A Simple Dynamical Model with Features of Convective Momentum Transport

Andrew Majda and Sam Stechmann

Courant Institute, New York University

(paper accepted to J. Atmos. Sci. on July 17, 2008)

NCAR IMAGe

Summer School: Geophysical Turbulence

July 18, 2008

2 important multiscale effects

$$\frac{\partial u}{\partial t} + u\partial_x u + w\partial_z u = \cdots$$

$$u = \bar{u} + u'$$

1. Eddy momentum flux

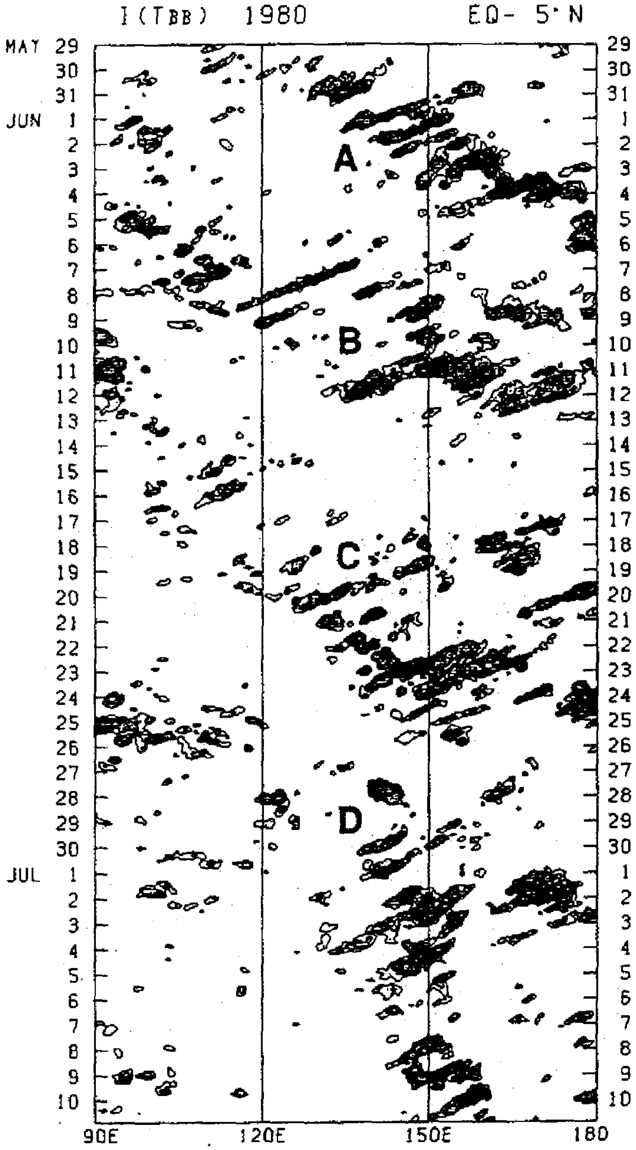
   "Convective momentum transport" (CMT)

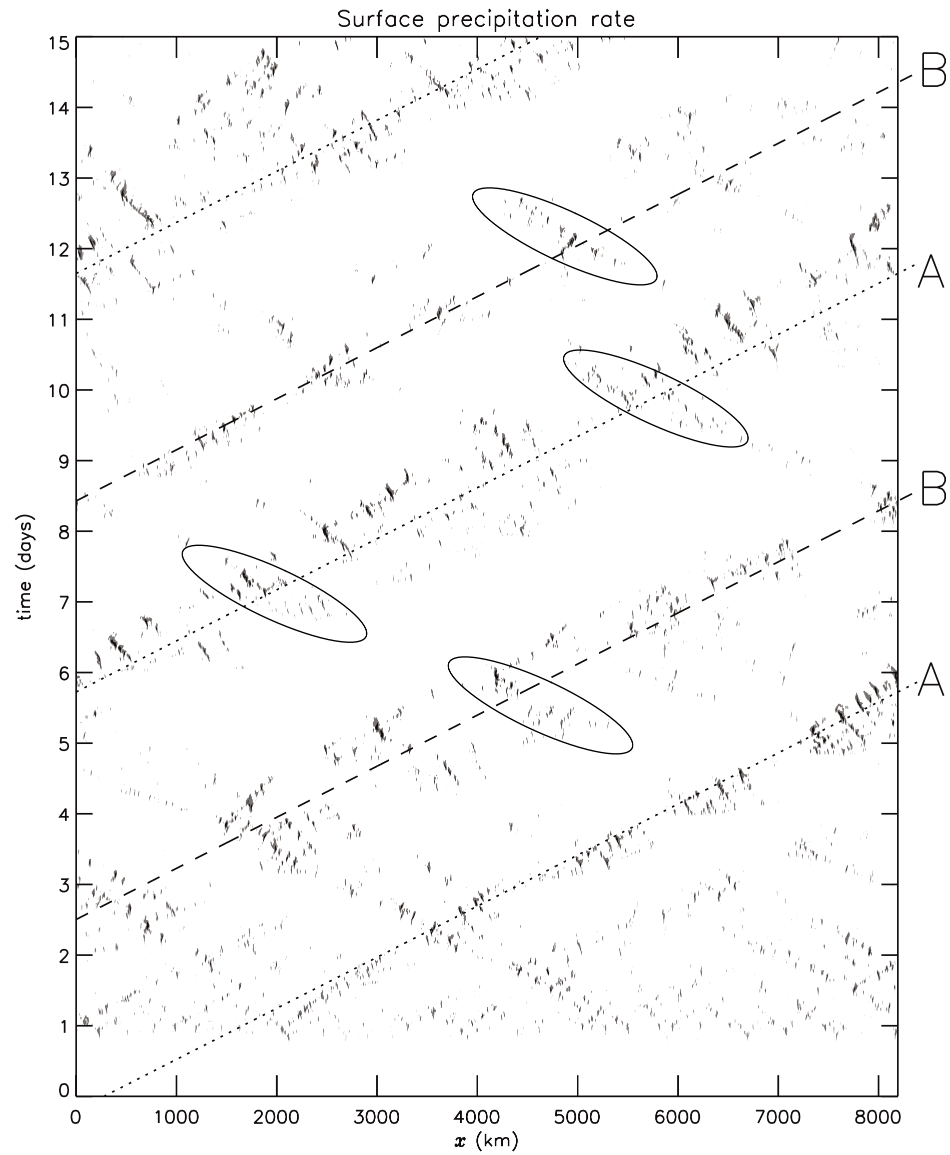
$$\frac{\partial \bar{u}}{\partial t} = -\partial_z \overline{w'u'} + \cdots$$

2. Background wind shear

$$\frac{\partial u'}{\partial t} + \bar{u}\partial_x u' + w'\partial_z \bar{u} = \cdots$$

Convectively coupled waves (CCWs)





observations from Nakazawa (1988)

simulations from Tulich et al. (2007)

Outline

- Background
  - Convective momentum transport (CMT)
  - Effect of background wind shear on convection/waves
- A multicloud model for convectively coupled waves (CCWs)
  - Observations of CCWs
  - A multicloud model for CCWs
- A simple dynamical model for convective wave–mean flow interaction
  - Derivation
  - Derivation using multiscale asymptotics
  - Results

Convective momentum transport (CMT)

$$\frac{\partial \bar{u}}{\partial t} = -\partial_z \overline{w'u'} + \cdots$$

Mesoscales and smaller:

Earlier this week:

- Multi-scale models for squall lines
- Superparameterization

Synoptic scales:

CMT from convectively coupled waves (CCWs)

- Can change velocity on the planetary scales (and MJO)
- Majda and Biello (2004), Biello and Majda (2005)

Convective momentum transport (CMT)

The importance of wave tilts:

- Need wave tilts to get nonzero CMT!
- Moncrieff (1992), Majda and Biello (2004), Biello and Majda (2005)

CMT illustration with diagnostic WTG model (Majda, 2007):

$$w' = S'_\theta$$

$$u'_x + w'_z = 0$$

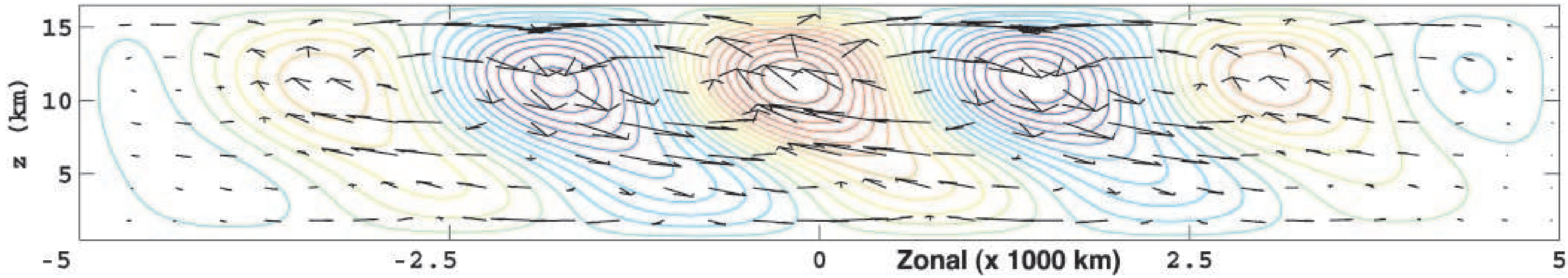
To solve:

- Specify the heat source S'_θ (this gives us w')
- Compute u' using the continuity equation
- Use w' and u' to compute the CMT: $-\partial_z \overline{w'u'}$

Convective momentum transport (CMT)

Assume heat source with 2 phase-lagged baroclinic modes:

$$S'_\theta = k\cos[kx - \omega t]\,\sqrt{2}\sin(z) + \alpha k\cos[k(x + x_0) - \omega t]\,\sqrt{2}\sin(2z)$$



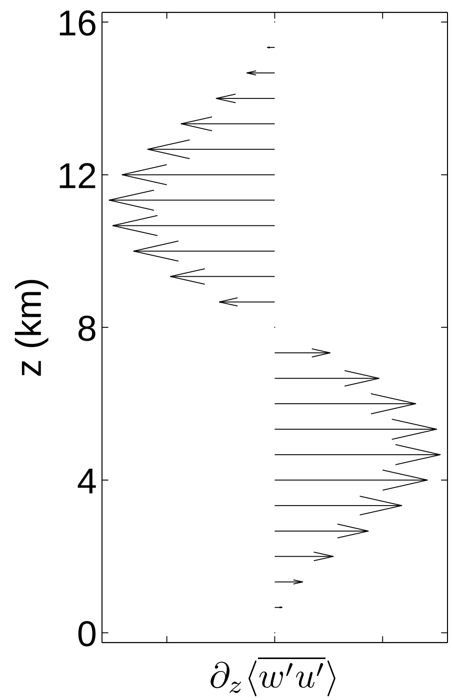
CMT affects first and *third* baroclinic modes:

$$\partial_z\langle\overline{w'u'}\rangle = \frac{3\alpha k}{2}\sin(kx_0)[\cos(z) - \cos(3z)]$$

Need stratiform heating ($\alpha \neq 0$)

