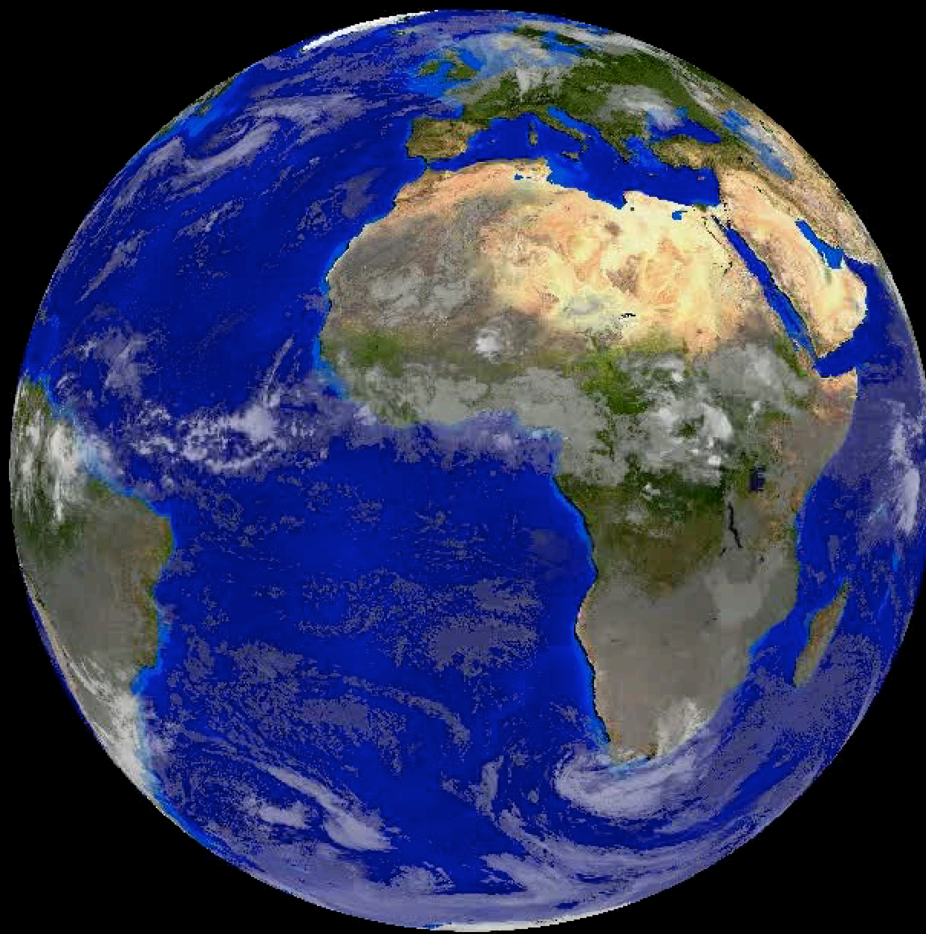
Geophysical Fluid Dynamics of the Earth



Jeffrey B. Weiss

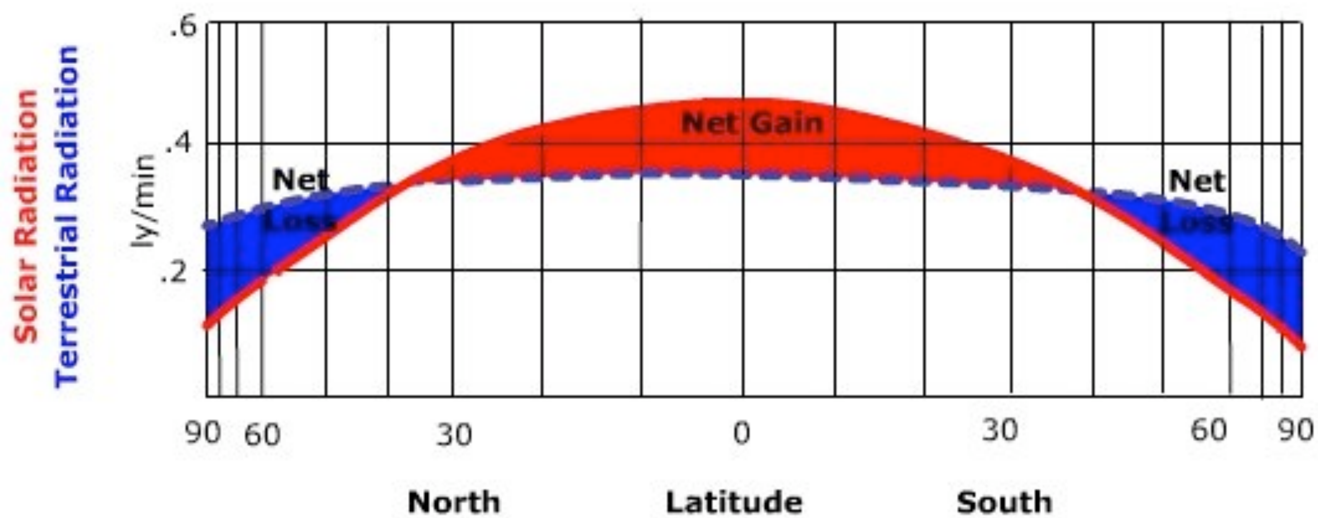
University of Colorado, Boulder

The Earth is a spinning sphere

- Coriolis force depends on latitude

$$f = 2\Omega \sin\theta$$

- solar flux depends on latitude



Michael Ritter,
http://www.uwsp.edu/geo/faculty/ritter/geog101/textbook/title_page.html

Atmosphere is turbulent

- global scale forcing:
  - $L \sim 10^7$ m
- dissipation:
  - $\nu_{air} \sim 10^{-5}$ m^2/s
- jet stream velocity:
  - $U \sim 10^2$ m/s
- $R_{atm} = 10^{14}$

