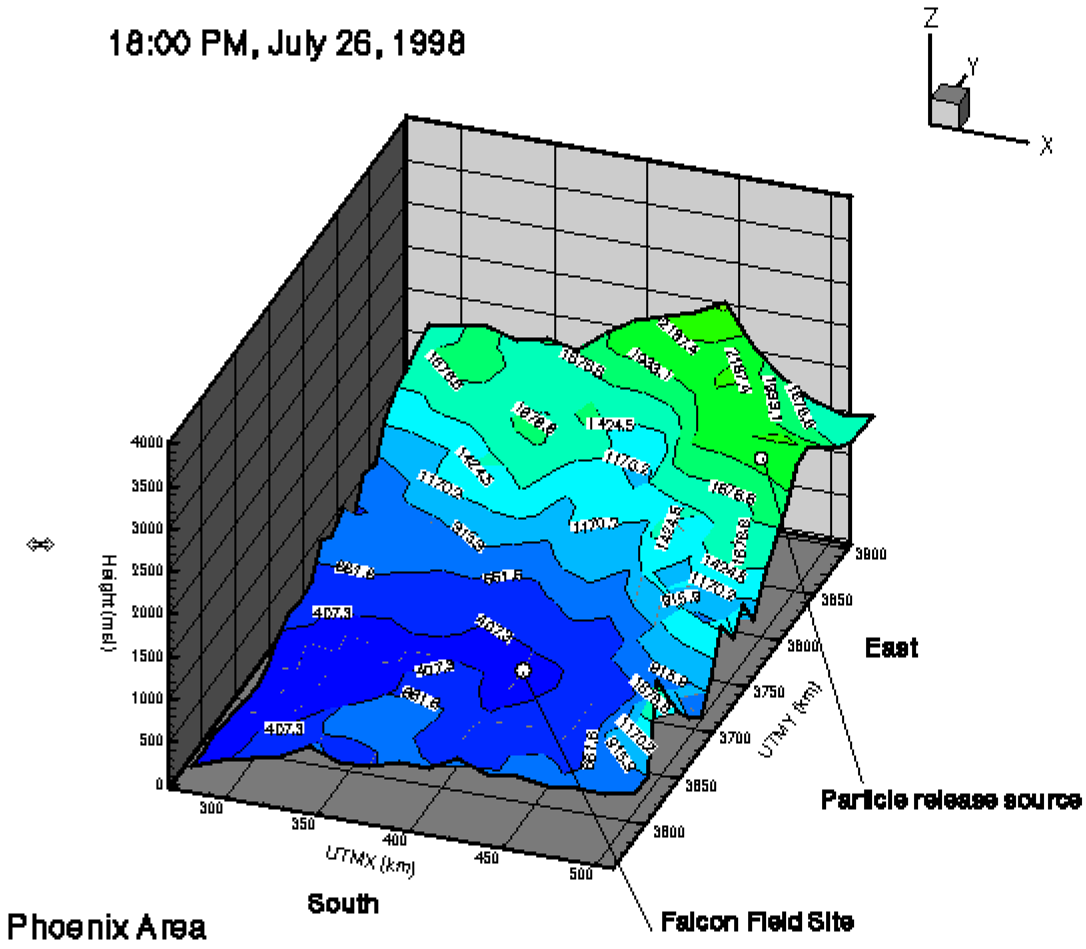
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Ozone (ppm)
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
Particles/cm**3
7/22/1994 0:00
7/23/1994 0:00
7/24/1994 0:00
7/25/1994 0:00
7/26/1994 0:00
7/27/1994 0:00
Date & Time
Ozone
Particles



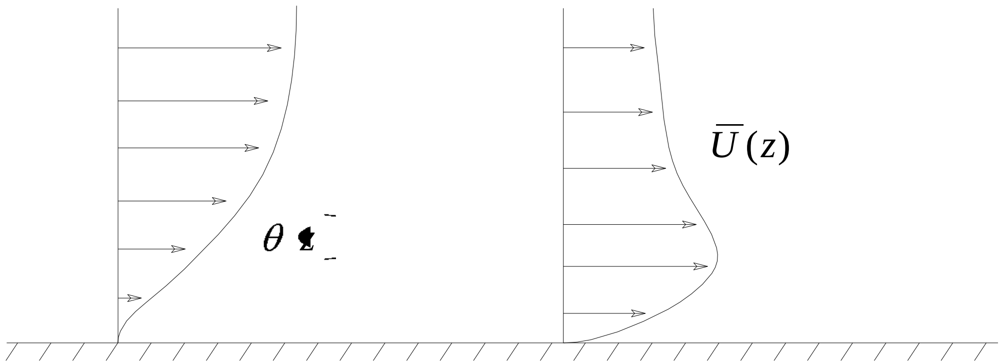
Ozone
Particles
Particles/cm^3
Ozone(ppm)
8
7
6
5
4
3
2
1
0
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
7/27/98 00:00
7/27/98 06:00
7/27/98 12:00
7/27/98 18:00
7/28/98 00:00
7/28/98 06:00
7/28/98 12:00
Date & Time