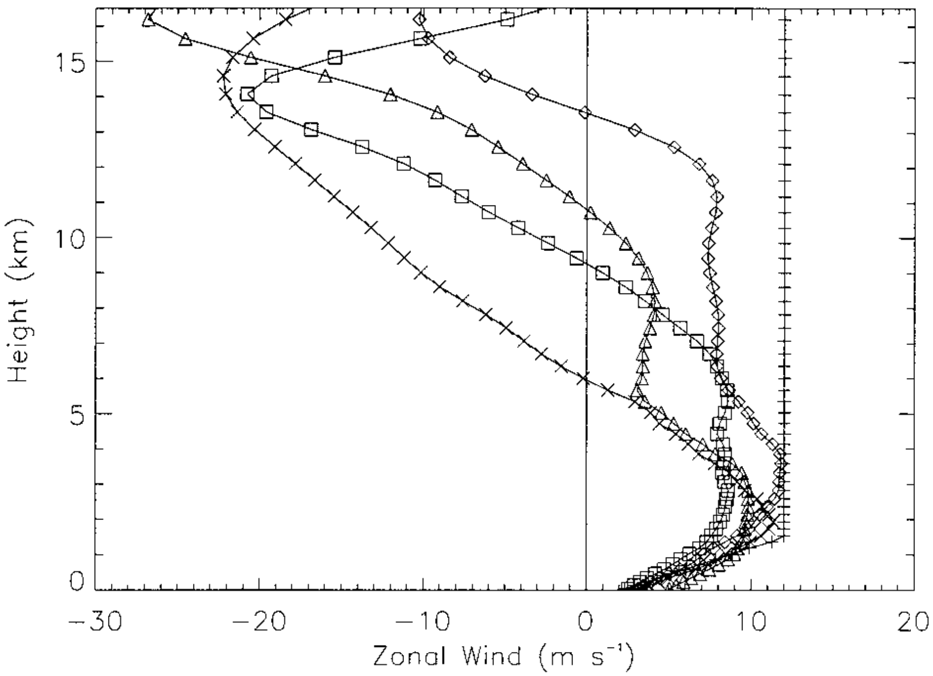
Need lag between vertical modes ($x_0 \neq 0$)

i.e., Need wave tilts!

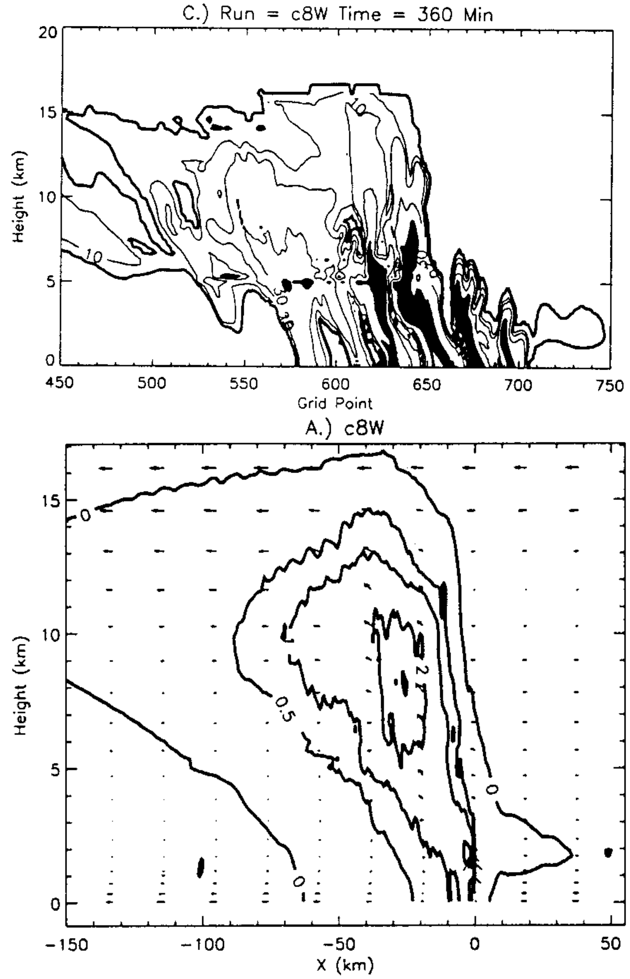


Effect of background wind shear on convection/waves

Effect of background wind shear on squall lines



Condensate



Low-level shear determines propagation direction

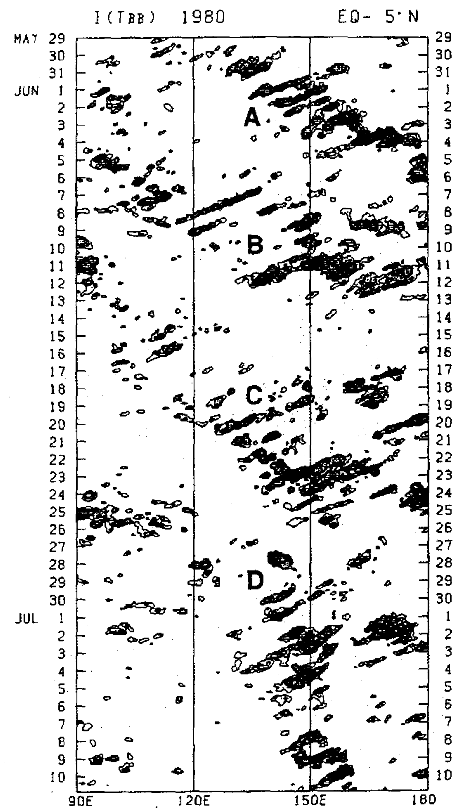
from Lucas et al. (2000)

Effect of background wind shear on convection/waves

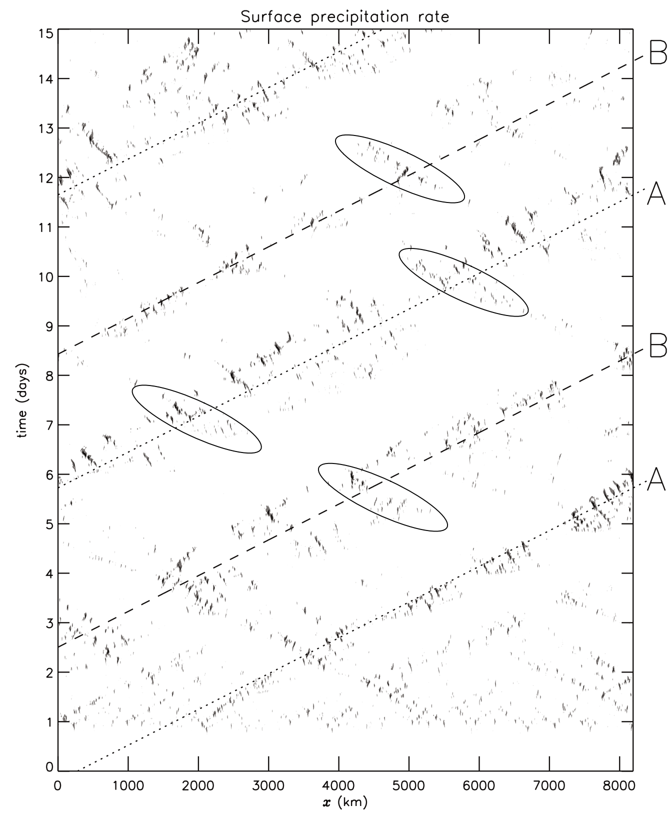
Effect of background wind shear on CCWs

Not many observations of this, but what is seen is ...

CCWs propagate in opposite direction of convective systems within them



from Nakazawa (1988)



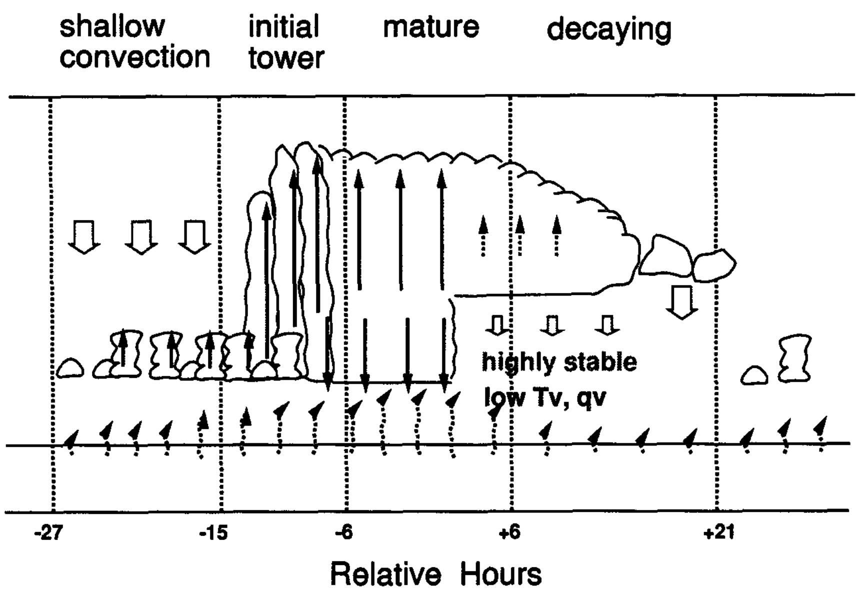
from Tulich et al. (2007)

This will be a consistency check

Outline

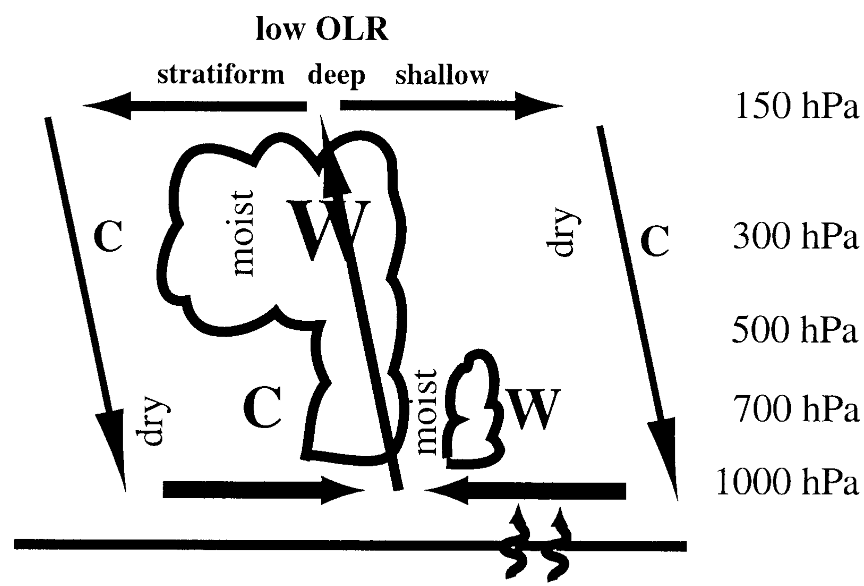
- Background
  - Convective momentum transport (CMT)
  - Effect of background wind shear on convection/waves
- A multicloud model for convectively coupled waves (CCWs)
  - Observations of CCWs
  - A multicloud model for CCWs
- A simple dynamical model for convective wave–mean flow interaction
  - Derivation
  - Derivation using multiscale asymptotics
  - Results

Observations of Convectively Coupled Waves (CCWs)



