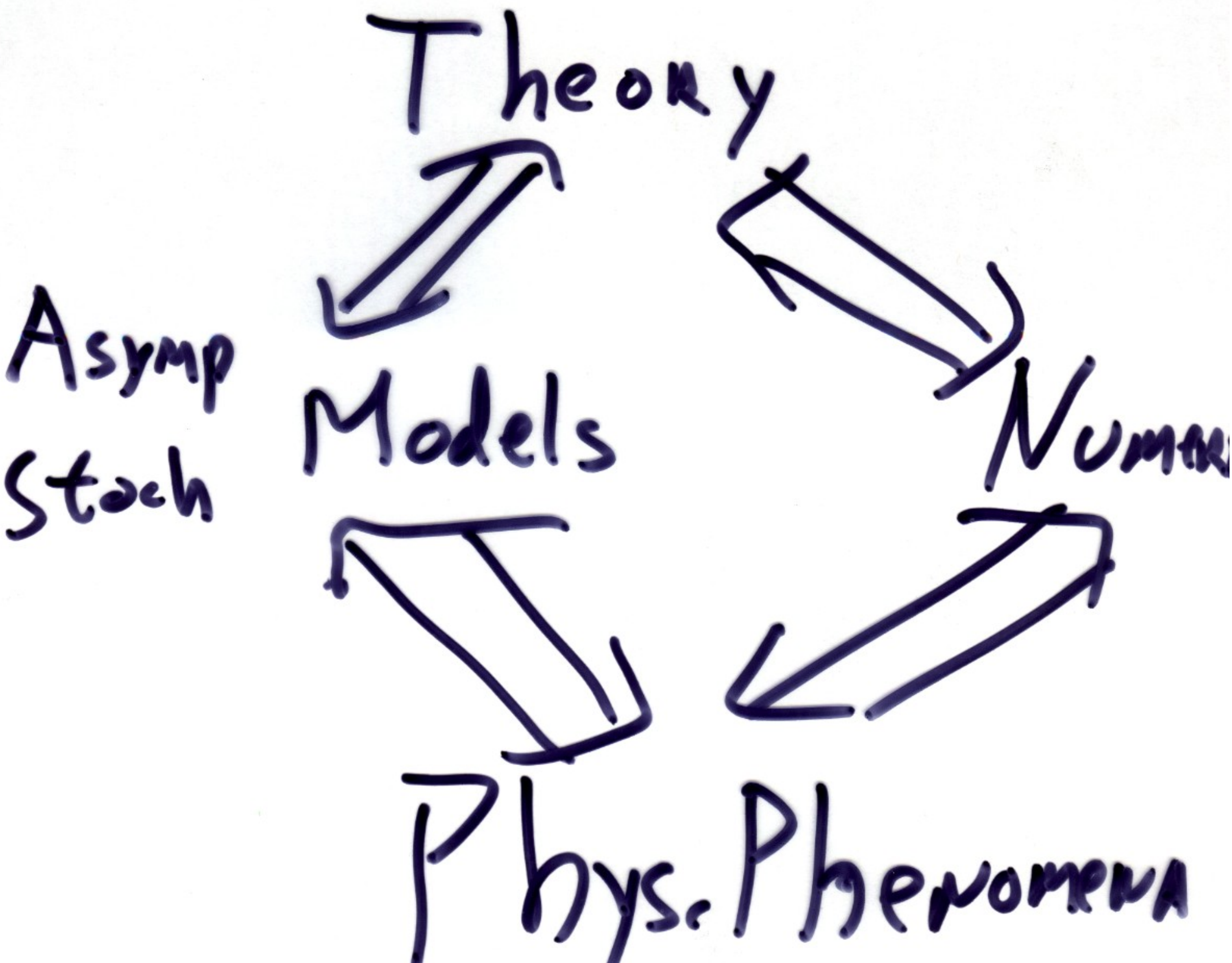
Introductory Lecture for Class

From "Waves, PDE's & Coarse-Grained Stochastic Models for Tropics"

Modern Appl. Math.



Applied and Theoretical Challenges for Multi-scale Hyperbolic PDE's in the Tropics

Andrew J. Majda
Morse Professor of Arts and Sciences
Department of Mathematics and
Climate, Atmosphere, Ocean Science
(CAOS)
Courant Institute of Mathematical
Sciences
New York University

Hierarchy of Organized Deep Convection over the Tropical Western Pacific

Mesoscale

Precipitating Clouds	1-10 km	<1hr
Cloud Ensemble	10-100 km	>hrs
Mesoscale Cloud Systems	500 km	hrs-days
"Super" Cloud Clusters	1000-2000km	days-week

Large Scale

Tropical Waves (Kelvin, Rossby-gravity)	Synoptic	week
Mean Circulations (Hadley, Walker)	Planetary	interseasonal
ENSO	Planetary	interannual