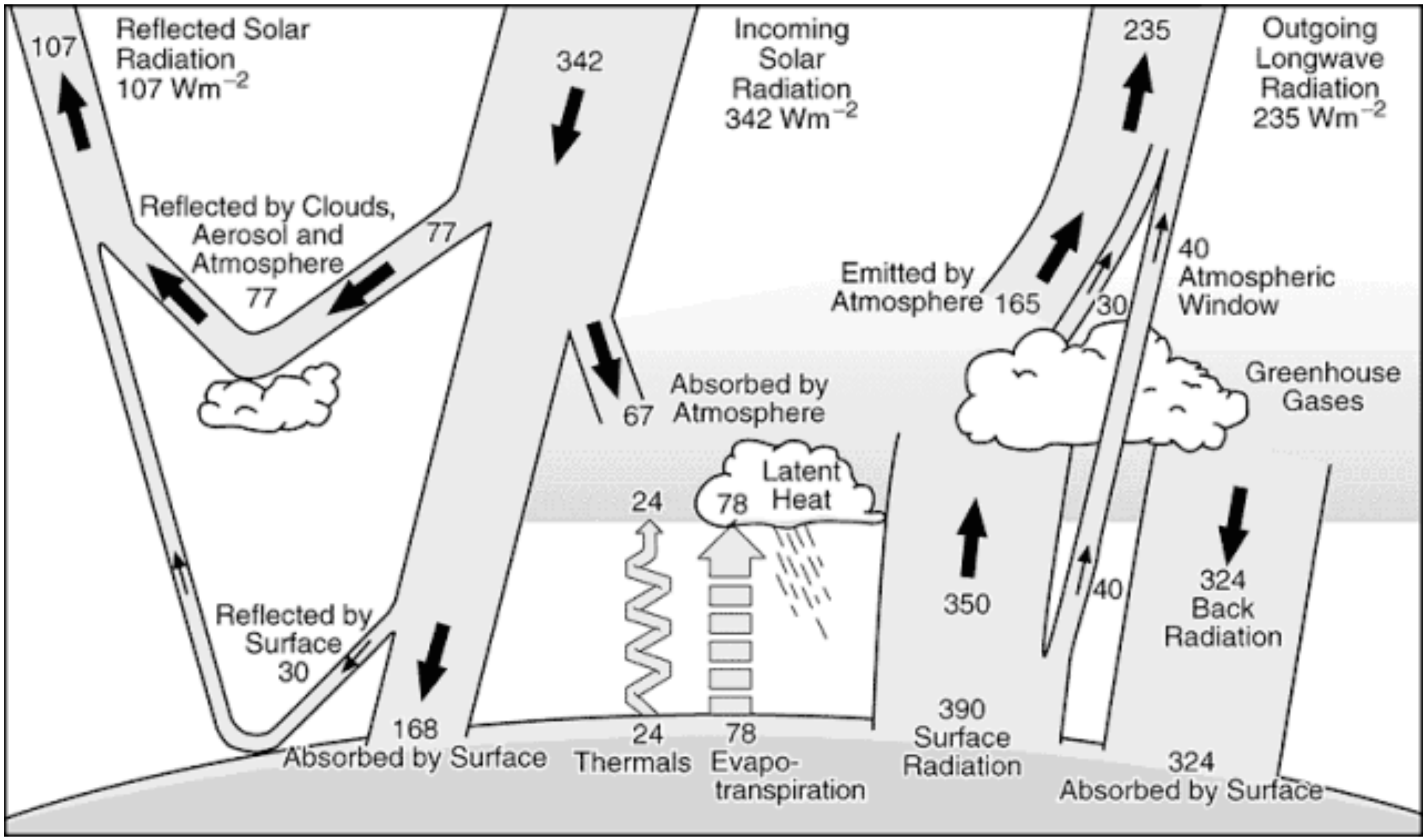
Direct simulation impossible

- Kolmogorov scale
  - $\eta = 1$ mm
- current global models:
  - ~ 1^o resolution ~ 100 km
- improvement needed: 10^8
- computer speed-up: $10^{32} \sim 2^{100}$
- Moore's Law: 200 years
- need to be smarter (or very patient)

Climate system self-organizes

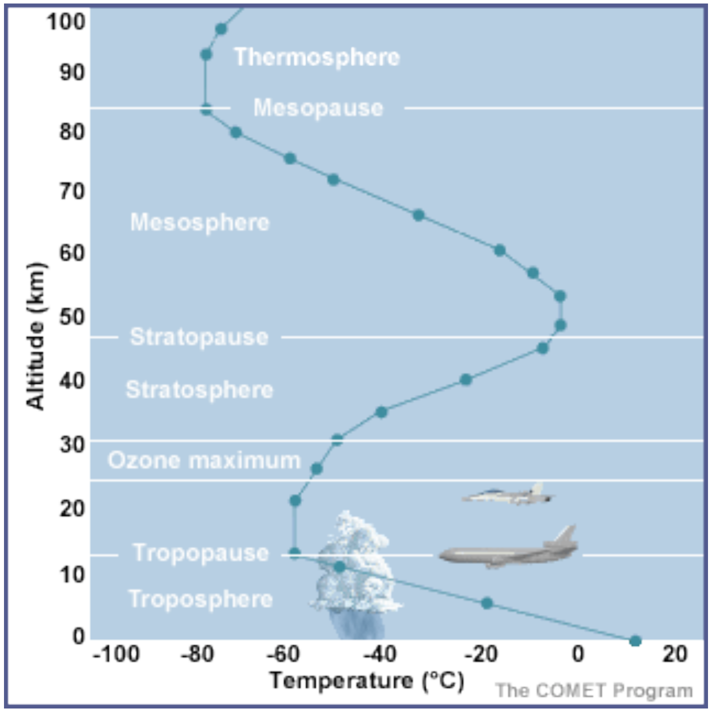
- layers:
  - troposphere
- circulations
  - Hadley cell
- spatially localized organization
  - ocean vortices
- spatio-temporal organization
  - El-Niño

Radiative forcing

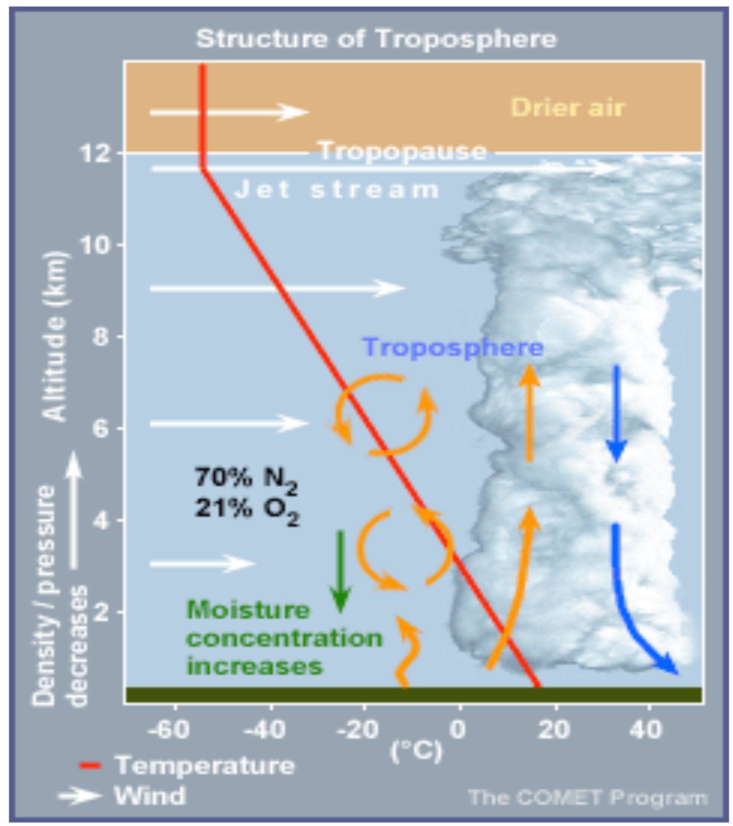


IPCC

Atmospheric Layers



- troposphere is weather layer
- height governed by
  - radiation
  - convection
  - dynamics



- tropopause dynamics interesting
- stratosphere stably stratified

planetary boundary layer

- directly feels surface
- 3d turbulence
- strong diurnal cycle
- surface layer
  - lower 10%
  - log wind profile