Schematics for the quasi 2-day variation in TOGA COARE

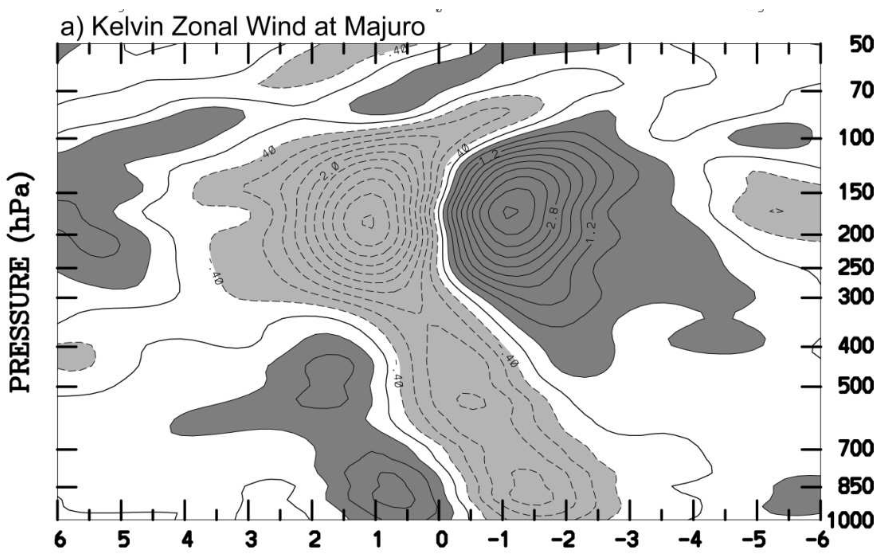
Takayabu et al. (1996)

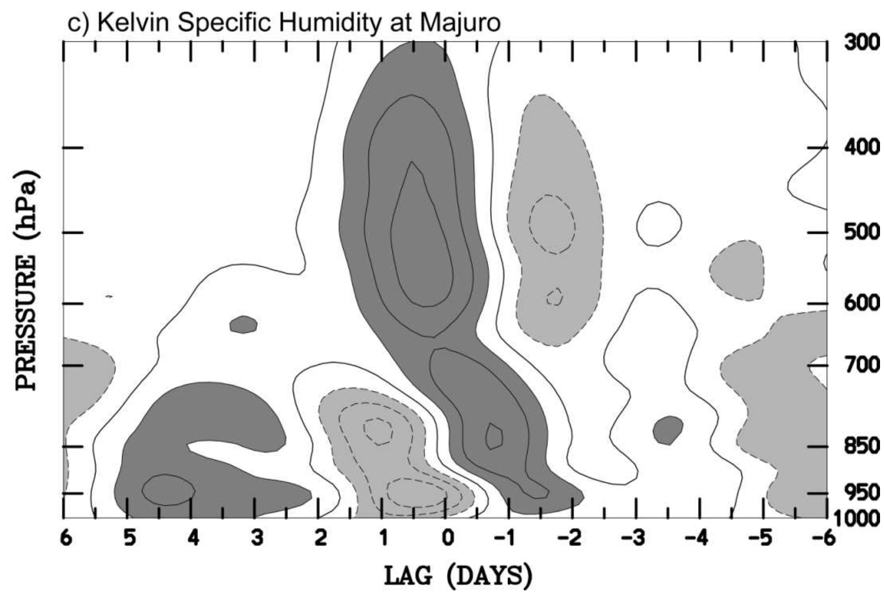


Straub and Kiladis (2003)

Progression from congestus to deep convective to stratiform clouds

Observations of Convectively Coupled Waves (CCWs)



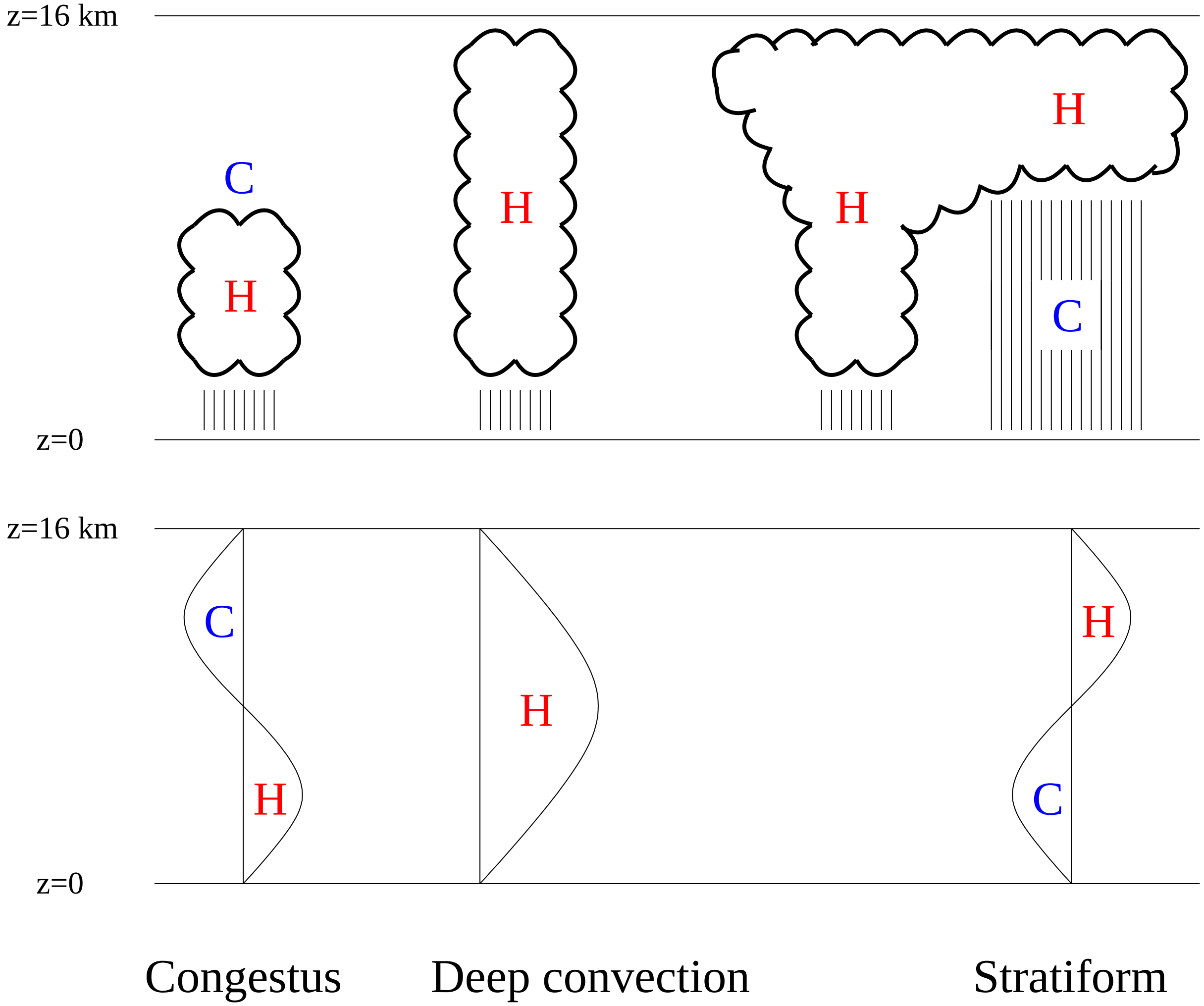


Kiladis et al. (2008)

Vertical tilts seen in velocity, moisture, temperature, etc.

so CCWs will have upscale CMT

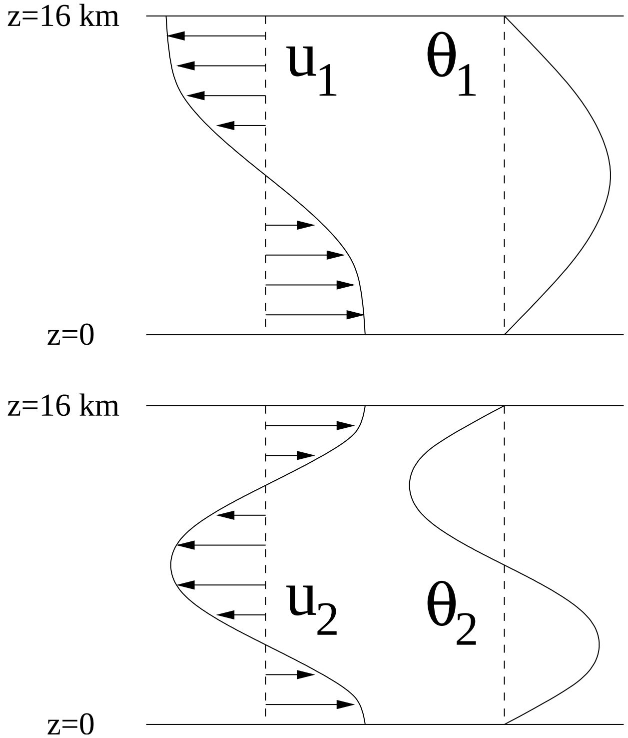
The Multicloud Model (Khouider and Majda 2006)
(a model for CCWs)



Two vertical baroclinic modes $\Rightarrow$ vertical tilts are possible

Equations of the multicloud model

Two linear shallow water systems, coupled through nonlinear source terms:



$$\begin{cases} \dfrac{\partial u_1}{\partial t} - \dfrac{\partial \theta_1}{\partial x} = -\dfrac{1}{\tau_u} u_1 \\ \dfrac{\partial \theta_1}{\partial t} - \dfrac{\partial u_1}{\partial x} = H_d - R_1 \end{cases}$$

$$\begin{cases} \dfrac{\partial u_2}{\partial t} - \dfrac{\partial \theta_2}{\partial x} = -\dfrac{1}{\tau_u} u_2 \\ \dfrac{\partial \theta_2}{\partial t} - \dfrac{1}{4}\dfrac{\partial u_2}{\partial x} = H_c - H_s - R_2 \end{cases}$$

$H_d =$ Deep convective heating

$H_c =$ Congestus heating

$R =$ Radiative cooling

$H_s =$ Stratiform heating

$+$ 4 more prognostic equations for θ_{eb}, q, H_s, H_c

$+$ diagnostic equations for some source terms

Equations of the multicloud model

Mathematical form: system of conservations laws with source terms

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial}{\partial x}\mathbf{f}(\mathbf{u}) = \mathbf{S}(\mathbf{u})$$

$$\mathbf{u} = (u_1, \theta_1, u_2, \theta_2, \theta_{eb}, q, H_s, H_c)$$

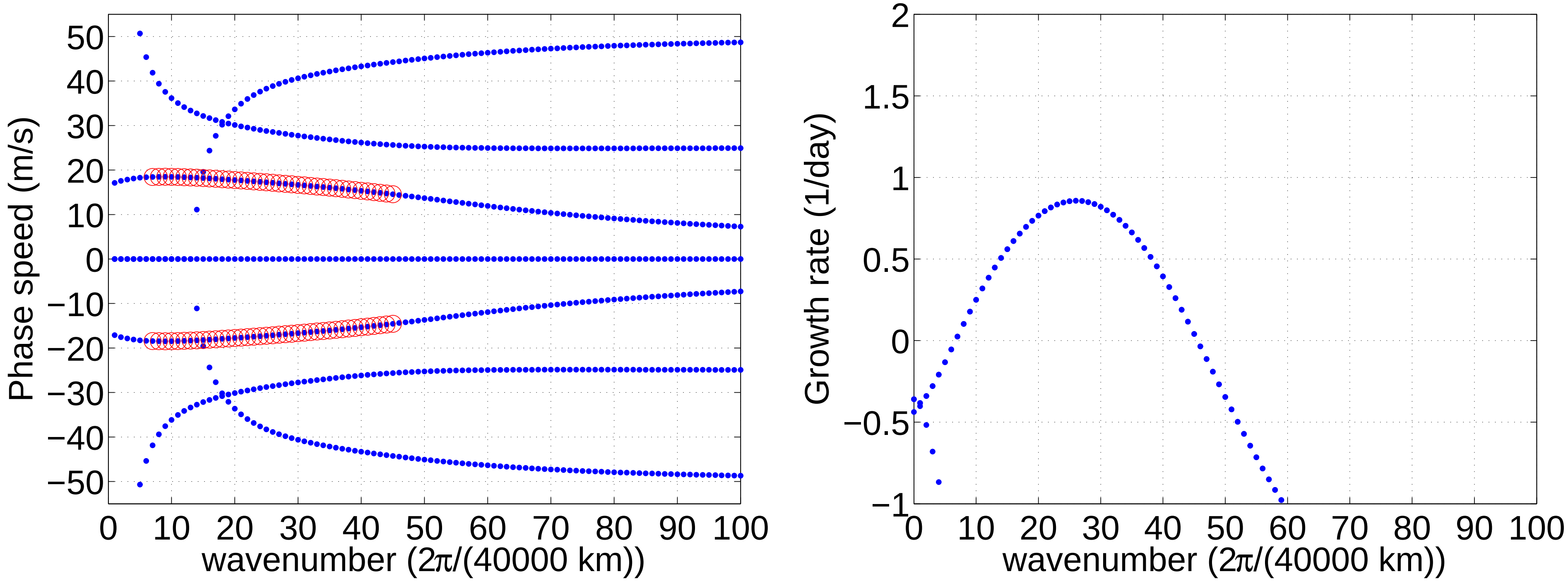
Source terms are parameterizations of physical processes such as convective heating, radiative cooling, evaporation, downdrafts