Hierarchy à la Nakazawa

in the tropical intraseasonal variability

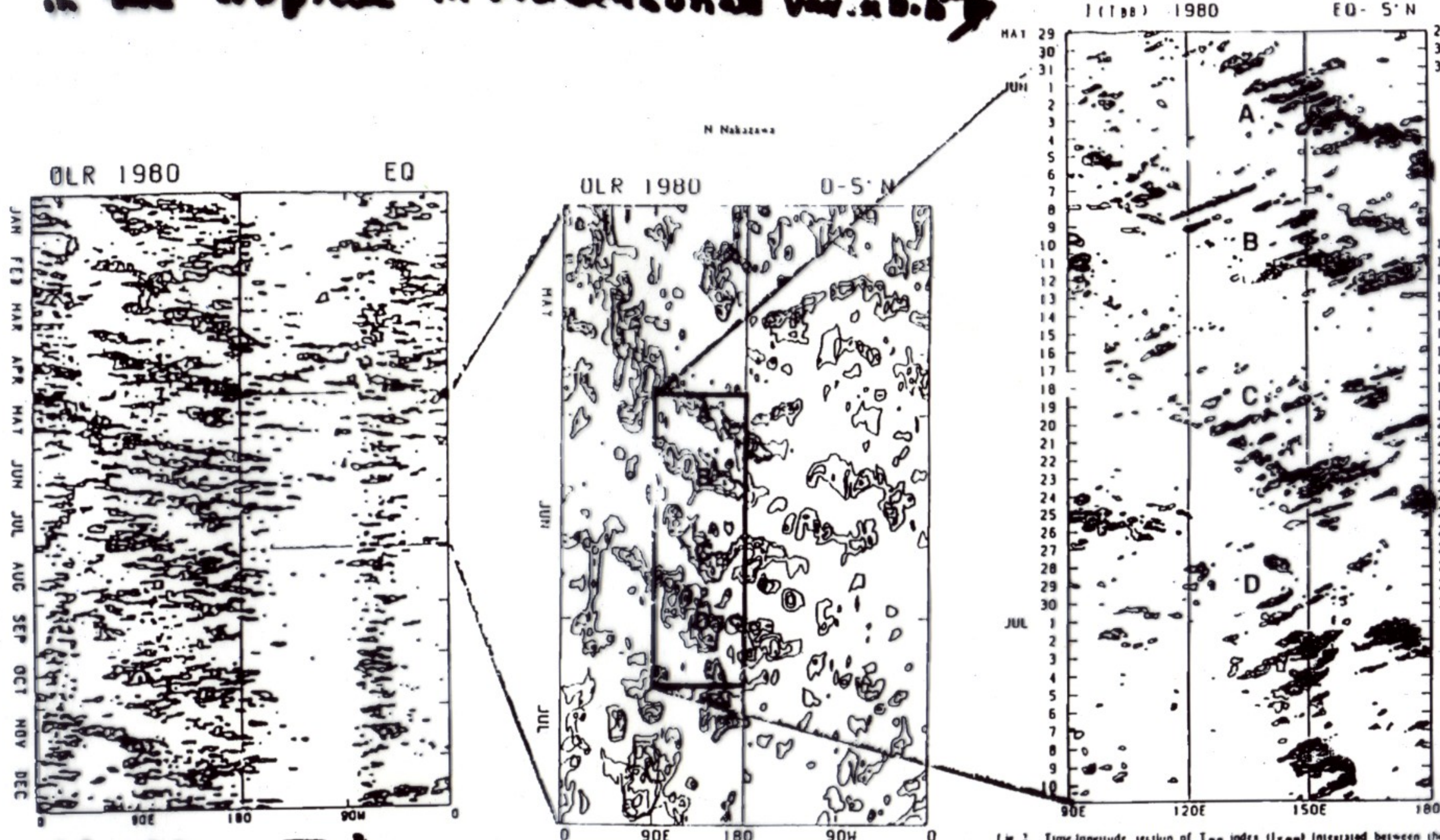


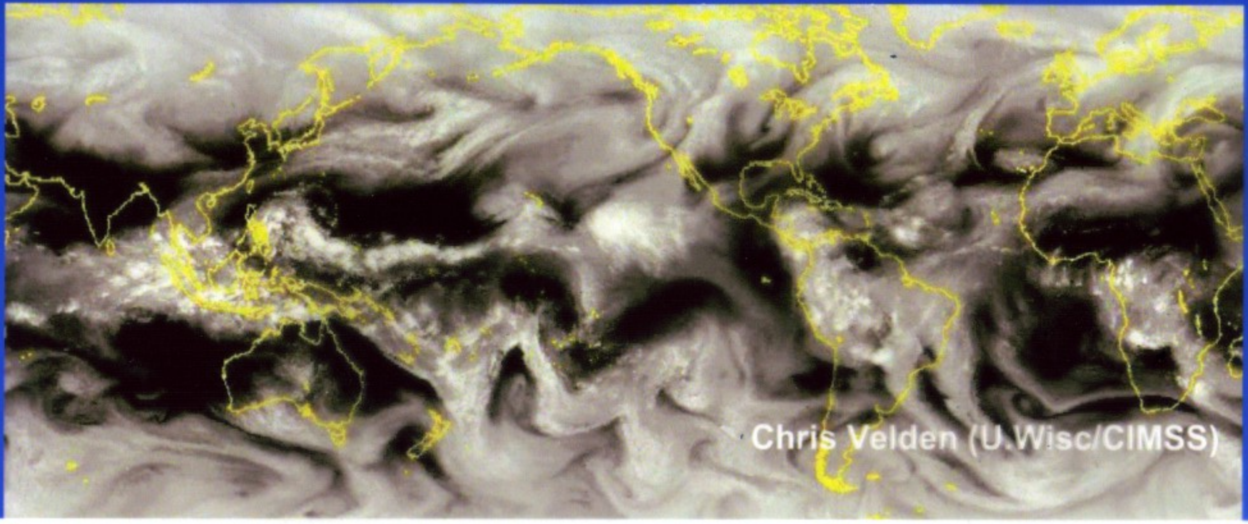
Fig 1. Time-longitude section of transient (seasonal trend removed) OLR averaged between the equator and 5°N from May to July in 1980. Negative (active convective) regions are contoured. Contour interval decrements of 30 Wm^{-2} starting at $-15\ Wm^{-2}$. Symbols A to D indicate super clusters