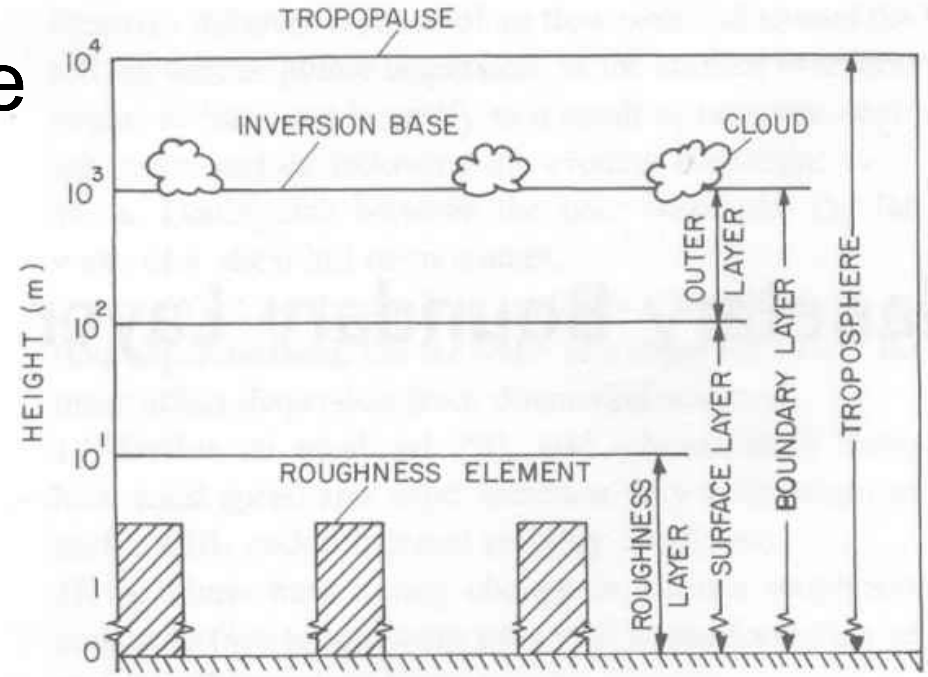
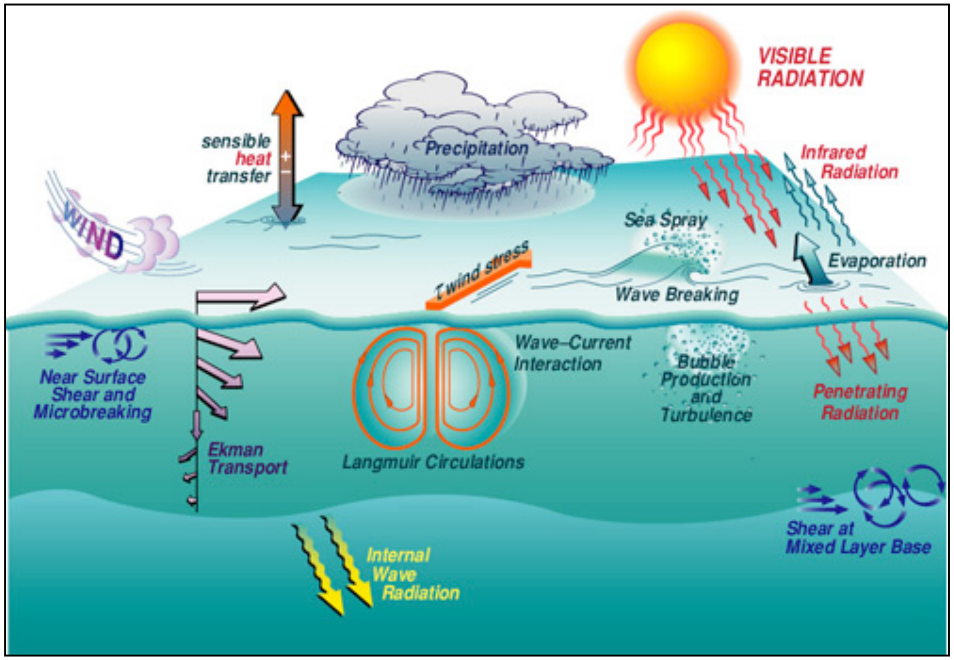


Figure 4.1 Schematic of the planetary boundary layer (PBL) as the lower part of the atmosphere.

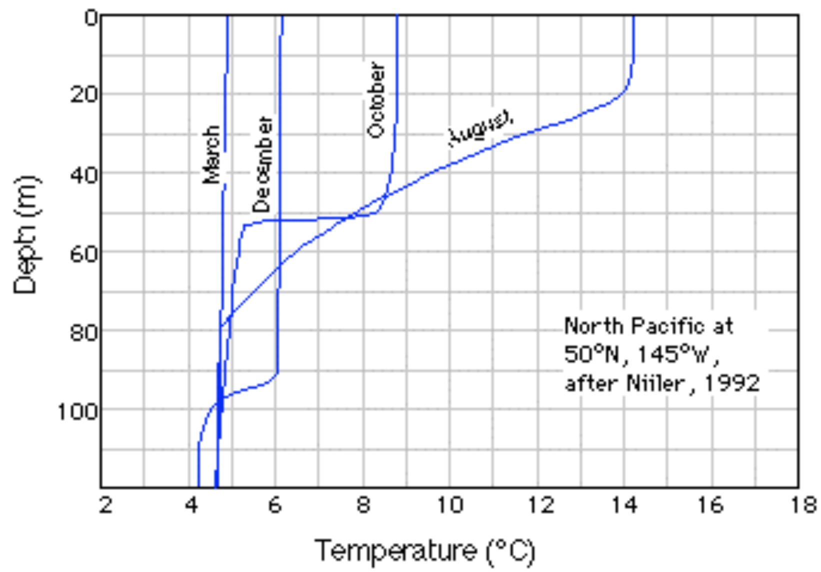
Arya, S.P. Air Pollution Meteorology and Dispersion