Simple exact solution: radiative–convective equilibrium

$$\mathbf{S}(\mathbf{u}) = 0, \qquad \text{i.e.,} \qquad H_d = R_1, \qquad \text{etc.}$$

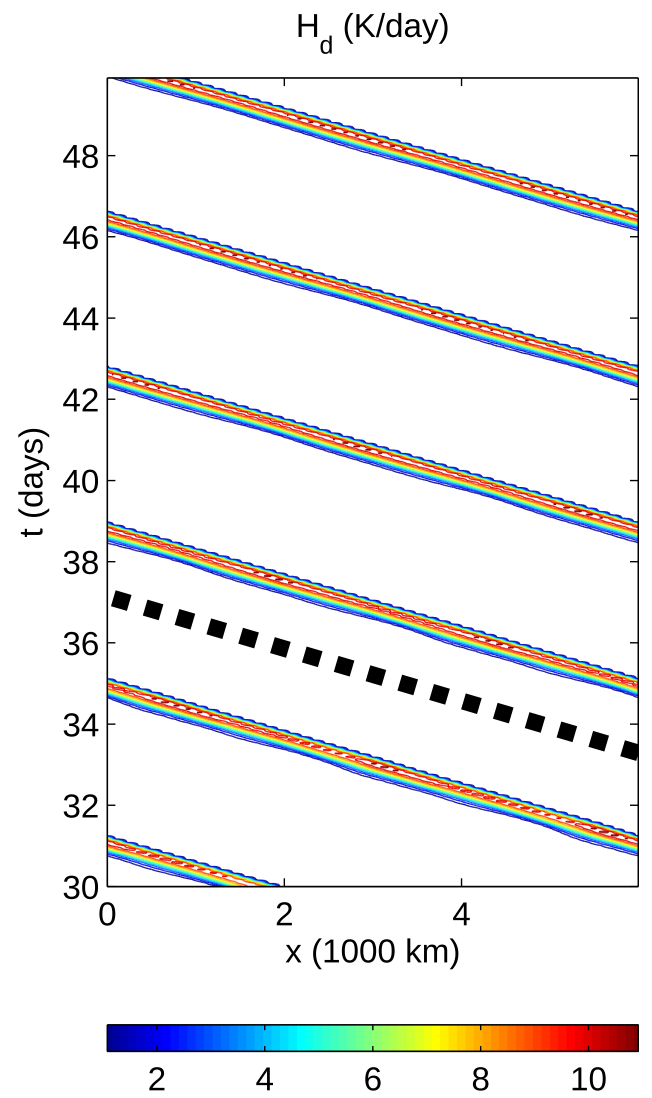
CCWs in the Multicloud Model

Linear theory



CCWs in the Multicloud Model

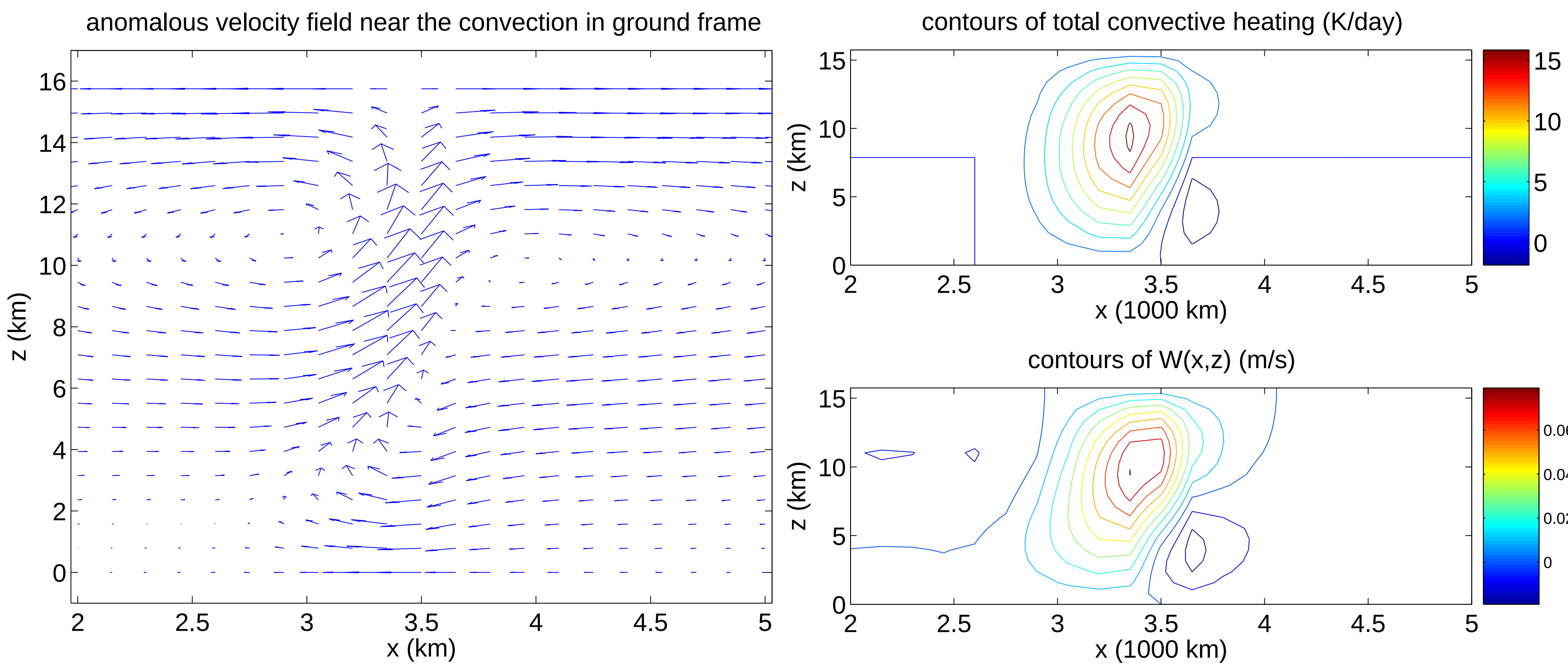
Nonlinear simulation



westward propagation at 18 m/s

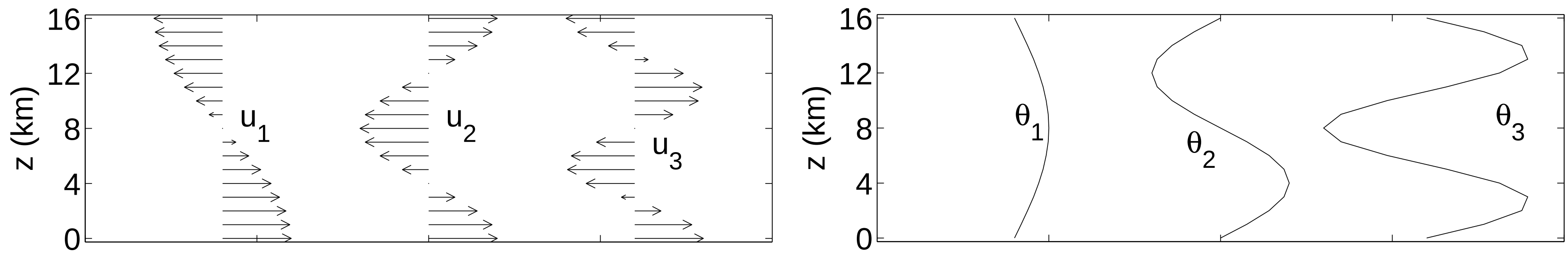
CCWs in the Multicloud Model

Nonlinear simulation



Modifications to the multicloud model

1. Add nonlinear advection and a 3rd baroclinic mode to capture key multiscale effects



[Stechmann, Majda, Khouider (2008) in press in journal *Theor. Comp. Fluid Dyn.*]

2. Use enhanced congestus closure of Khouider and Majda (2008)

3. Make congestus like a lower tropospheric version of deep convection

$$H_c = \alpha_c \frac{\Lambda - \Lambda^*}{1 - \Lambda^*} Q_c, \qquad \text{where} \qquad Q_c = \frac{1}{\tau_{conv}} (\theta_{eb} - a_0'(\theta_1 + \gamma_2' \theta_2))^+$$

Choose big value of γ_2' for a lower tropospheric CAPE closure

Outline

- Background
  - Convective momentum transport (CMT)
  - Effect of background wind shear on convection/waves
- A multicloud model for convectively coupled waves (CCWs)
  - Observations of CCWs
  - A multicloud model for CCWs
- A simple dynamical model for convective wave–mean flow interaction
  - Derivation
  - Derivation using multiscale asymptotics
  - Results

Dynamic model for convective wave–mean interaction

$$\frac{\partial \bar{U}}{\partial T} + \frac{\partial}{\partial z}\langle\overline{w'u'}\rangle = 0$$

$$\frac{\partial u'}{\partial t} + \bar{U}\frac{\partial u'}{\partial x} + w'\frac{\partial \bar{U}}{\partial z} + \frac{\partial p'}{\partial x} = S'_{u,1}$$

(with similar equations for other variables)

Key features of the model:

- Eddy flux convergence of wave momentum, $\partial_z\langle\overline{w'u'}\rangle$, feeds the mean flow $\bar{U}$
- Advection of the waves u' by the mean flow $\bar{U}$
- Mean flow time scale $T = \epsilon^2 t$ is longer than that for the waves

Multiscale asymptotic derivation of model

Need convectively coupled waves with *tilts* to have nonzero $\partial_z\langle\overline{w'u'}\rangle$

Derivation of convective wave–mean equations

Start with multicloud model with nonlinear advection:

$$\frac{\partial u_2}{\partial t} - \frac{\partial \theta_2}{\partial x} = -\frac{1}{\tau_u} u_2 - 2\sqrt{2}\,\bar{U}_3 \frac{\partial u_1}{\partial x}$$
$$= S_u + A_u$$
$$\frac{\partial \theta_2}{\partial t} - \frac{1}{4}\frac{\partial u_2}{\partial x} = H_c - H_s - R_2 - \frac{1}{2\sqrt{2}}\left[(u_1 - \bar{U}_3)\frac{\partial \theta_1}{\partial x} - (\theta_1 - 9\bar{\Theta}_3)\frac{\partial u_1}{\partial x} + 8\bar{\Theta}_4 \frac{\partial u_2}{\partial x}\right]$$
$$= S_\theta + A_\theta$$