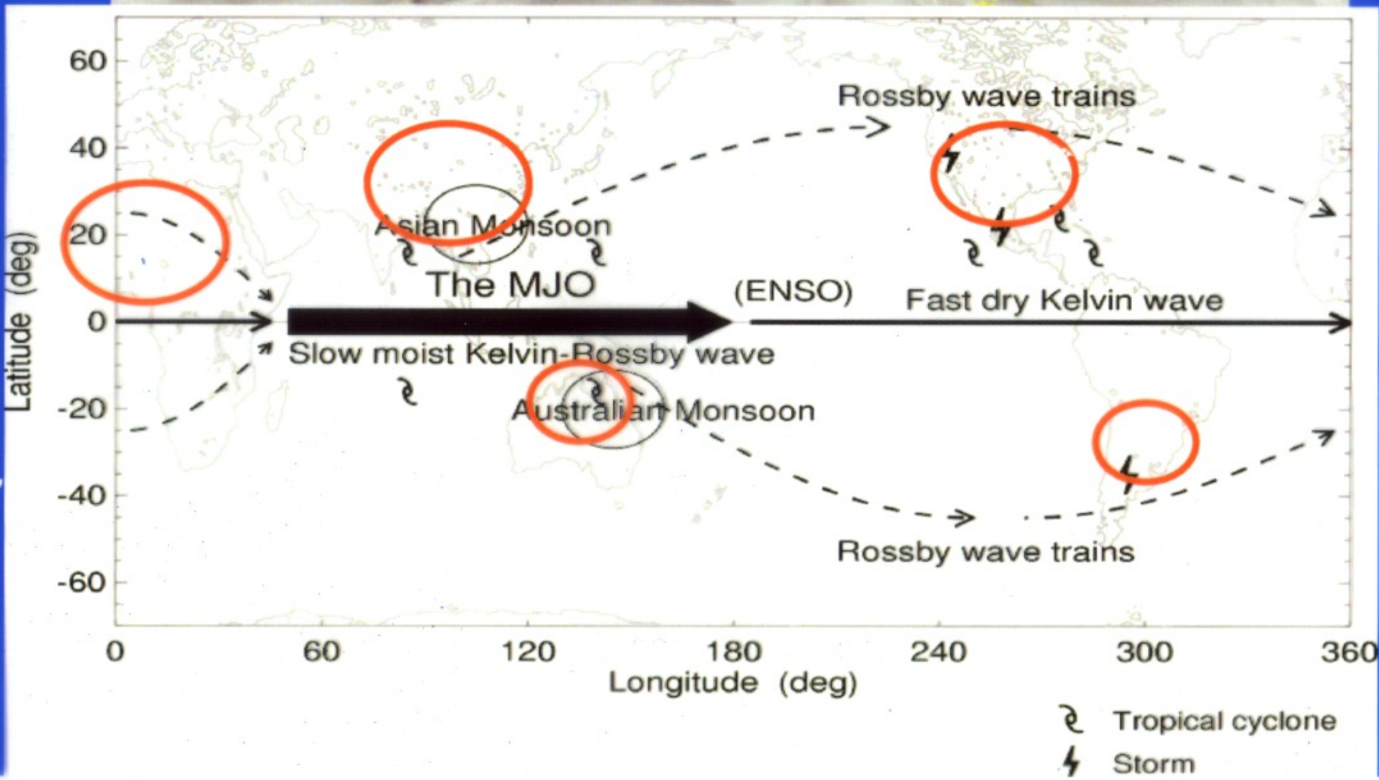
Fig 2. Time-longitude section of T_{BB} index (I_{TBB}) integrated between the equator and 5°N obtained from the 3-hourly GMS IR data from 29 May 00Z to 10 July 21Z, 1980. Symbols A to D denote the same super cluster as in Fig 1. Contour interval is 10, and shading denotes the region where values are greater than 20

Madden-Julian waves	Super-Clusters	Cloud Clusters
(Eastward)	(Eastward)	(Westward)
40-60 days	2000 km, 10 days	10^2 km, 1 day
10^4 km		