- free atmosphere
  - affected by rotation/stratification

ocean mixed layer



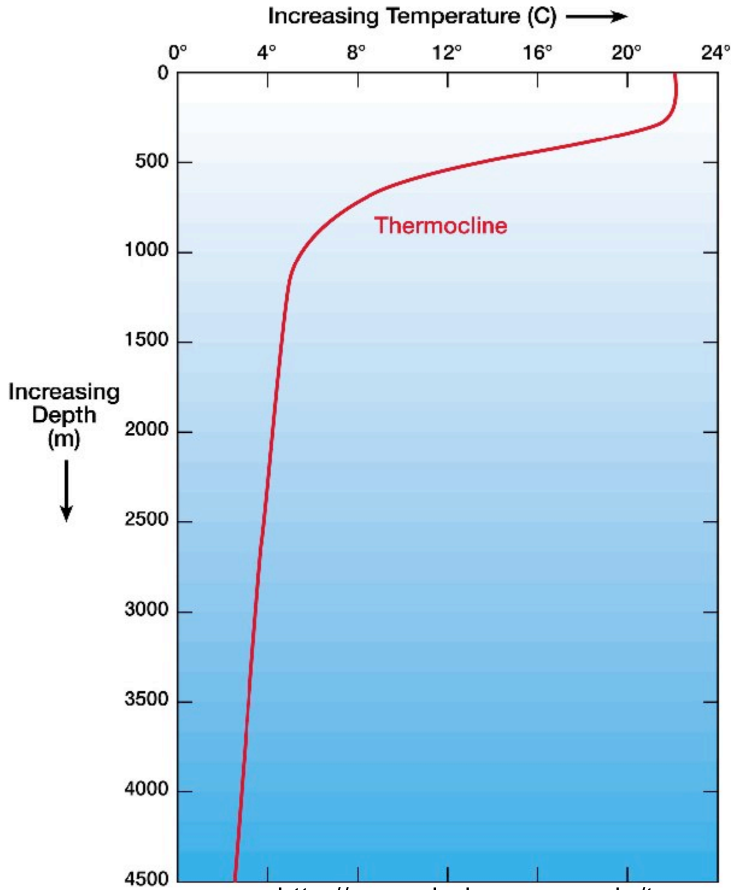
http://hpl.umces.edu/ocean/sml_main.htm



- directly feels atmosphere
- mixed by wind
- stabilized by solar heating

deep ocean

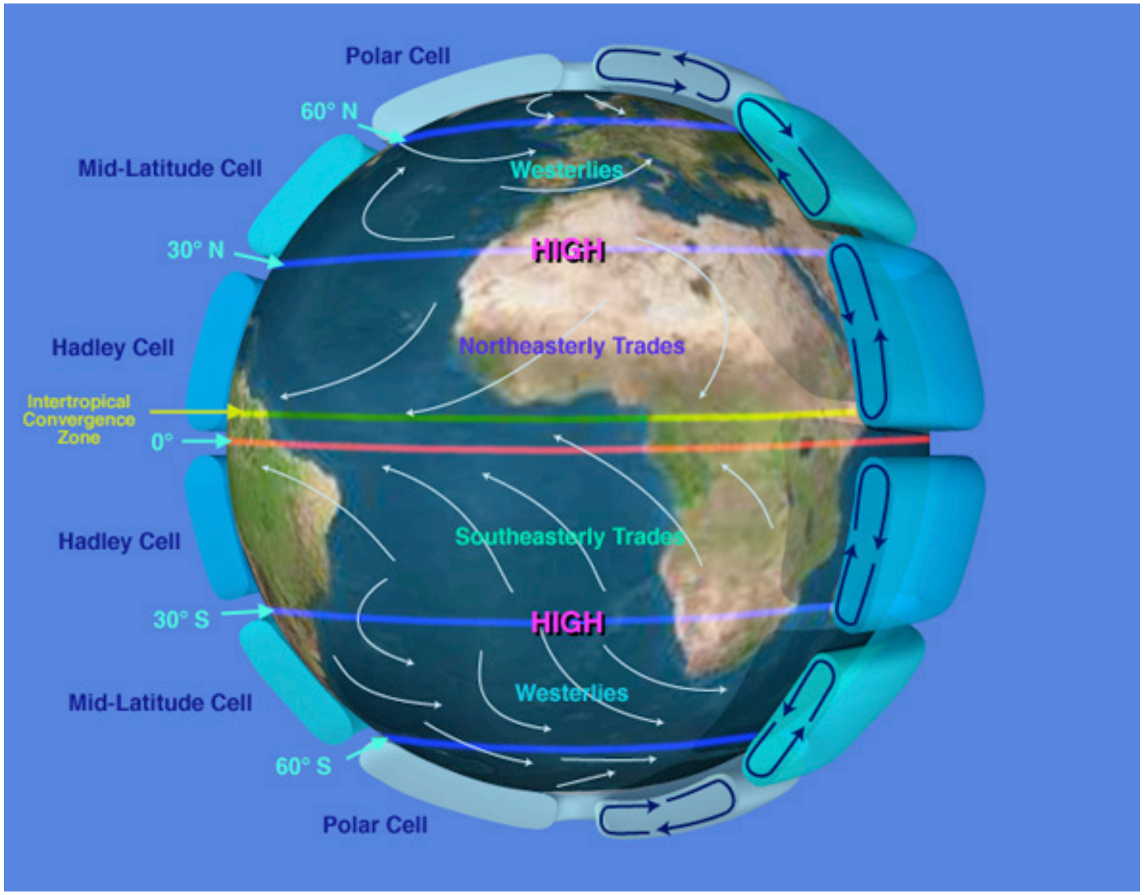
- thermocline
  - 50 - 1000 m
- below stably stratified
- slow currents



http://www.windows.ucar.edu/tour

Large scale circulations

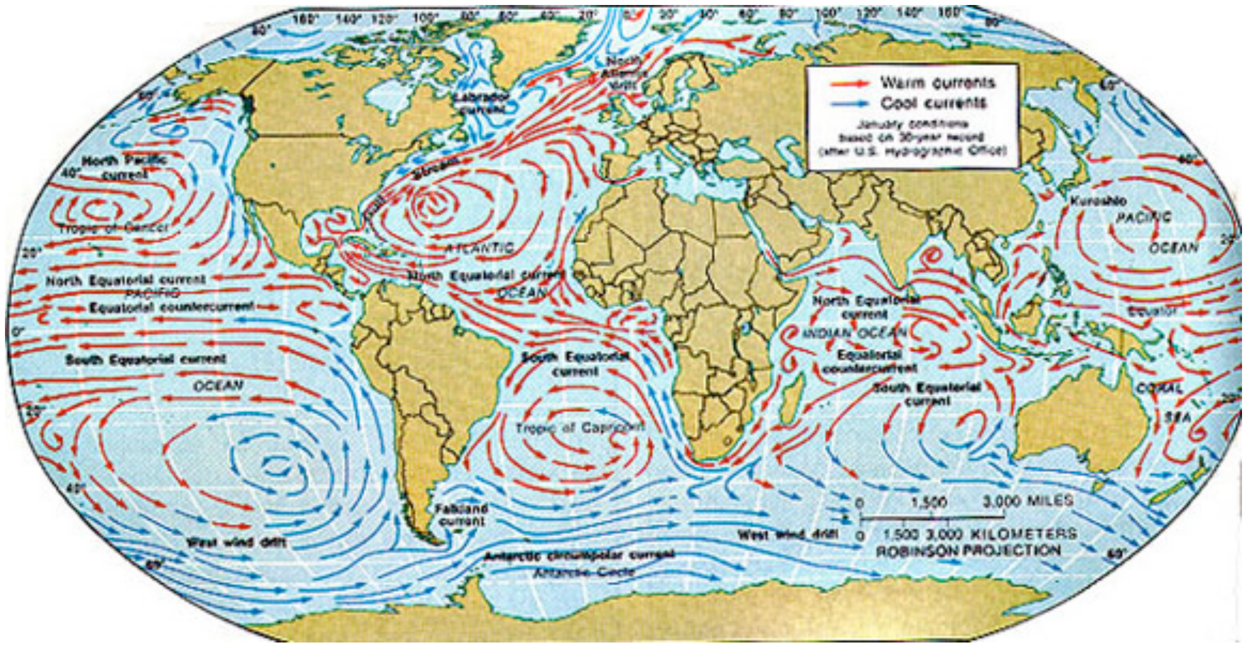
- 3-cell model
  - trades
  - westerlies
  - ITCZ
  - deserts
  - frontal zone
- shifts seasonally



NASA JPL

wind-driven ocean circulation

- winds + basins = gyres



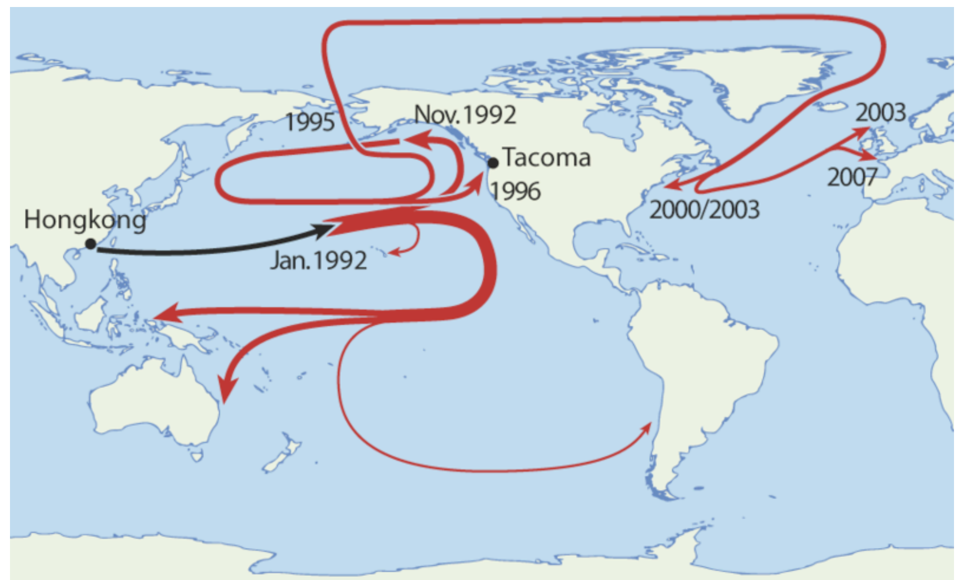
geographyalltheway.com

- horizontal
- large geostrophic component
- timescales: months

serendipitous tracers

- 1990: 20,000 Nike shoes washed into Pacific Gyre
- 1992: 29,000 bathtub toys
- locations from beachcombers