Apply space-time average to obtain mean equations:

$$\frac{\partial \bar{U}_2}{\partial T} = \langle \overline{A_u} \rangle$$
$$\frac{\partial \bar{\Theta}_2}{\partial T} = \langle \overline{S_\theta} \rangle + \langle \overline{A_\theta} \rangle, \qquad \text{where } T = \epsilon^2 t \text{ is a longer time scale}$$

Caveat: Mean momentum source terms $\langle \overline{S_u} \rangle$ are dropped because convective momentum transport (CMT) is explicitly resolved by the eddy flux divergence, $\langle \overline{A_u} \rangle$

Asymptotic derivation of wave–mean equations

Overview

Multiscale ansatz:

$$u = \bar{U}(z, \epsilon^2 t) + \epsilon\, u'(x, z, t, \epsilon^2 t) + O(\epsilon^2),$$

Insert ansatz into equations of motion:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} + \frac{\partial p}{\partial x} = S_u.$$

Collect terms at each order, average, and apply secular growth condition to get:

$$\boxed{\frac{\partial \bar{U}}{\partial T} + \frac{\partial}{\partial z}\langle\overline{w'u'}\rangle = 0}$$

$$\boxed{\frac{\partial u'}{\partial t} + \bar{U}\frac{\partial u'}{\partial x} + w'\frac{\partial \bar{U}}{\partial z} + \frac{\partial p'}{\partial x} = S'_{u,1}}$$

(with similar equations for other variables)

Asymptotic derivation of wave–mean equations

Details for u equation

x: synoptic length scale (1500 km)

t: synoptic time scale (8 hours)

$T = \epsilon^2 t$: intraseasonal time scale (30 days)

Space and time averaging:

$$\bar{f}(z,t,T) = \lim_{L\to\infty} \frac{1}{2L} \int_{-L}^{L} f(x,z,t,T)\, dx$$

$$f'(x,z,t,T) = f(x,z,t,T) - \bar{f}(z,t,T)$$

$$\langle f\rangle(x,z,T) = \lim_{\tilde{T}\to\infty} \frac{1}{2\tilde{T}} \int_{-\tilde{T}}^{\tilde{T}} f(x,z,t,T)\, dt$$

Full temporal derivative of $f(t,\epsilon^2 t)$ is $\partial_t f + \epsilon^2 \partial_T f$

Asymptotic derivation of wave–mean equations

Details for u equation

Multiscale ansatz:

$$u = \bar{U}(z, \epsilon^2 t) + \epsilon\, u'(x, z, t, \epsilon^2 t) + \epsilon^2 u_2 + O(\epsilon^3),$$

Insert ansatz into equations of motion:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} + \frac{\partial p}{\partial x} = S_u.$$

Leading order terms at $O(\epsilon)$:

$$\partial_t u' + \partial_x(2\bar{U}u') + \partial_z(w'\bar{U}) + \partial_x p' = S'_{u,1}$$

This is the equation for the fluctuations u'

Asymptotic derivation of wave–mean equations

Details for u equation

Next order terms at $O(\epsilon^2)$:

$$\partial_t u_2 + \partial_T \bar{U} + \partial_x (u'^2 + 2\bar{U} u_2) + \partial_z (\overline{w'u'}) + \partial_x p_2 = S_{u,2}. \tag{1}$$

Apply zonal average to get equation for means:

$$\partial_t \bar{u}_2 + \partial_T \bar{U} + \partial_z (\overline{w'u'}) = 0$$

Suppressing secular growth of the higher order terms (see Majda refs):

- Ansatz assumed $\epsilon^2 u_2$ has a magnitude of $O(\epsilon^2)$
- This must be maintained or else the asymptotic ordering of the ansatz would be destroyed
- For a time-dependent equation of the form $\partial \bar{u}_2 / \partial t = F(t)$, secular growth in time is avoided if and only if $\langle F \rangle = 0$

Thus secular growth of $\bar{u}_2$ is avoided if

$$\partial_T \bar{U} = -\partial_z \langle \overline{w'u'} \rangle$$

Dynamic model for convective wave–mean interaction

$$\frac{\partial \bar{U}}{\partial T} + \frac{\partial}{\partial z}\langle\overline{w'u'}\rangle = 0$$

$$\frac{\partial u'}{\partial t} + \bar{U}\frac{\partial u'}{\partial x} + w'\frac{\partial \bar{U}}{\partial z} + \frac{\partial p'}{\partial x} = S'_{u,1}$$

(with similar equations for other variables)

Key features of the model:

- Eddy flux convergence of wave momentum, $\partial_z\langle\overline{w'u'}\rangle$, feeds the mean flow $\bar{U}$
- Advection of the waves u' by the mean flow $\bar{U}$
- Mean flow time scale $T = \epsilon^2 t$ is longer than that for the waves

Multiscale asymptotic derivation of model

- Intraseasonal time scale of $\bar{U}$ appears self-consistently

Multi-scale Model

$$\partial_T \bar{U} = -\partial_z \langle \overline{w'u'} \rangle$$

$$\partial_T \bar{\Theta} = -\partial_z \langle \overline{w'\theta'} \rangle + \langle \overline{S_{\theta,2}} \rangle$$

$$\partial_z \bar{P} = \bar{\Theta}$$

$$\begin{aligned} \partial_t u' + \bar{U}\partial_x u' + w'\partial_z \bar{U} + \partial_x p' &= S'_{u,1} \\ \partial_t \theta' + \bar{U}\partial_x u' + w'\partial_z \bar{U} + w' &= S'_{\theta,1} \\ \partial_z p' &= \theta' \\ \partial_x u' + \partial_z w' &= 0 \end{aligned}$$

Source terms are interactive using the multicloud model

Numerical methods

Equations take the form:

$$\frac{\partial u'}{\partial t} = f(u', \bar{U})$$

$$\frac{\partial \bar{U}}{\partial T} = g(u', \bar{U})$$

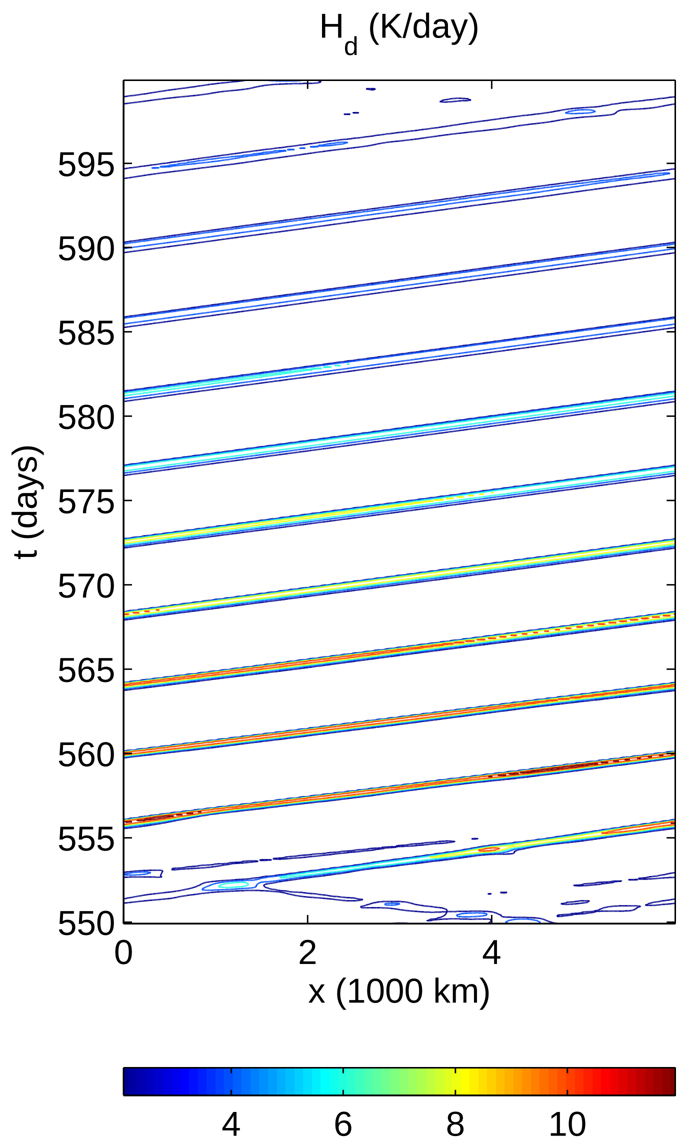
Two different time steps: $\Delta t \ll \Delta T$, with $\Delta T = 10\Delta t$ used here

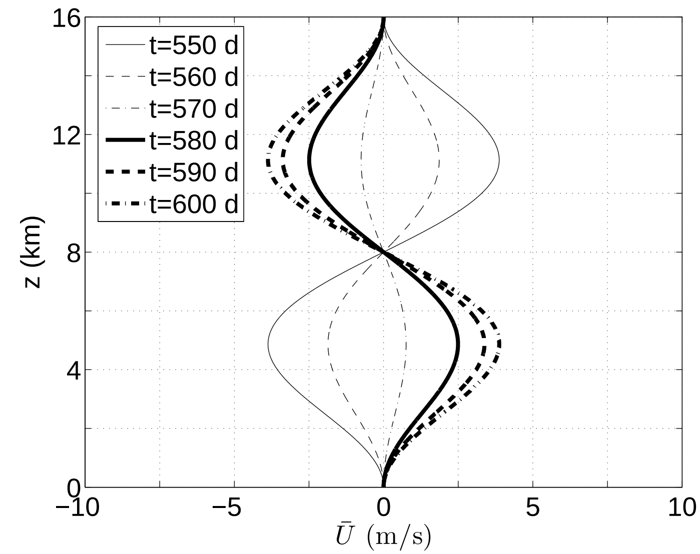
Numerical method:

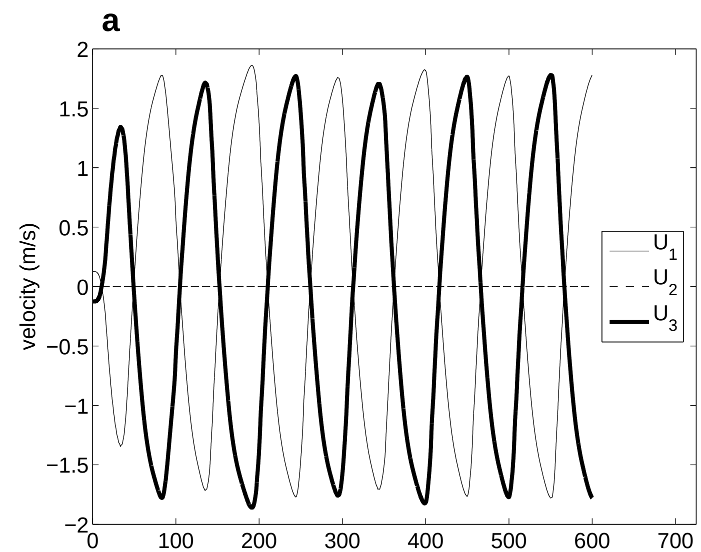
1. $\bar{U}(t_0)$ frozen, $u'(t_0) \to u'(t_0 + \Delta t) \to u'(t_0 + 2\Delta t) \cdots \to u'(t_0 + \Delta T)$
2. Compute $g(u', \bar{U})$, which involves time and space averages such as $\langle \overline{w'u'} \rangle$, over the long time interval $\Delta T = 10\Delta t$,
3. $u'(t_0 + \Delta T)$ frozen, $\bar{U}(t_0) \to \bar{U}(t_0 + \Delta T)$ using $g(u', \bar{U})$ from Step 2.