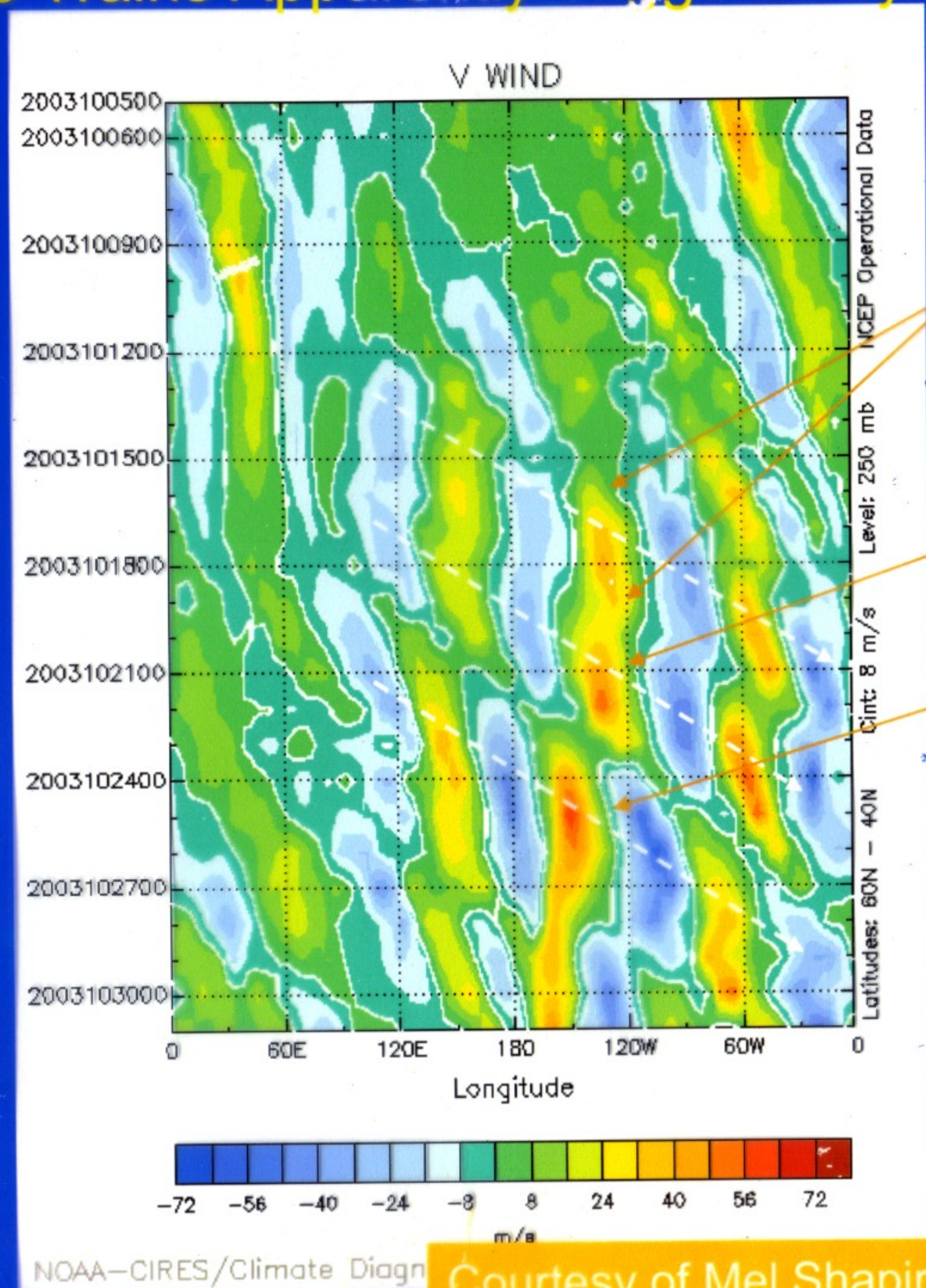
Tropical – Extratropical Interaction and weather

Water Vapor Channel





Rossby Wave Trains Apparently Triggered by Tropical Convection



BC's flood of the Century (18.5")

Western WA Flood (Seattle 1-day record)

CA Wild Fires (downslope winds)

Courtesy of Mel Shapiro and David Parsons

I. Equatorial Shallow Water with Topography

$$\begin{aligned} \eta_t + [(1+\eta-h)u]_x + [(1+\eta-h)v]_y &= 0 \\ u_t + uu_x + vu_y + \eta_x - yv &= 0 \\ v_t + uv_x + vv_y + \eta_y + yu &= 0. \end{aligned}$$

Length and Time Scales

$$L = (\frac{c}{\beta})^{\frac{1}{2}}, \quad T = \frac{L}{c}, \quad c = \sqrt{gH},$$

H equivalent height

I. Linear Equatorial Waves

Kelvin Wave

$$\begin{aligned}\eta &= K(x-t)e^{-\frac{y^2}{2}} \\ u &= K(x-t)e^{-\frac{y^2}{2}} \\ v &= 0,\end{aligned}$$