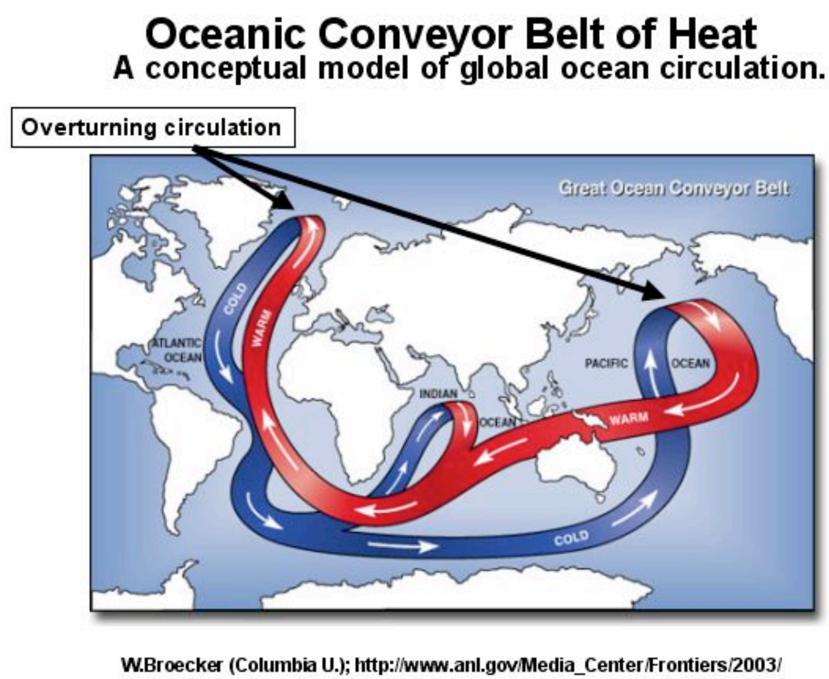
Jim Ingraham / NOAA-Fisheries



http://en.wikipedia.org/wiki/Image:Friendly_Floatees.png

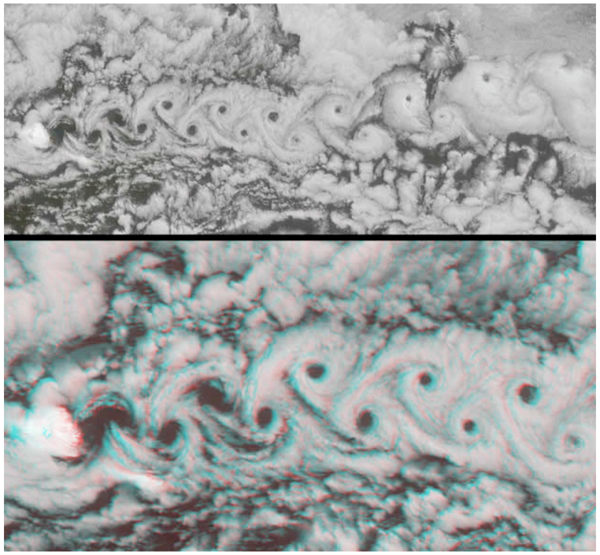
thermohaline circulation

- density driven vertical overturning
- forcing:
  - small scale intermittent convection
  - uncertain vertical mixing
- timescale: 1000 yrs
- large climate impact

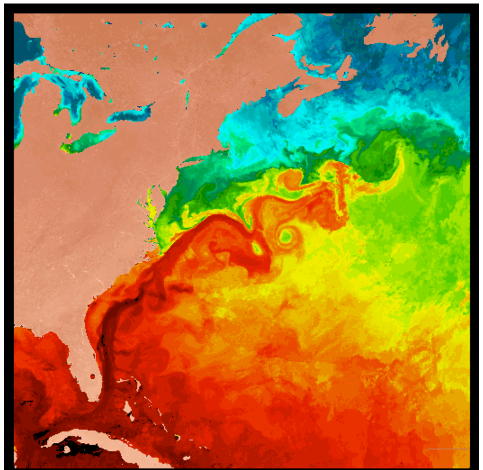


spatially localized features

- Coherent structures
  - vortices, jets, fronts
- rotation and stratification suppress vortex stretching
- inverse cascade to large scales



http://eosweb.larc.nasa.gov/HPDOCS/misr/misr_images/arctic_vortex.jpg

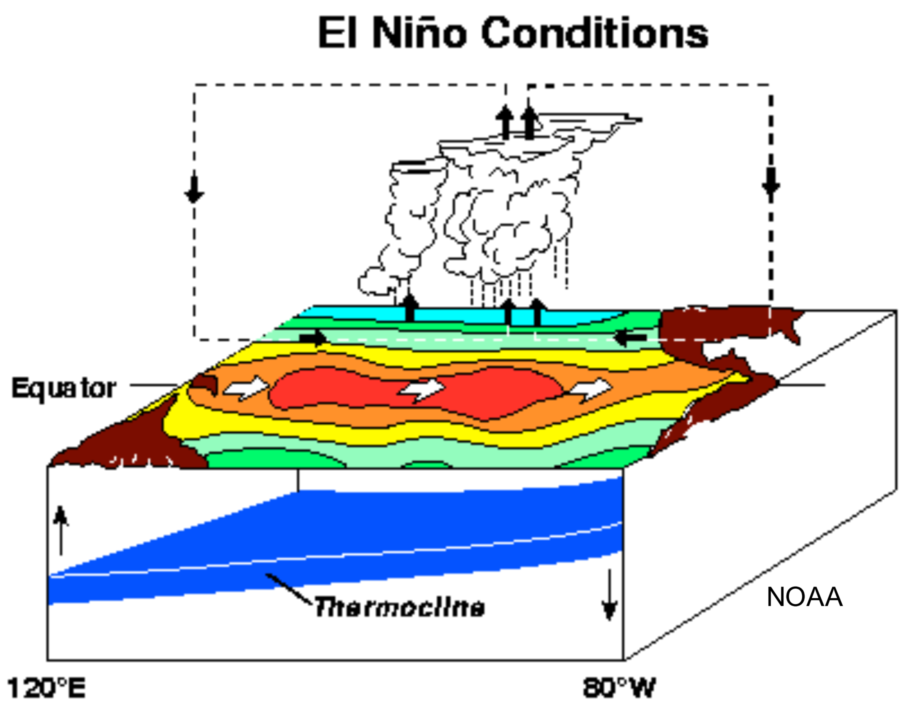


Gulf-Stream and mesuscale eddies pictured on a color-enhanced NOAA/AVHRR image (downloaded from http://seawifs.gsfc.nasa.gov/SEAWIFS/IMAGES/eastcoast.gif).

Figure 1.3.1

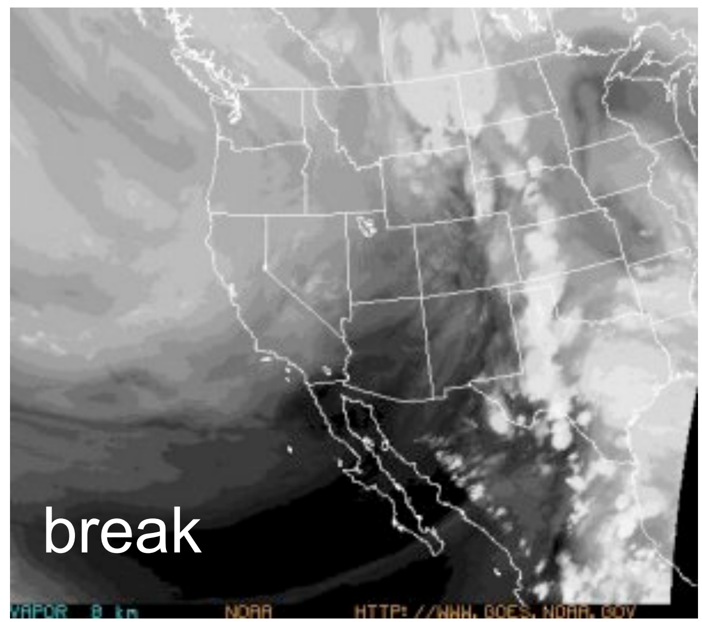
Spatio-temporal phenomena

- El Niño, Monsoons, NAO, MJO, PDO, …
- As records lengthen, expect to see more
- can have extreme multi-scale components



Monsoon

- seasonal circulation associated with land-sea contrast
- complex intermittency
- experience the North American Monsoon



http://www.wrh.noaa.gov/fgz/science/monsoon.php?wfo=fgz

Balances

- often find balances between subset of forces
- provide insight into dynamics
- departures important even if small
- asymptotic analysis gives reduced models