See Grabowski (2004) and Majda (2007) for other examples and references for this technique

Regular intraseasonal oscillations





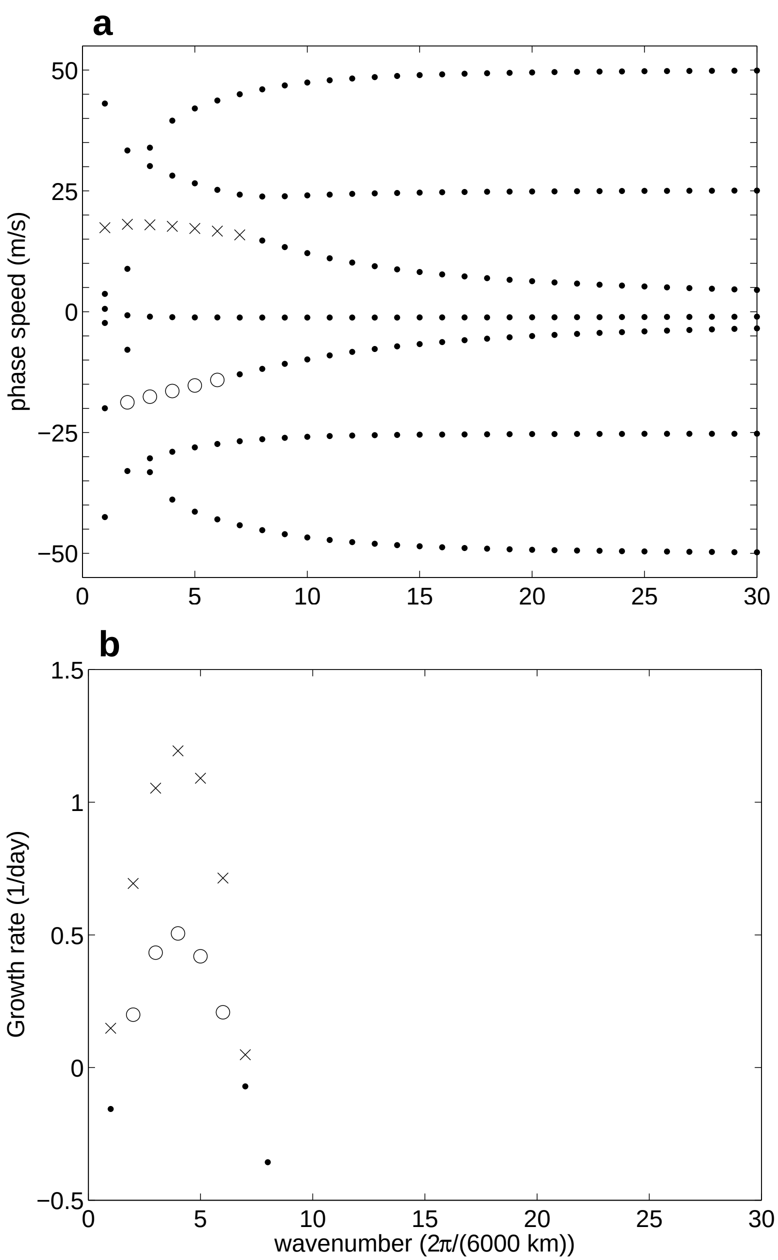


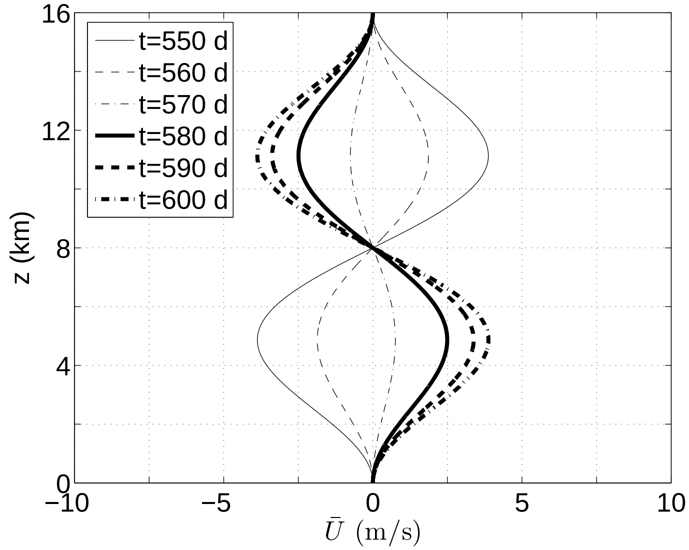
Wave strengthens or decays depending on mean wind

First downscale, then upscale CMT

Linear Stability Theory

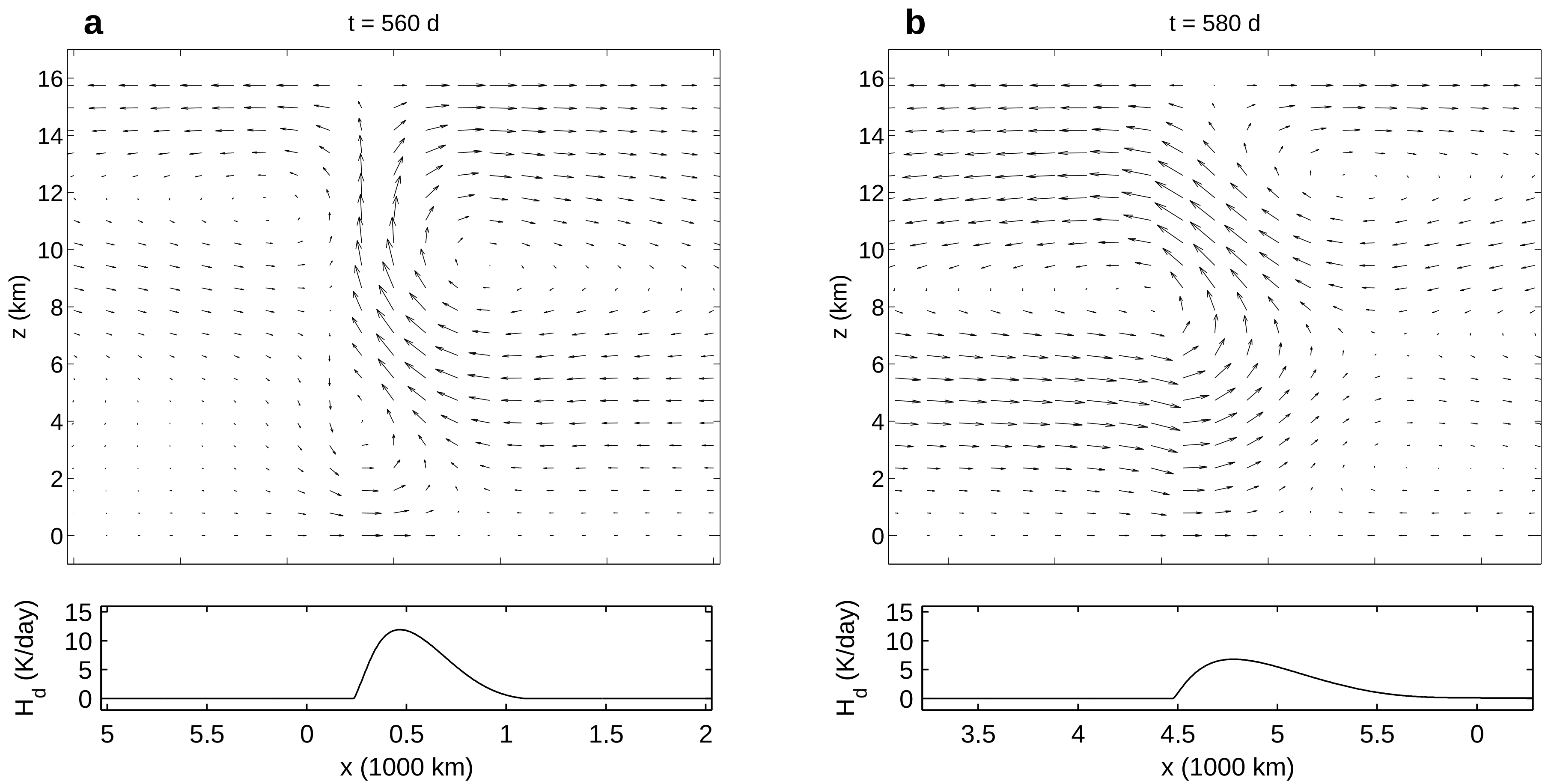
$t = 550$ days





- Linear theory corroborates simulations
- Eastward-propagating convectively coupled waves favored at $t = 550$ days

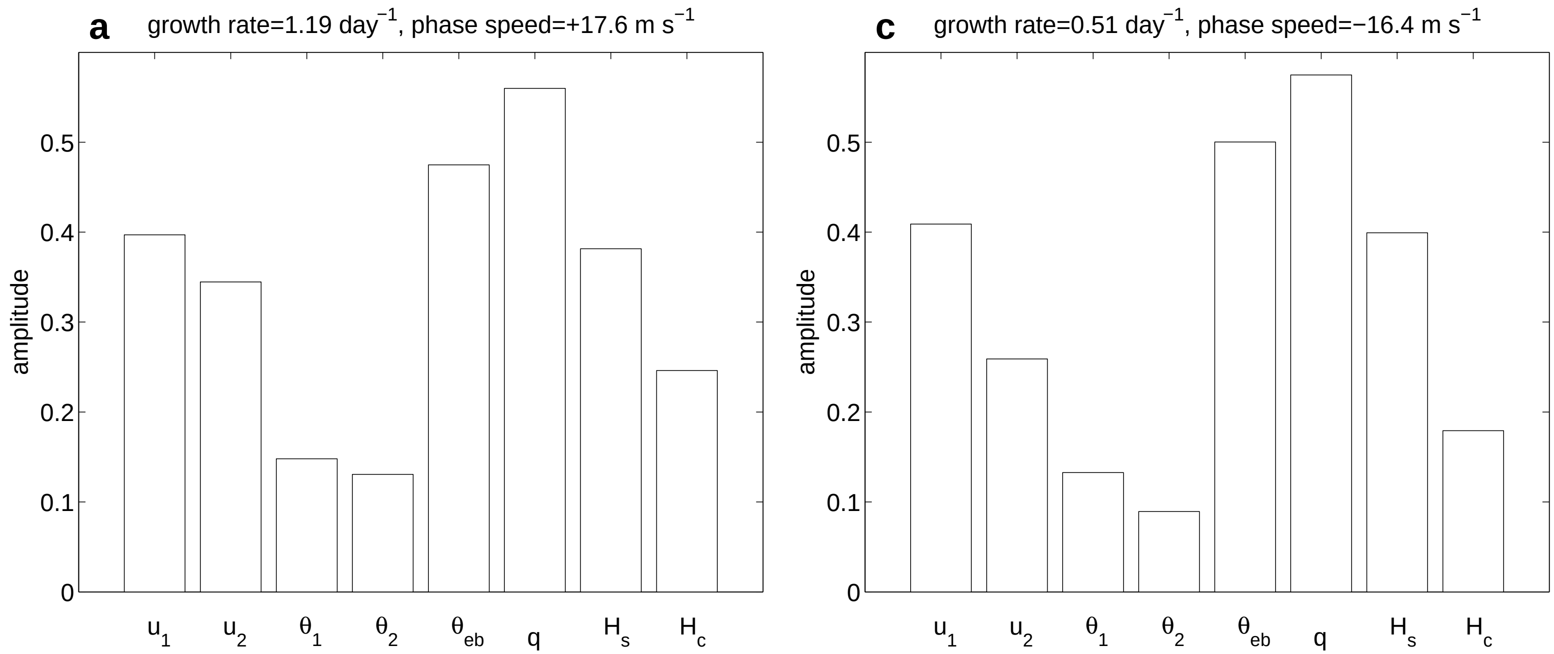
Snapshots of nonlinear waves in different mean shears