K an arbitrary function

Dispersive Equatorial Waves

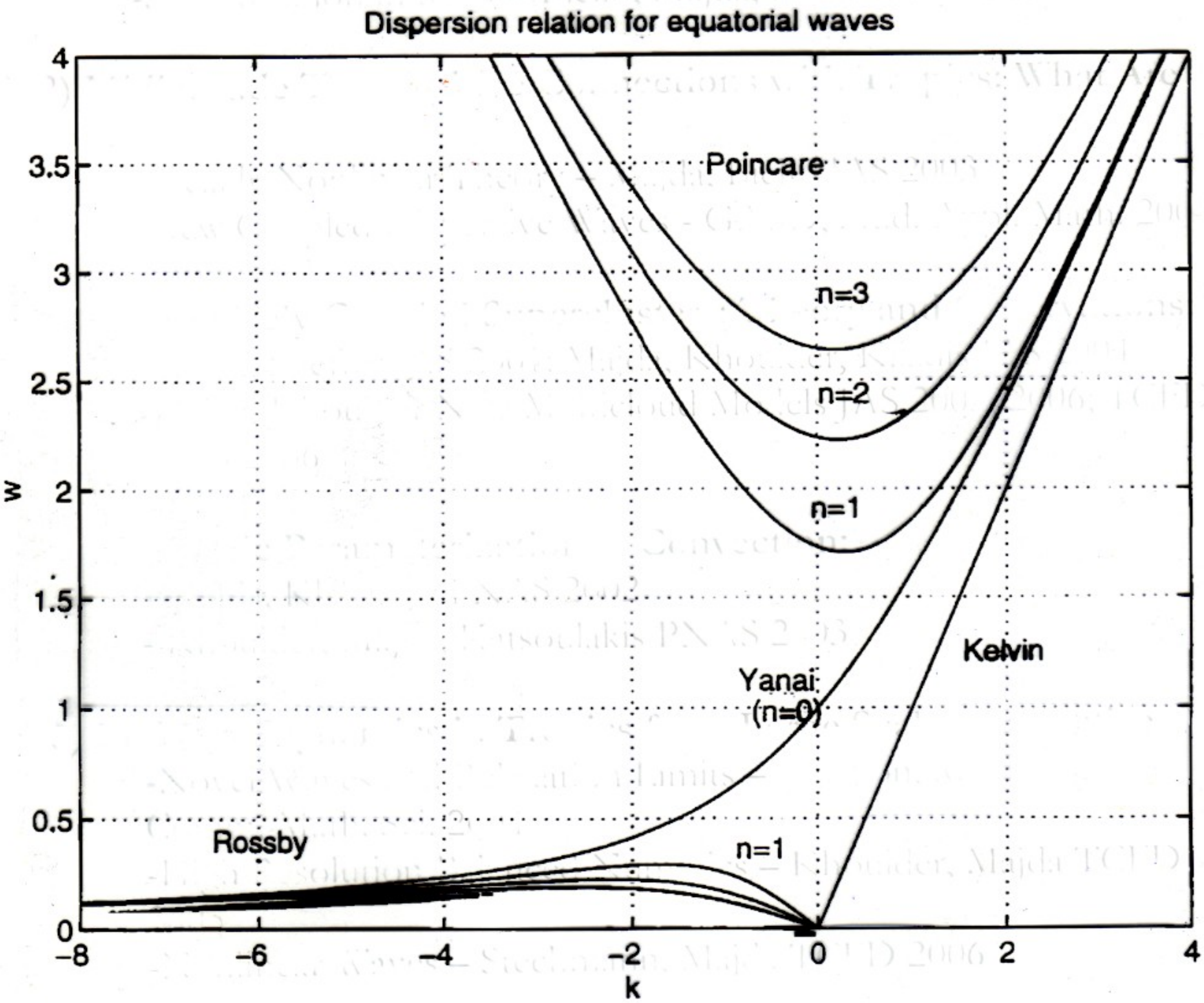
$$\begin{aligned}\eta &= \left[\frac{y}{k-\omega}H_n(y) + \frac{\omega}{\omega^2-k^2}H_n'(y)\right]e^{i(kx-\omega t)}e^{-\frac{y^2}{2}} \\ u &= \left[\frac{y}{k-\omega}H_n(y) + \frac{k}{\omega^2-k^2}H_n'(y)\right]e^{i(kx-\omega t)}e^{-\frac{y^2}{2}} \\ v &= iH_n(y)e^{i(kx-\omega t)}e^{-\frac{y^2}{2}},\end{aligned}$$