Hydrostatic Balance

- aspect ratio α = *H/L*
- Froude number *Fr* = *U/NH*
- non-rotating Boussinesq equations for departures about basic state *b(z)*
- vertical momentum eqn

$$Fr^2\alpha^2\frac{Dw}{Dt} = -\frac{\partial\phi}{\partial z} + b'$$

- $\alpha^2 \, Fr^2 << 1$ gives hydrostatic balance
  - $U^2/L^2 \, N^2 << 1$
  - $t_{buoy}^2 << t_{adv}^2$

- large scale troposphere
  - $N \sim 10^{-2}$/s, $H \sim 10$ km, $L \sim 1000$ km, $U \sim 10$ m/s
  - $Fr \sim 0.1$ $\alpha \sim 0.01$
  - $\alpha^2 Fr^2 \sim 10^{-6}$
  - fails at fronts and convection
- large scale ocean
  - $N \sim 10^{-2}$/s, $H \sim 1$ km, $L \sim 1000$ km, $U \sim 0.1$m/s
  - $Fr \sim 0.01$ $\alpha \sim 0.001$
  - $\alpha^2 Fr^2 \sim 10^{-10}$
  - hydrostatic down to small horizontal scales
  - fails at deep convection sites
    - preconditioning: Fr near unity
    - plumes: H/L large

Geostrophic balance

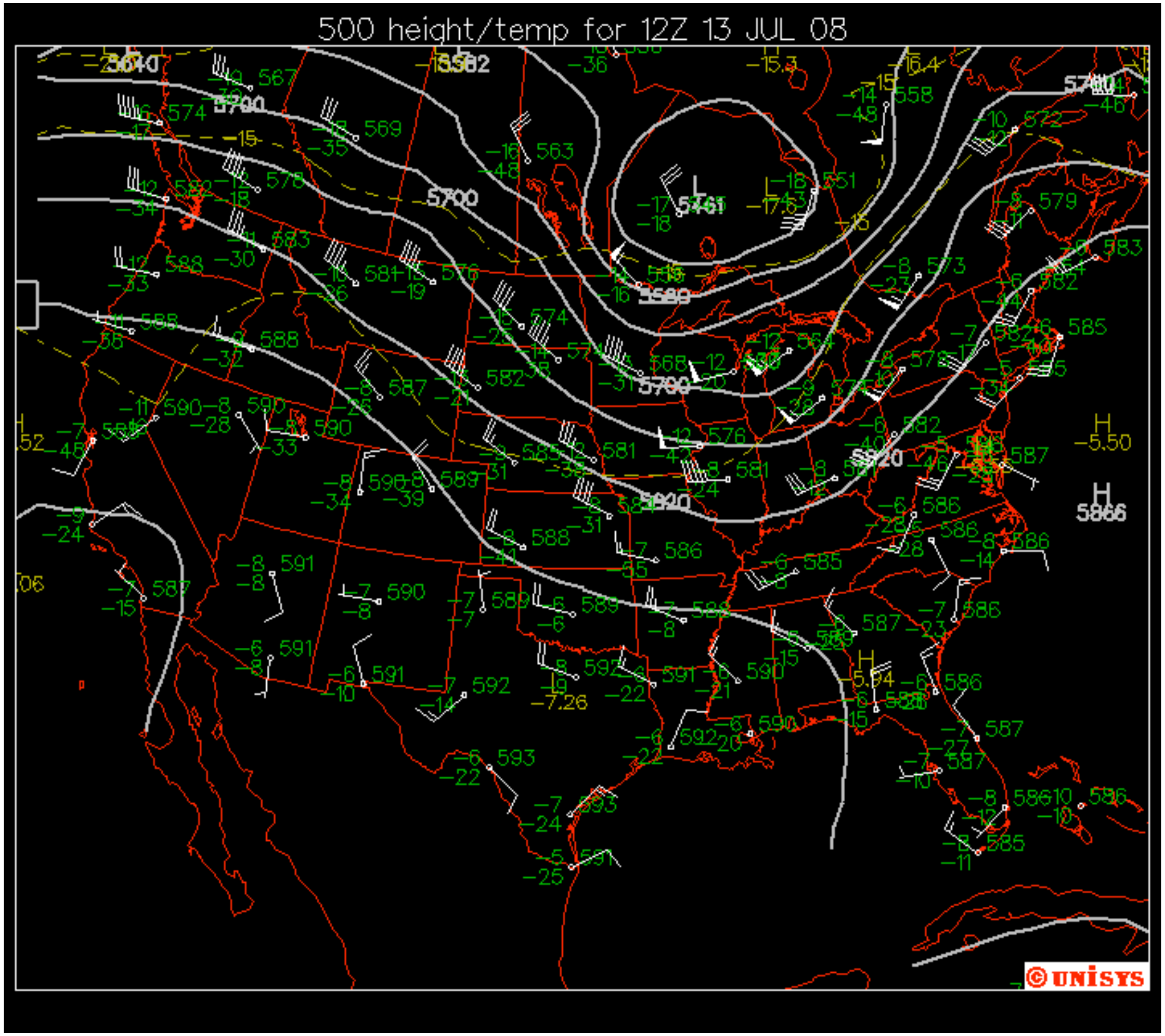
- Rossby number: Ro = U/ f L
- horizontal momentum eqn

$$\frac{D\mathbf{u}}{Dt} + \mathbf{f} \times \mathbf{u} = -\frac{1}{\rho}\nabla p$$

scaling: $Ro\, fU \quad fU$

- Ro << 1: geostrophy
  - flow along isobars
  - diagnostic relation

upper air map



thermal wind

- combine geostrophic and hydrostatic
- obtain wind shear

$$\mathbf{f} \times \mathbf{u} = -\nabla_h \phi \quad \frac{\partial \phi}{\partial z} = b'$$

$$\mathbf{f} \times \frac{\partial \mathbf{u}}{\partial z} = -\nabla_h b'$$

- midlatitudes
  - heating decreases with latitude
  - eastward wind increases with height
  - jet intensification

turbulent cascades

- compare homogeneous isotropic 3d and 2d
- energy injection at large scale
- dissipation at small scale
- intermediate: inertial scales, only depend on energy flux ε
- focus on spectra: $E = \int E(k)dk$
- assume local in k and universal

$$E(k) = g(\varepsilon, k)$$

- dimensional analysis

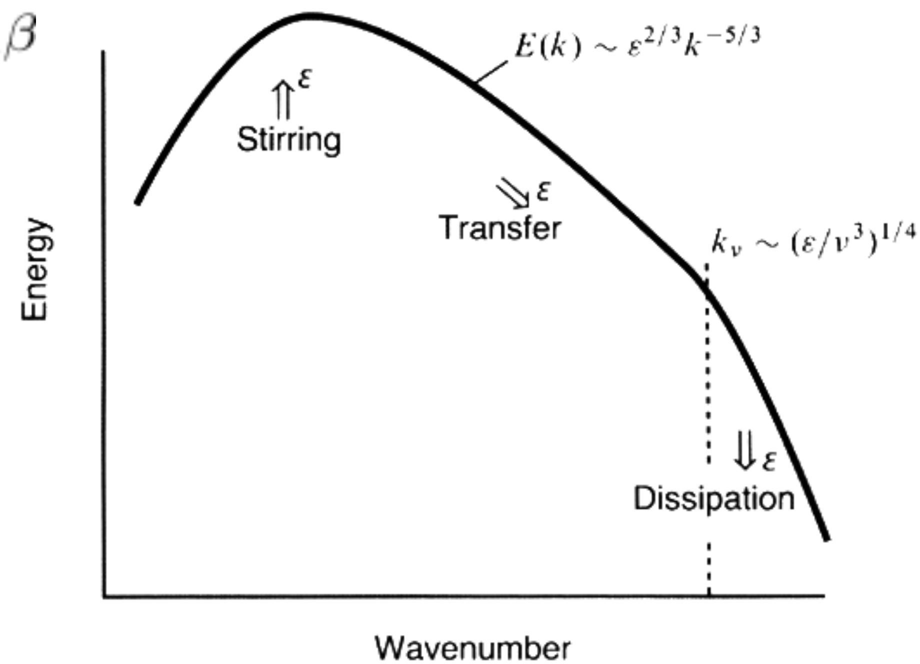
$$k \sim 1/L$$

$$E \sim L^2/T^2$$

$$E(k) \sim EL \sim L^3/T^2$$

$$\varepsilon \sim E/T \sim L^2/T^3$$

- $E(k) = g(\varepsilon, k) = \mathcal{K}\varepsilon^{\alpha}k^{\beta}$
  - T requires $\alpha = 2/3$
  - L requires $\beta = -5/3$