silicate glass
soot
carbon cenospheres
J.Anderson & X. Hua
15.0kV 12.8mm ×18.0k SE(U) 11/19/03 12:23
3.00um

Dispersion of Air Pollutants



Parameterization of Vertical Mixing



$$\frac{\partial \overline{q^2/2}}{\partial t} + U_j \frac{\partial \overline{q^2/2}}{\partial x_j} = \underbrace{-\overline{u_i u_j}\frac{\partial U_i}{\partial x_j}}_{P} + \underbrace{\overline{bw}}_{B} - \frac{\partial\left(\overline{u_j\, q^2/2} + \overline{p u_j}\right)}{\partial x_j} - \underbrace{\varepsilon}_{D}$$

Flux Richardson Number

$$R_f = \frac{-\overline{bw}}{-\overline{u_i u_j}\frac{\partial \overline{U_i}}{\partial x_j}}$$

Gradient Richardson Number

$$R_{ig} = \frac{N^2}{\left(\frac{\partial \overline{U}}{\partial z}\right)^2}$$

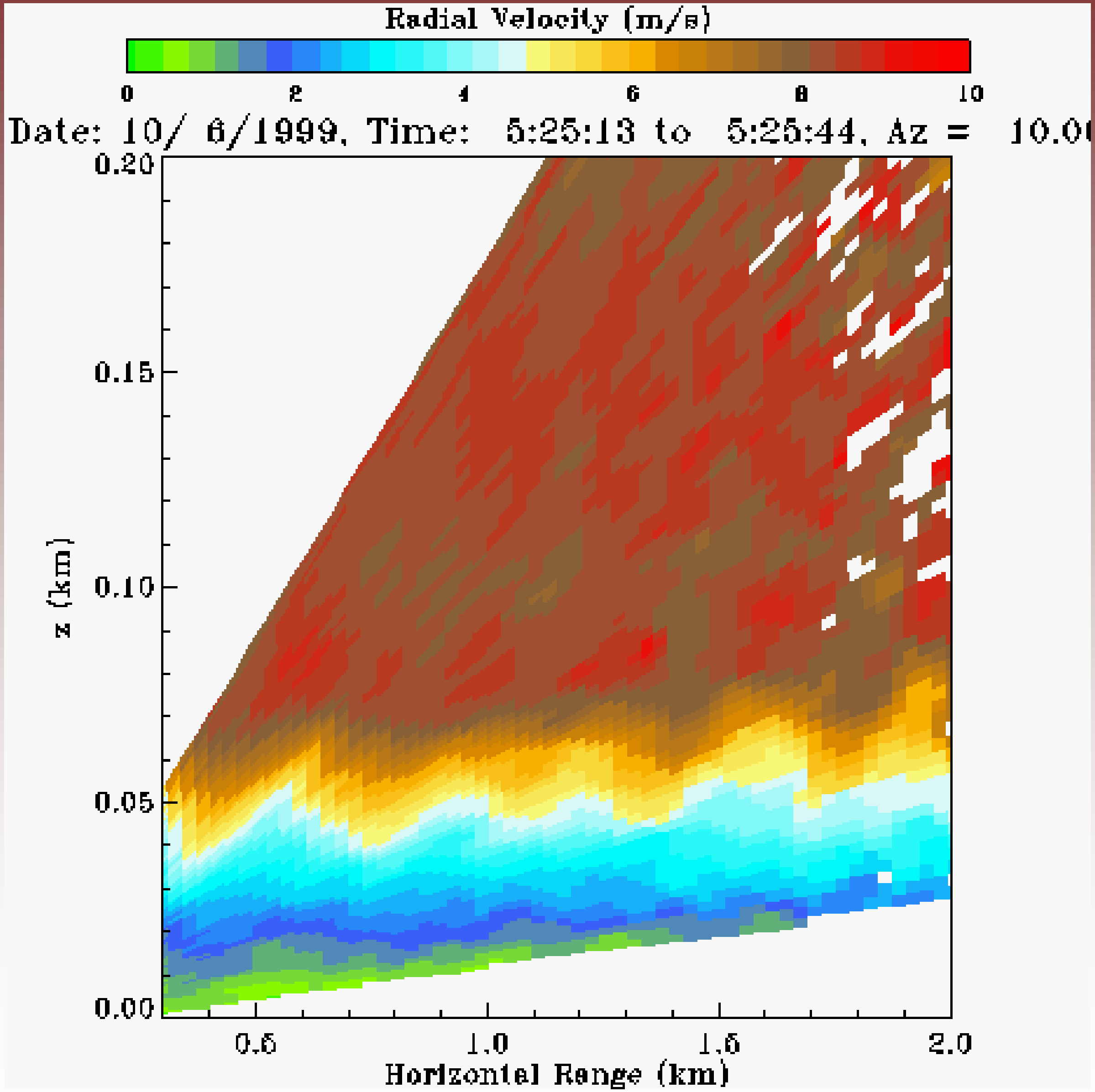
Diffusion Coefficients

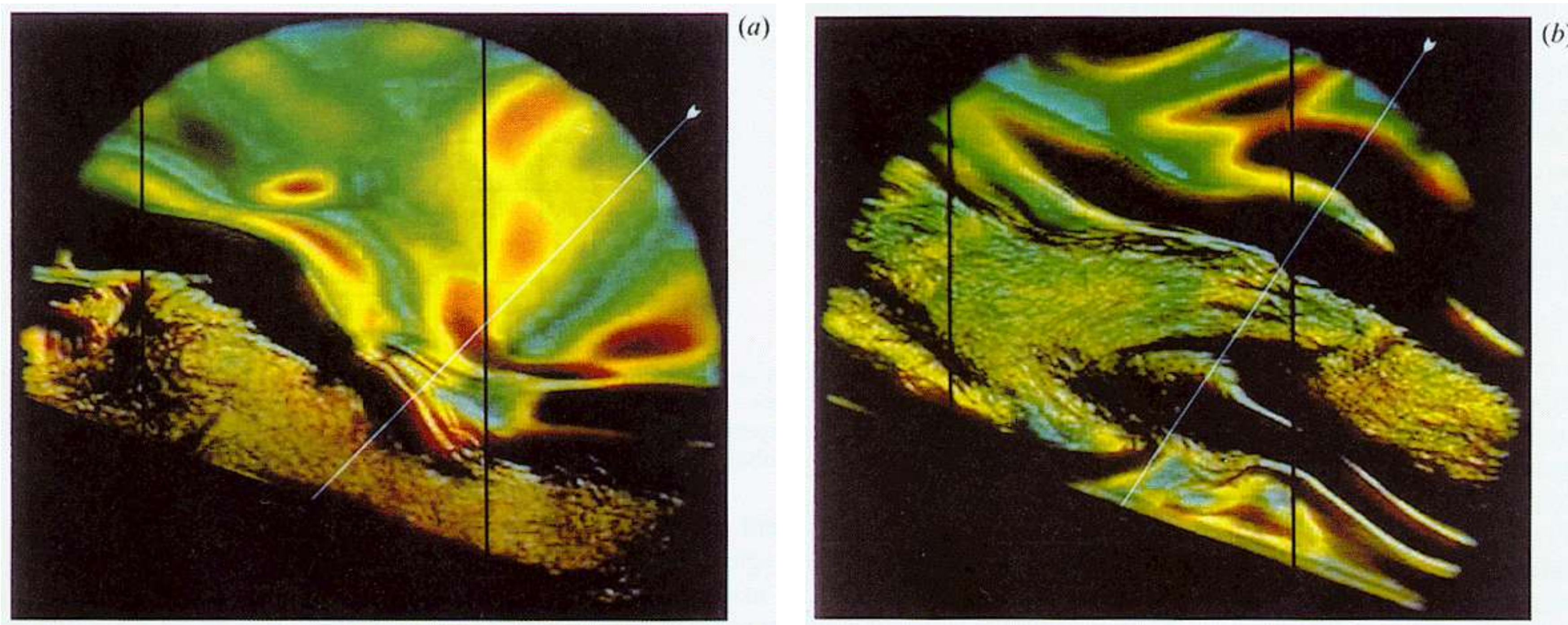
$$-\overline{u_i u_j} = K_m\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