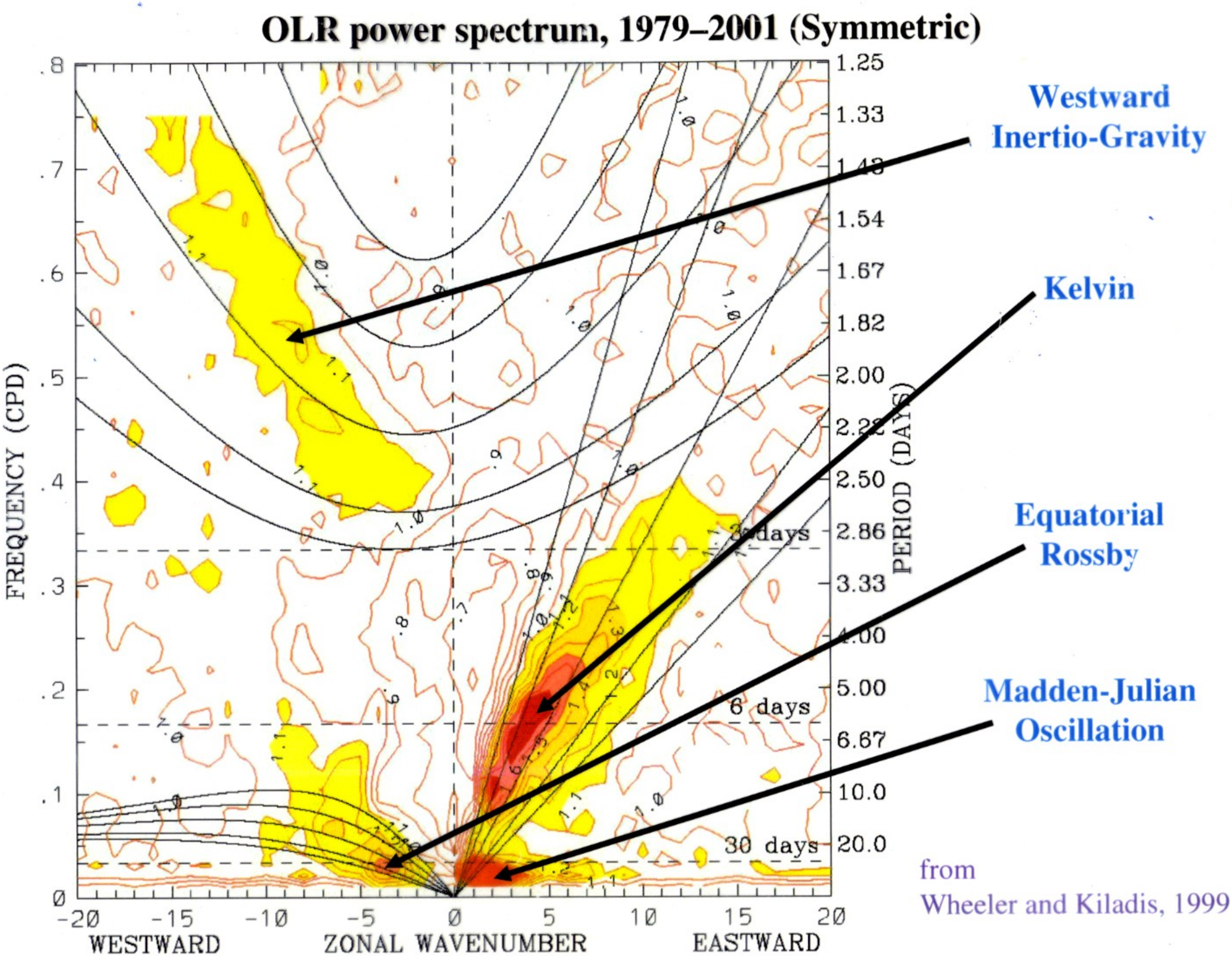
$H_n(y)$ Hermite polynomial of order n

$\omega = W(k)$ satisfies the dispersion relation

$$-2n + \frac{(\omega+k)\left(\omega^2 - k\omega - 1\right)}{\omega} = 0.$$

Dispersion Relation for Equatorial Waves



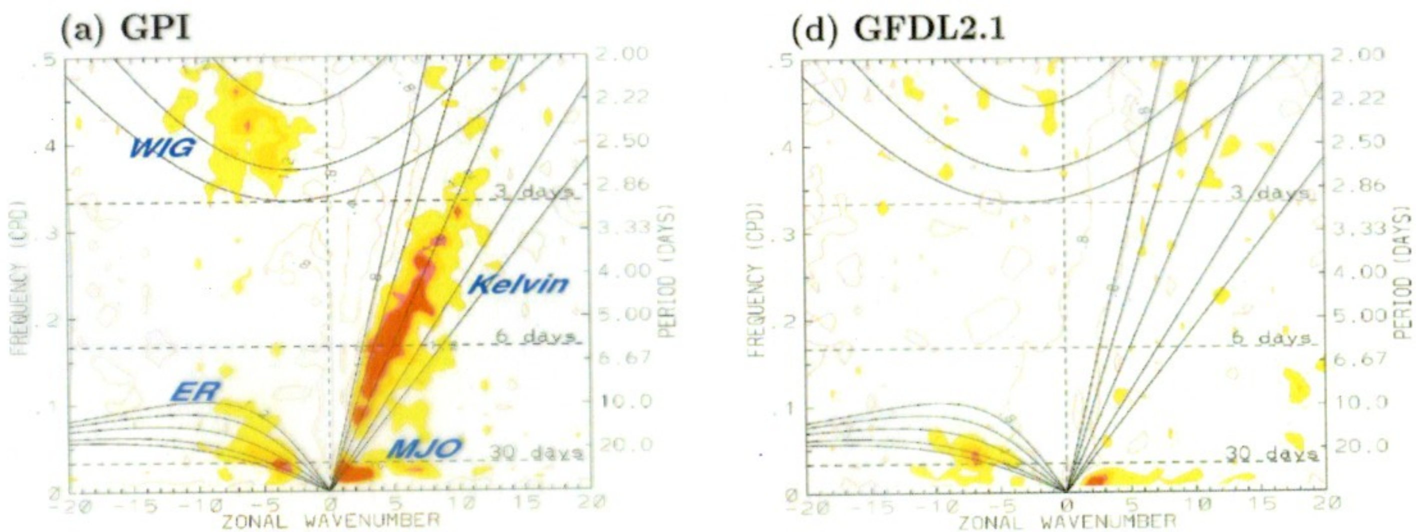


OLR power spectrum, 1979–2001 (Symmetric)
FREQUENCY (CPD)
PERIOD (DAYS)
WESTWARD
ZONAL WAVENUMBER
EASTWARD
3 days
6 days
30 days
Westward Inertio-Gravity
Kelvin
Equatorial Rossby
Madden-Julian Oscillation
from Wheeler and Kiladis, 1999

From Lin et al. (2006)

Left: Observations

Right: GCM



Basic References

A. J. Majda, Introduction to PDEs and Waves for the Atmosphere and Ocean, (Chapter 9) 2005, Courant Lecture Series #8 Publ. Amer. Math. Soc.

Special Issues

#1) Theoretical Developments in Tropical Meteorology
Vol. 20, #5-6, November 2006
Theoretical & Computational Fluid Dynamics

#2) Atmospheric Convection and Wave Interactions: Convective Life Cycles and Scale Interactions in Tropical Waves
Dynamics of Atmospheres & Oceans
Vol. 42 2006

Current Research Directions: Applied Math and Tropical Dynamics

1) Hierarchical Multi-Scale Modeling:

-Majda, Klein JAS 2003; Moisture, Majda JAS 2006 A, B; Klein, Majda TCFD 2006

-MJO Theory - Biello, Majda PNAS 2004, JAS 2005; DAO, TCFD 2006

-Superrotation in MJO – Biello, Majda, Moncrieff JAS 2006

2) Midlatitude Troposphere Connections with Tropics: What Are Routes?