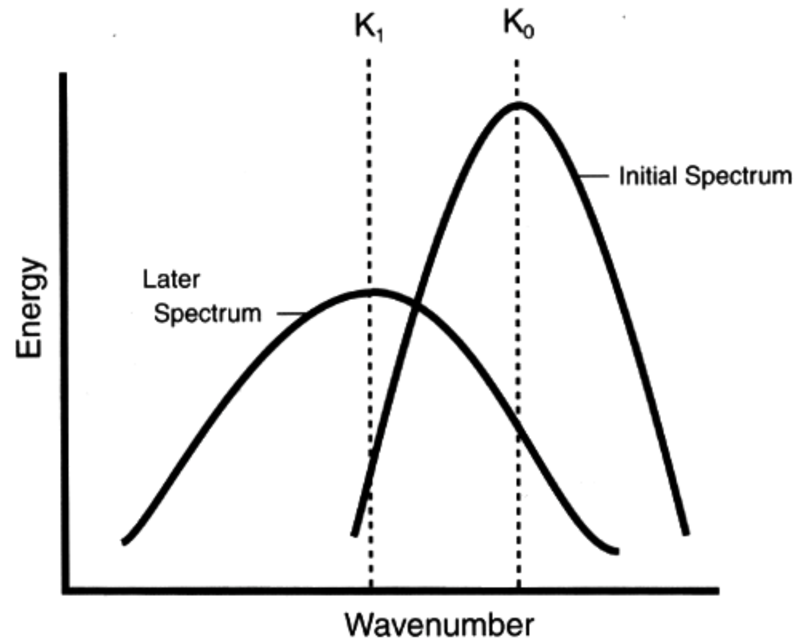
Vallis, Atmospheric and Oceanic Fluid Dynamics, 2006

2d cascades

- In 2d, two inviscid invariants
  - energy E and enstrophy Z
  - in 3d vortex stretching creates enstrophy
  - $Z(k) = k^2 E(k)$
- centroid wavenumber $k_c = \int kE(k), \quad E = 1$
- width $W = \int (k - k_c)^2 E(k) = Z - k_c^2$
- initial narrow E(k) spreads

$$dW/dt > 0 \Rightarrow dk_c^2/dt < 0$$

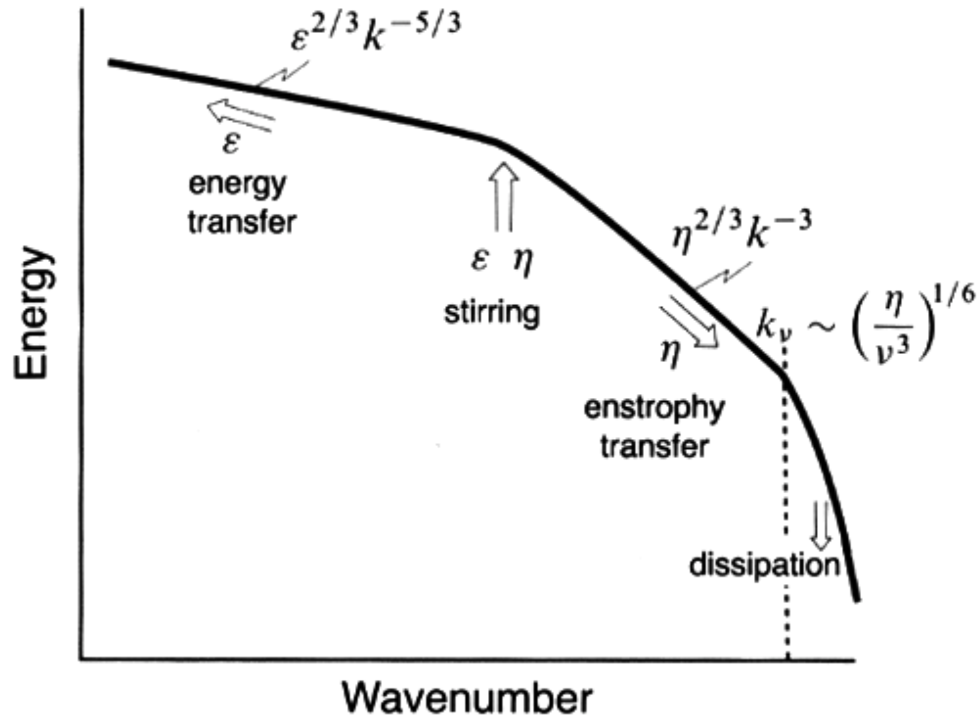
- energy cascades to large scales: inverse



Vallis, Atmospheric and Oceanic Fluid Dynamics, 2006

- similar argument: enstrophy to small scales: direct

- energy inertial range to large scales
- enstrophy inertial range to small scales
- energy inertial ranges same scaling as 3d
- need mechanism to remove energy at large scales, e.g. Ekman drag
- enstrophy flux
  - $\eta \sim Z/T \sim 1/T^3$
  - $E(k) \sim \eta^{2/3} k^{-3}$



Vallis, Atmospheric and Oceanic Fluid Dynamics, 2006

cascades on 2d β-plane

- vorticity equation on 2d β-plane
  - f = β y

$$\frac{\partial \zeta}{\partial t} + \mathbf{u} \cdot \nabla \zeta + \beta v = 0$$

- Rossby waves

$$\zeta = A e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}, \quad A \ll 1$$

- dispersion relation

$$\omega = \frac{-\beta k_x}{k_x^2 + k_y^2}$$

- scaling

$$\underbrace{\frac{\partial \zeta}{\partial t} + \mathbf{u} \cdot \nabla \zeta}_{\frac{U^2}{L^2}} + \underbrace{\beta v}_{\beta U} = 0$$

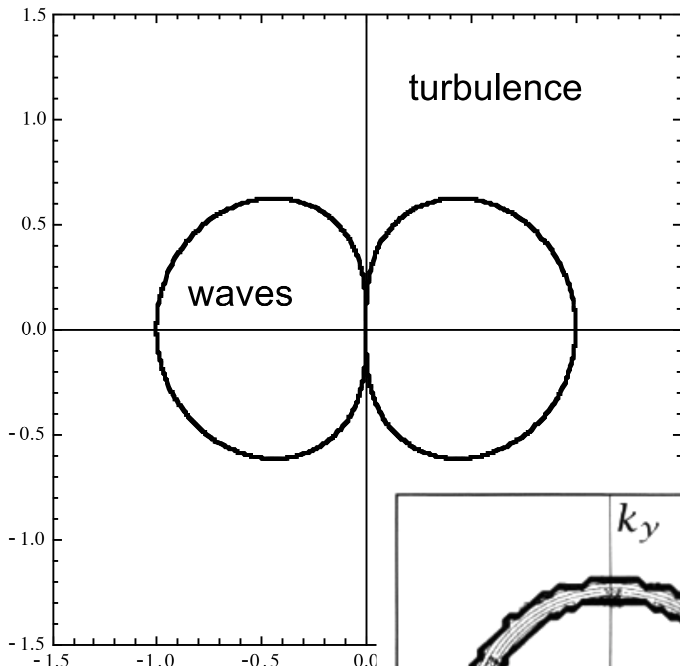
- small scales
  - advection and turbulence dominate
- crossover: Rhines scale