Strong front (rear) inflow for strong (weak) CCW

Weaker CCW has less vertical tilt

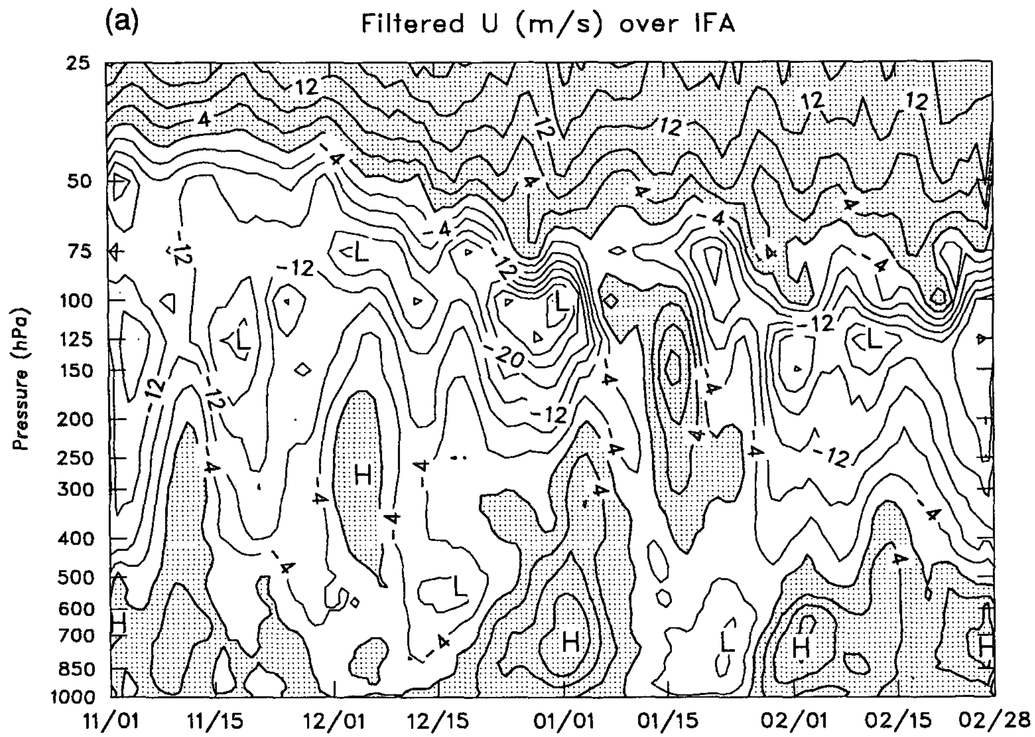
Linear Theory



u_2, θ_2, H_c stronger for favorable, eastward-propagating wave

I.e., favorable, eastward-propagating wave has more vertical tilt

The MJO and Westerly Wind Bursts



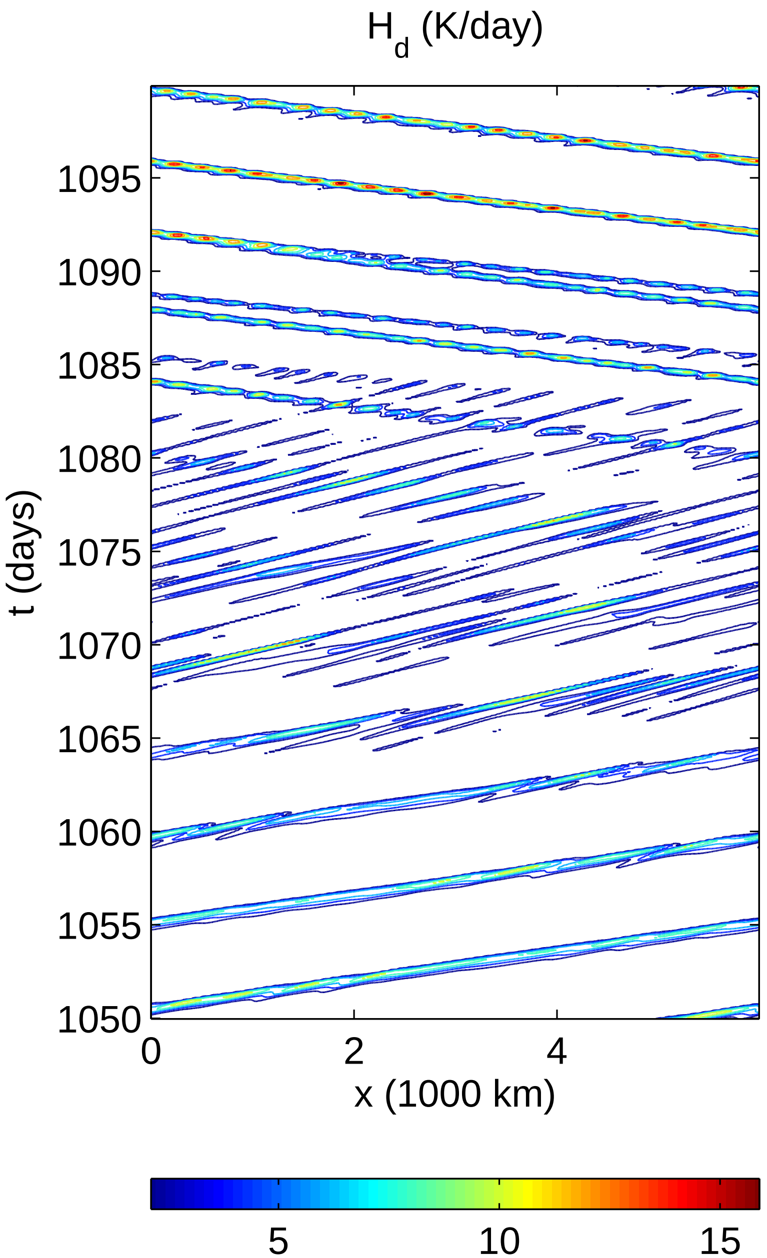
Surface westerlies begin at 12/15

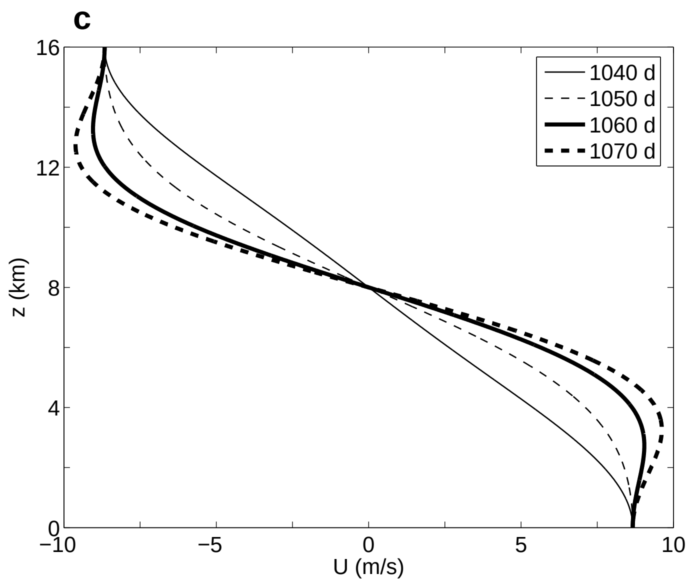
Strong westerly wind burst develops aloft at 01/01

Majda & Biello (2004), Biello & Majda (2005): multi-scale diagnostic model shows that
WWB develops from upscale CMT from eastward-moving CCWs

Obs. of WWB/CMT/CCWs also support this (Tung & Yanai, 2002; Masunaga et al., 2006)

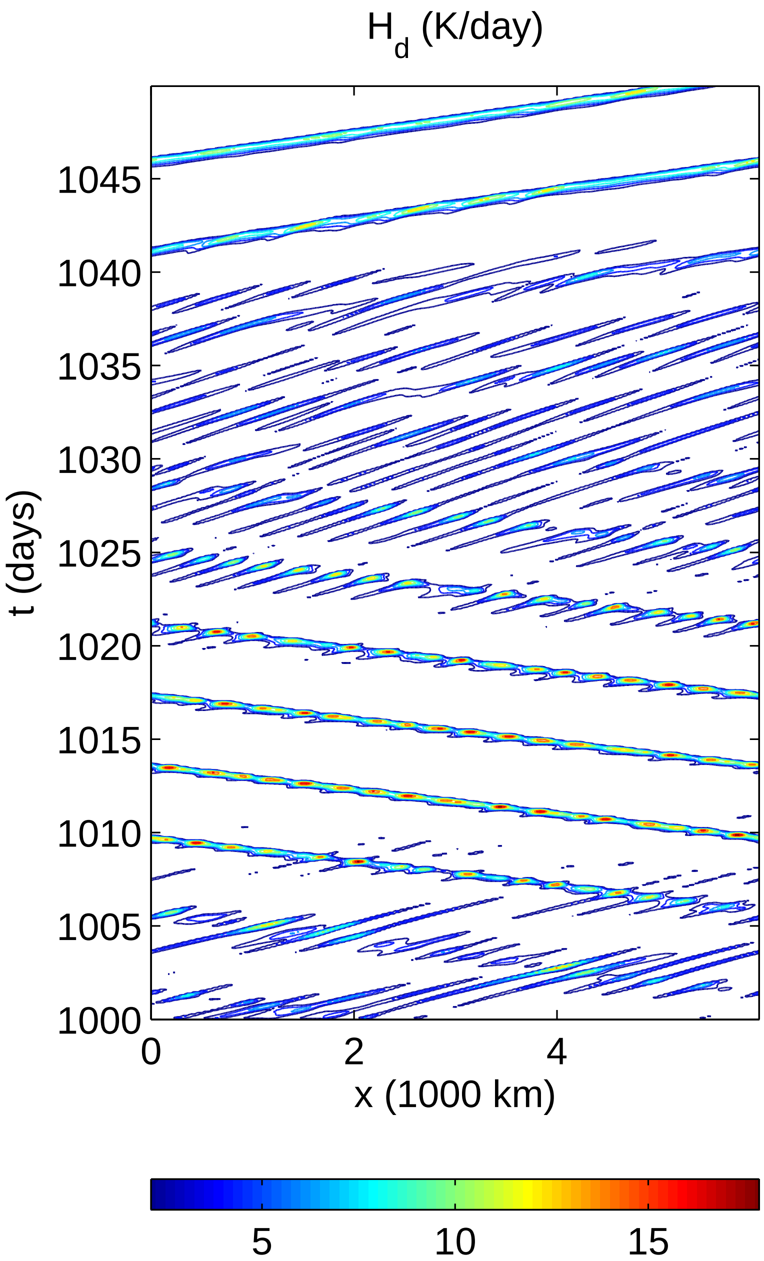
Westerly Wind Burst Intensification

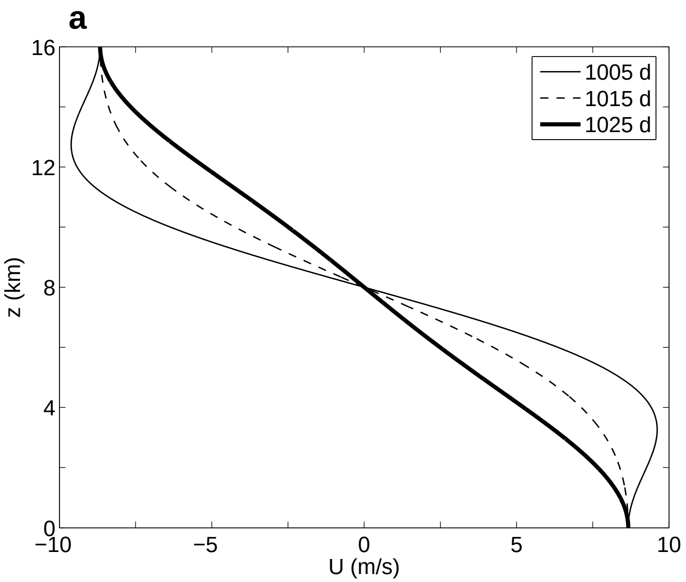




- Climate base state with low-level westerlies
- Upscale CMT from eastward-moving CCWs accelerates WWB aloft