-Weakly Nonlinear Theory – Majda, Biello JAS 2003

-New Coupled Dispersive Waves - GAFD, Stud. Appl. Math. 2004

3) Convectively Coupled Superclusters: Theory and Observations:

-Majda, Shefter JAS 2001; Majda, Khouider, Kiladis JAS 2004

-Majda, Khouider New Multicloud Models JAS 2005, 2006; TCFD, DAO 2006

4) Stochastic Parameterization of Convection:

- Majda, Khouider PNAS 2002
- Khouider, Majda, Katsoulakis PNAS 2003

5) Moisture Dynamics in Tropics from Large Scale Perspective

-Novel Waves and Relaxation Limits – Frierson, Majda, Pauluis Comm. Math. Sci. 2004

-High Resolution Balanced Numerics – Khouider, Majda TCFD 2005

-Nonlinear Waves – Stechmann, Majda TCFD 2006

6) Novel Singular Limits with Fast Variable Coefficients

-Fast Wave Averaging for the Equatorial Shallow Water Equations - Dutrifoy, Majda Comm. PDE 2006

-The Dynamics of Equatorial Long Waves; A singular Limit with Fast Variable Coefficients - Dutrifoy, Majda Comm. Math. Sci. 2006

http://www.math.nyu.edu/faculty/majda/

Self-similarity in tropical convection

Squall lines

Zipser 1969

Zipser et al 1981

Two-day waves

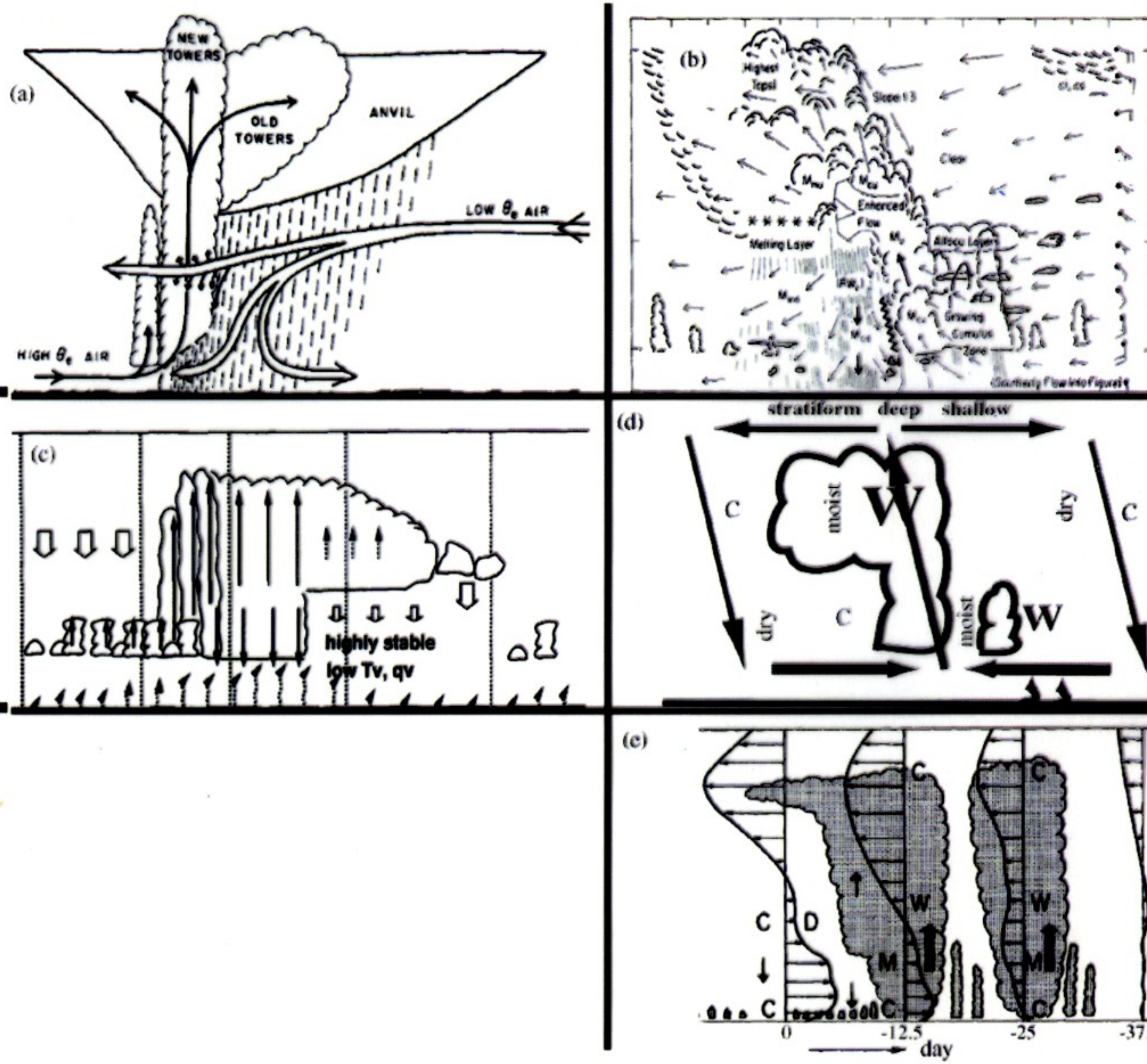
Takayabu et al 1996

C. C. Kelvin waves

Straub and Kiladis 2003

Madden–Julian Osc.