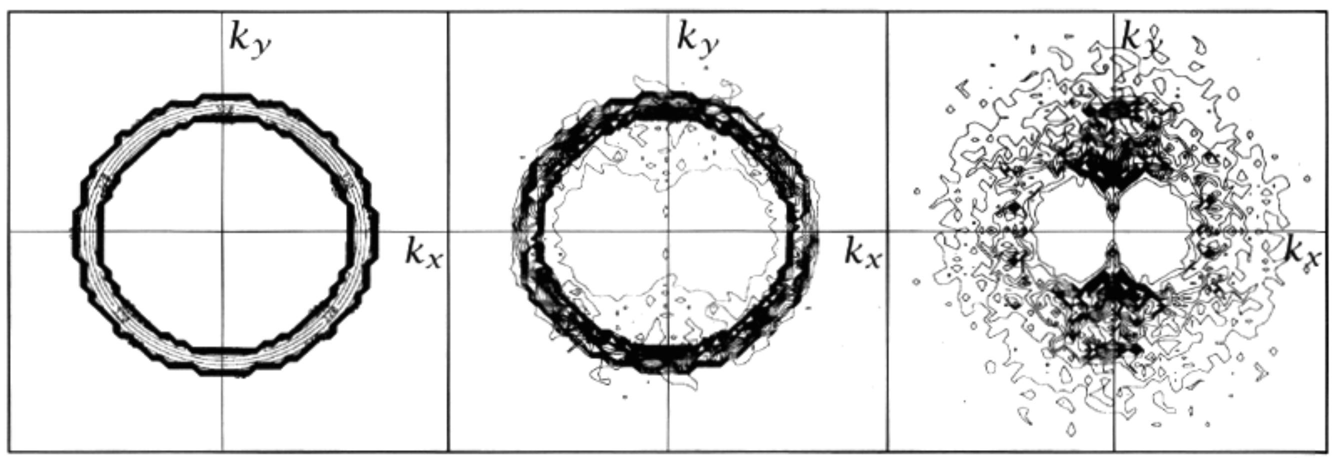
$$L_R \sim \sqrt{U/\beta}$$

- large scales, β dominates
  - waves inhibit cascades

- anisotropy of Rossby waves
  - advection freq *Uk* ~ Rossby wave freq ω

$$k_x = k_R \cos^{3/2}\theta, \quad k_y = k_R \sin\theta \cos^{1/2}\theta$$

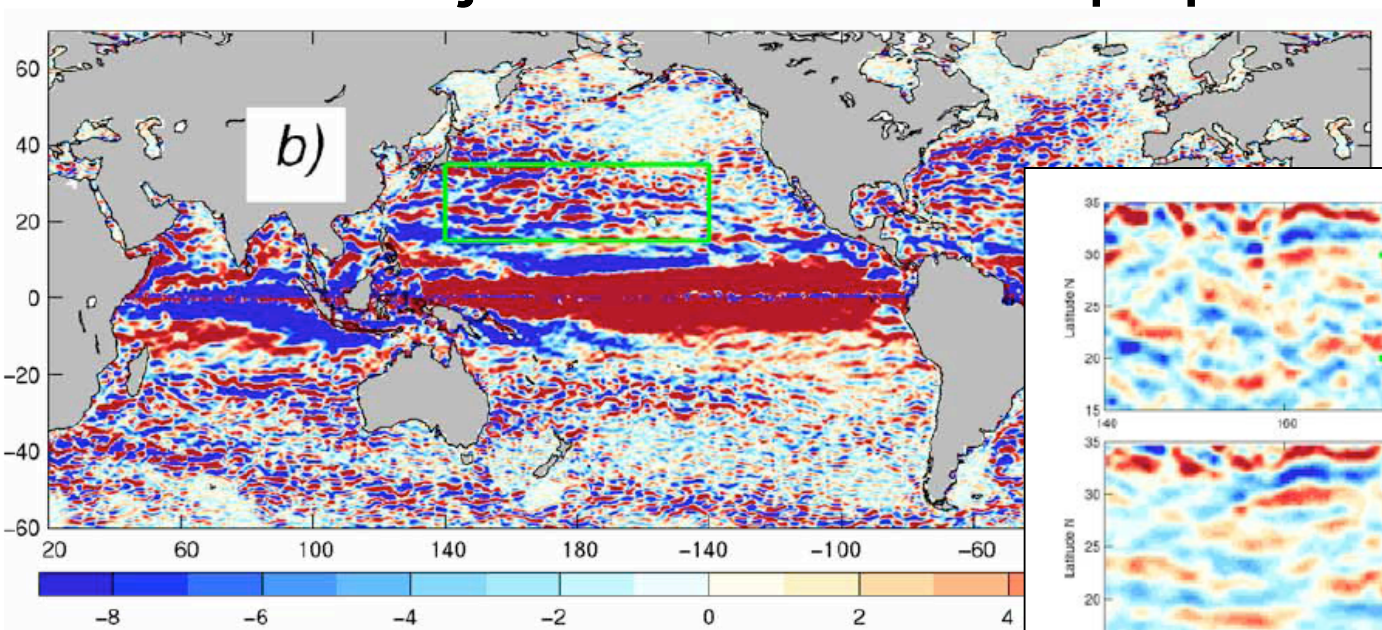


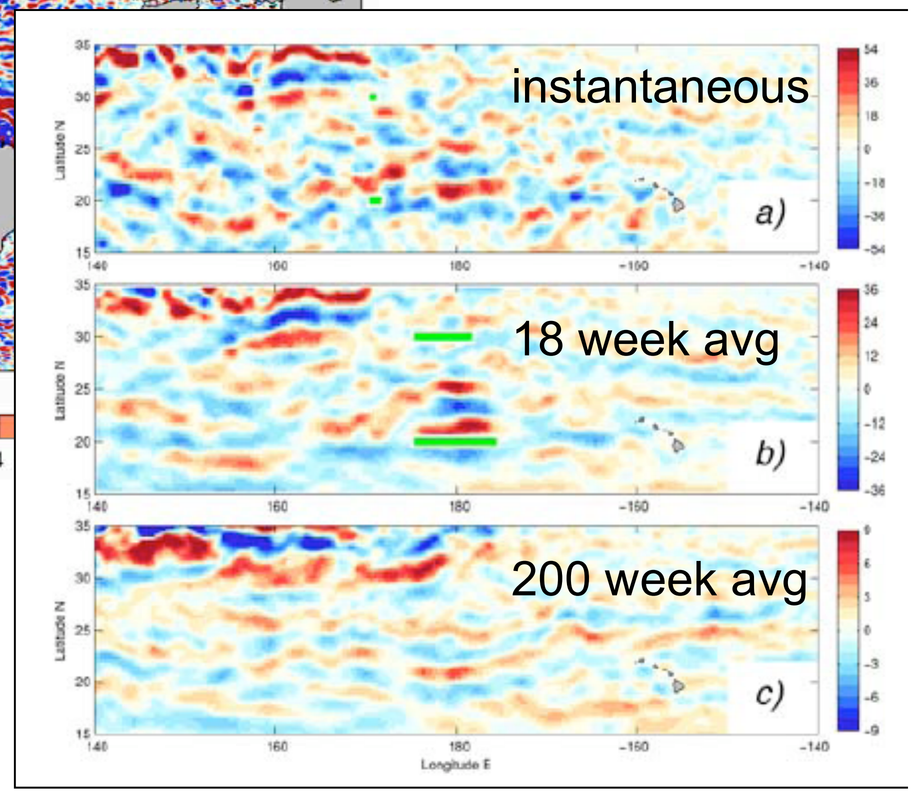


Vallis, Atmospheric and Oceanic Fluid Dynamics, 2006

ocean jets

- zonal jets observed to populate ocean





Maximenko, et al 2005