e.g.. $-\overline{uw} = K_m \frac{\partial \overline{U}}{\partial z}$

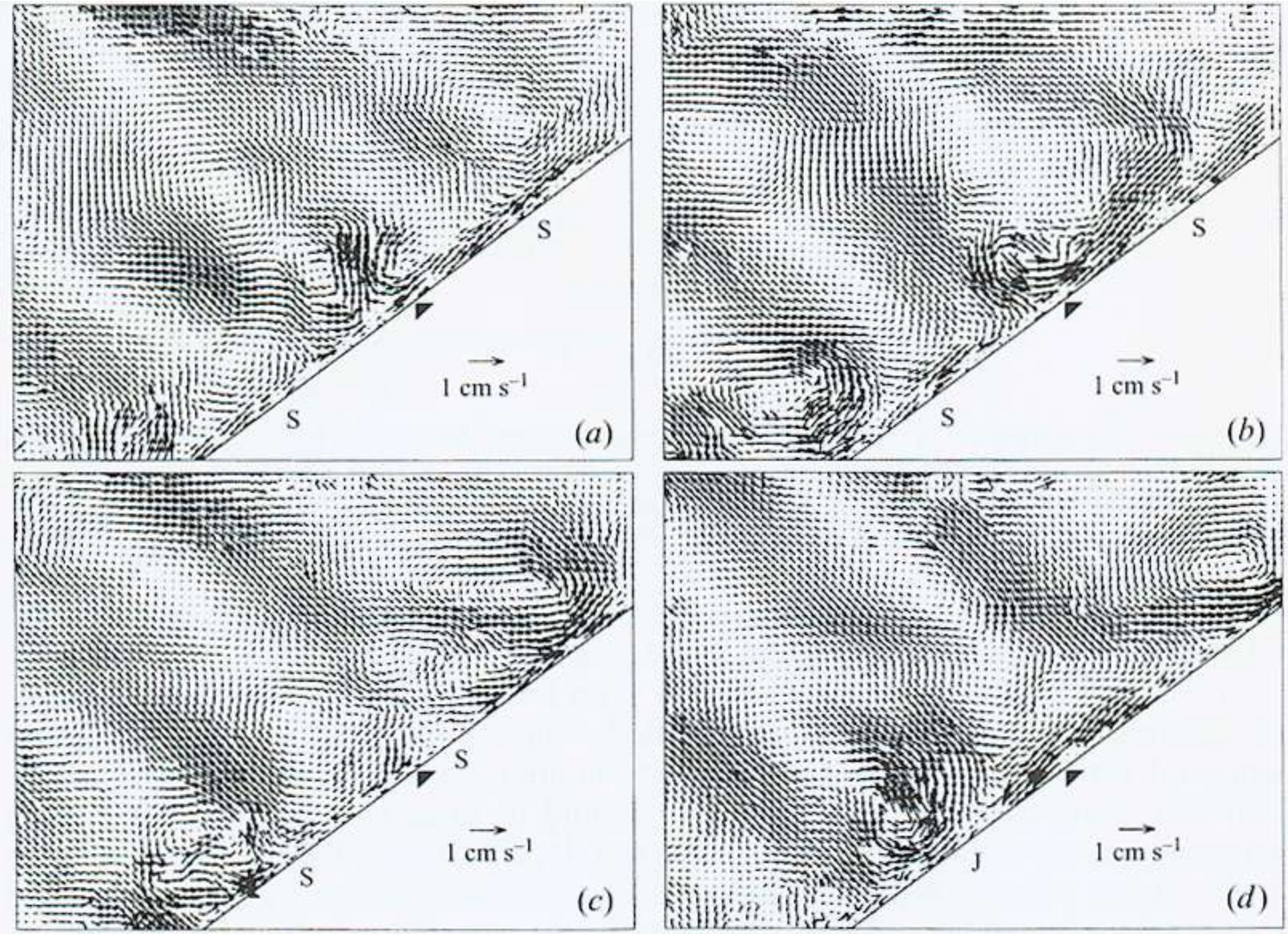
$$-\overline{b u_i} = K_h\left(\frac{\partial \overline{b}}{\partial x_i}\right)$$

ENTRAINMENT





Schlieren video images showing the on-slope and off-slope initiation of instabilities depending on γ. The experimental conditions are a^* = 2 cm, N = 0.921 rad s^{-1}, $\mathcal{R}$ = 20. (a) α = 45 , γ = 2.07; (b) α = 56 , γ = 2.43. The vertical lines in each figure are 10 cm apart. The oblique thin white line indicates the centre-line of the incident wave ray. From De Silva et al. (1997, JFM, 350,1-27)


S
S
1 cm s⁻¹
(a)
S
S
1 cm s⁻¹
(b)
S
S
1 cm s⁻¹
(c)
J
1 cm s⁻¹
(d)