Irregular intraseasonal oscillation with multiscale waves

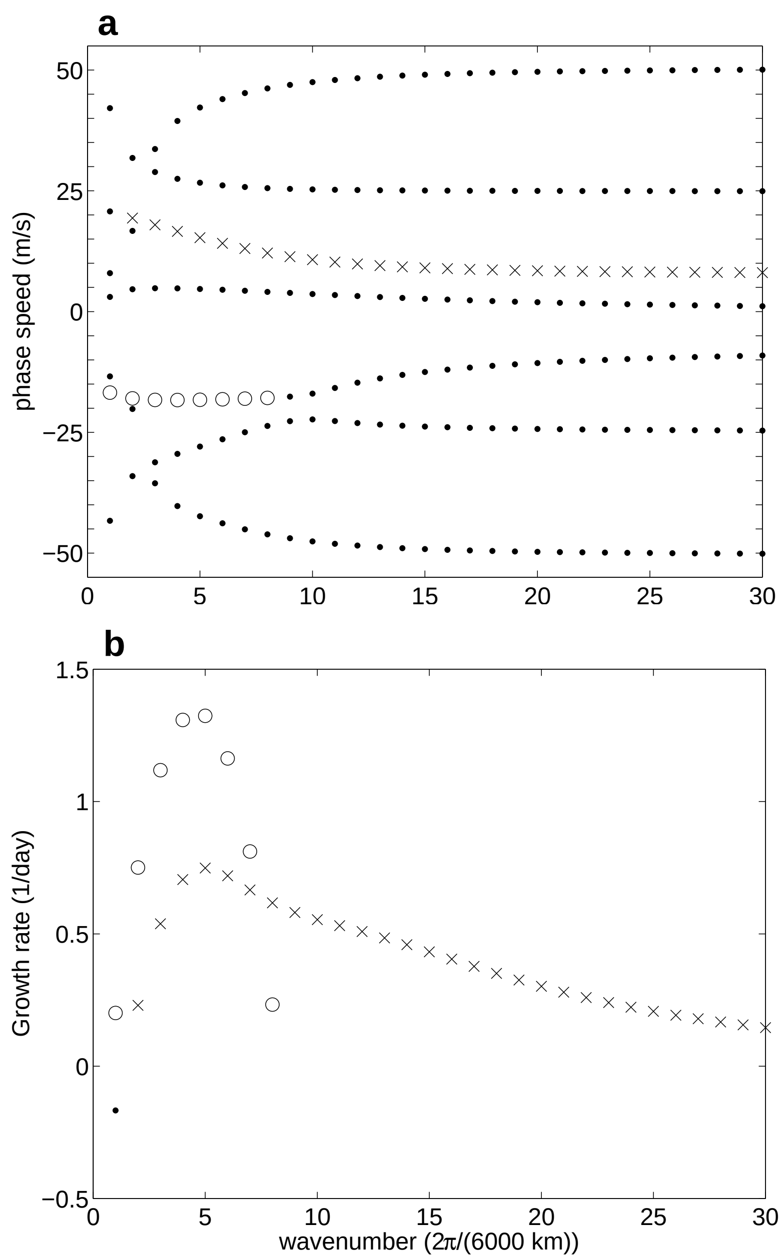


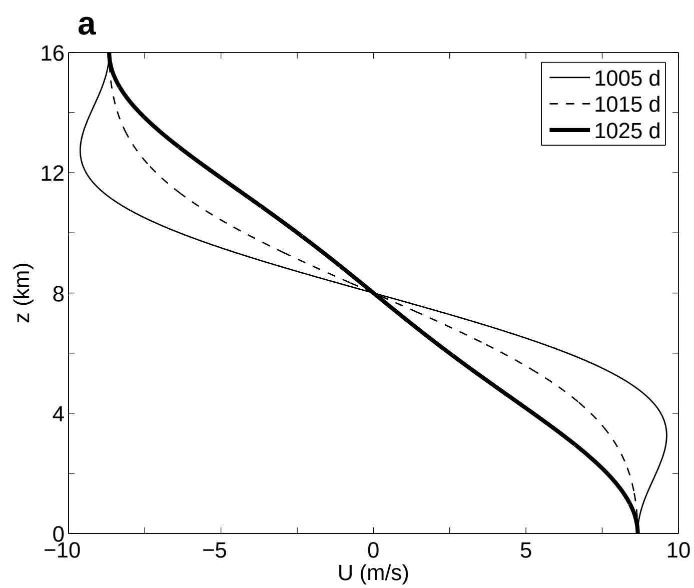


- Climate base state with low-level westerlies
- Either coherent or multiscale waves depending on mean wind

Linear Stability Theory

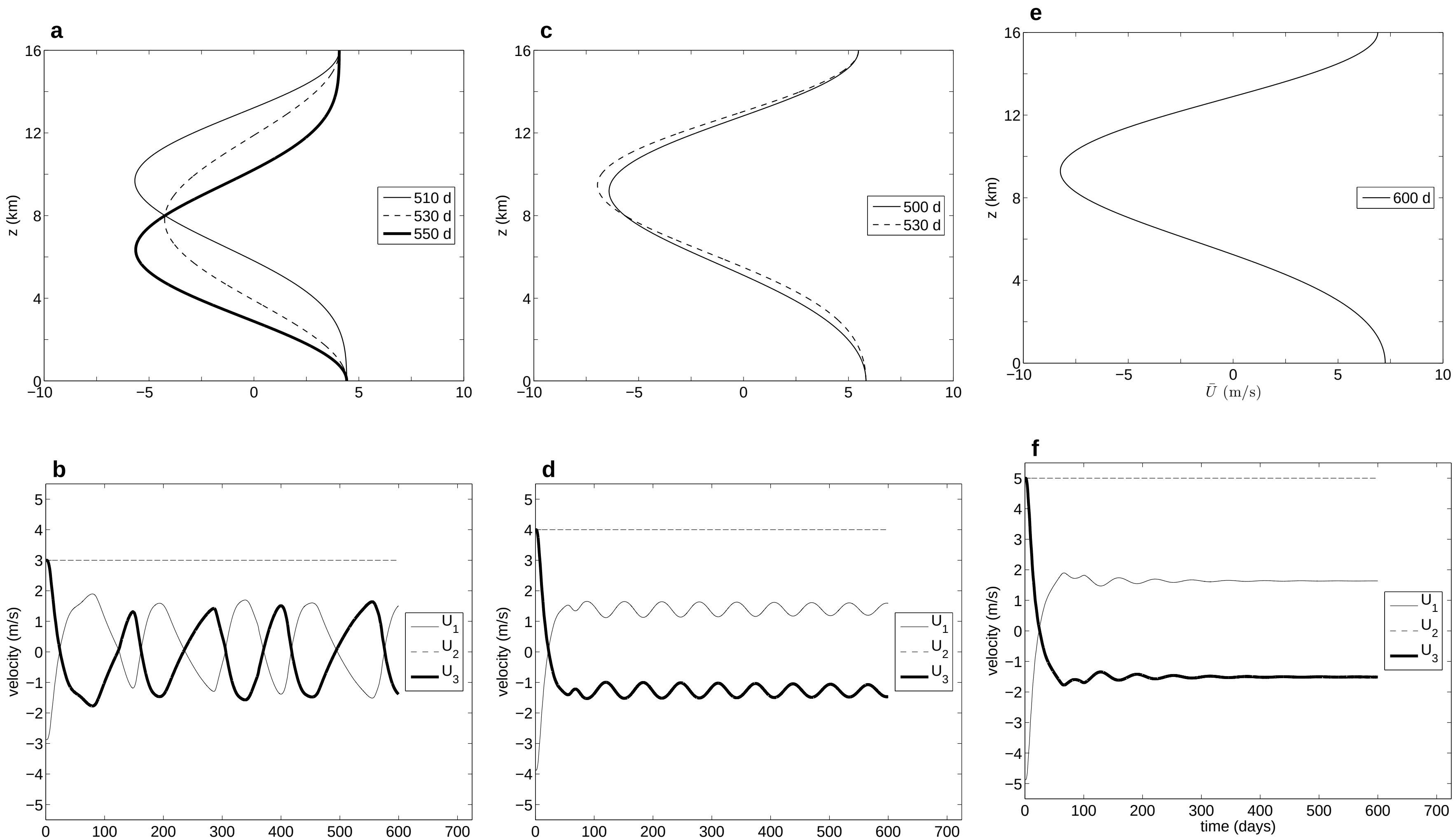
$t = 1005$ days





- Westward-propagating CCWs favored at larger scales
- Eastward-propagating CCWs favored at smaller scales

Irregular intraseasonal oscillations with Hopf bifurcation



Climate base state with mid-level easterly jet

Dynamics depend on strength of $\bar{U}_2$

Cloud-Resolving Model (CRM) simulations of CCWs:

What is the role of CMT from mesoscale convection?

Results vary depending on strength of momentum damping:

$$\frac{\partial u}{\partial t} = -\frac{1}{\tau}u + \cdots$$

- Held et al. (1993): No momentum damping: Long-time oscillation develops
  - Is this due to CMT interactions or stratospheric interactions?
- Grabowski & Moncrieff (2001): Weak momentum damping: CCWs develop with significant CMT
- Tulich et al. (2007): Stronger momentum damping: CCWs develop with little or no CMT
- Held et al. (1993): Intense momentum damping: Convection shut down except at a few grid points

Summary

- Dynamic model for convective wave–mean flow interactions
  - Two-way interactions
  - Asymptotic derivation and intraseasonal time scale
- Regular intraseasonal oscillations
  - CMT is first downscale, then upscale
  - Linear stability theory
- Irregular intraseasonal oscillations
  - Use different climate base states
  - One case:
    - westerly wind burst intensification as in MJO
    - either coherent or multiscale waves depending on mean wind
  - Another case: Hopf bifurcation