Lin and Johnson 1996



Source: Mapes et al. 2006 DAO

New Multiscale Models
and
Self-Similarity in Tropical Convection

Majda 2007, *J. Atmos. Sci.*

Multiscale Models with Moisture
and
Systematic Strategies for Superparameterization

Majda 2007, *J. Atmos. Sci.* (in press)

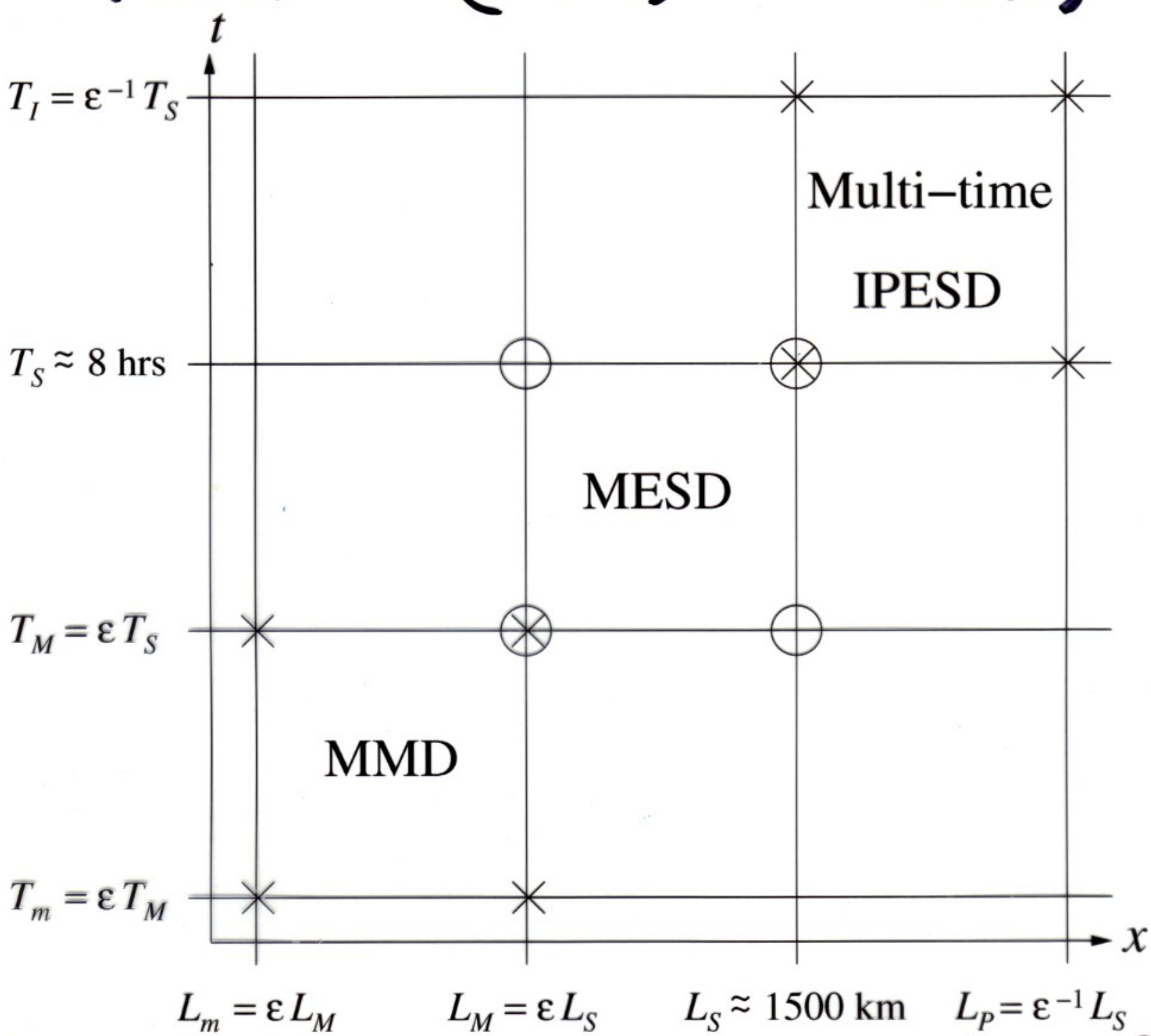
Assumptions:

1. Low Froude Number, $\mathbf{u}_h = \epsilon \mathbf{u}_{h,1}$
2. Weak Temperature Gradient, $\theta = \epsilon \theta_1, p = \epsilon p_1$

Hierarchical Multi-Scale Models
for Theory, Observations, and Numerical Strategies

Majda & Klein (2003); Klein & Majda (2006); Majda (2006a,b)

(JAS) (TCFD) (JAS)



IPESD – MJO – Appl.: Biello, Majda PNAS04

IPESD – Intraseasonal Planetary Equatorial Synoptic Scale Dynamics JAS05

MESD – Mesoscale Equatorial Synoptic Dynamics DAO06

MMD – Microscale Mesoscale Dynamics (B, M, Moncrieff) ↔ JAS07

(superrotation)

The Interlocking

IPESD Multi-scale
MESD Models

derived under Two universal Assumptions:

#1) Low Froude #

$$\varepsilon = \frac{|u|}{|c_g|}, \quad c_g = 50\,ms^{-1}$$

#2) Weak Temp. Gradient

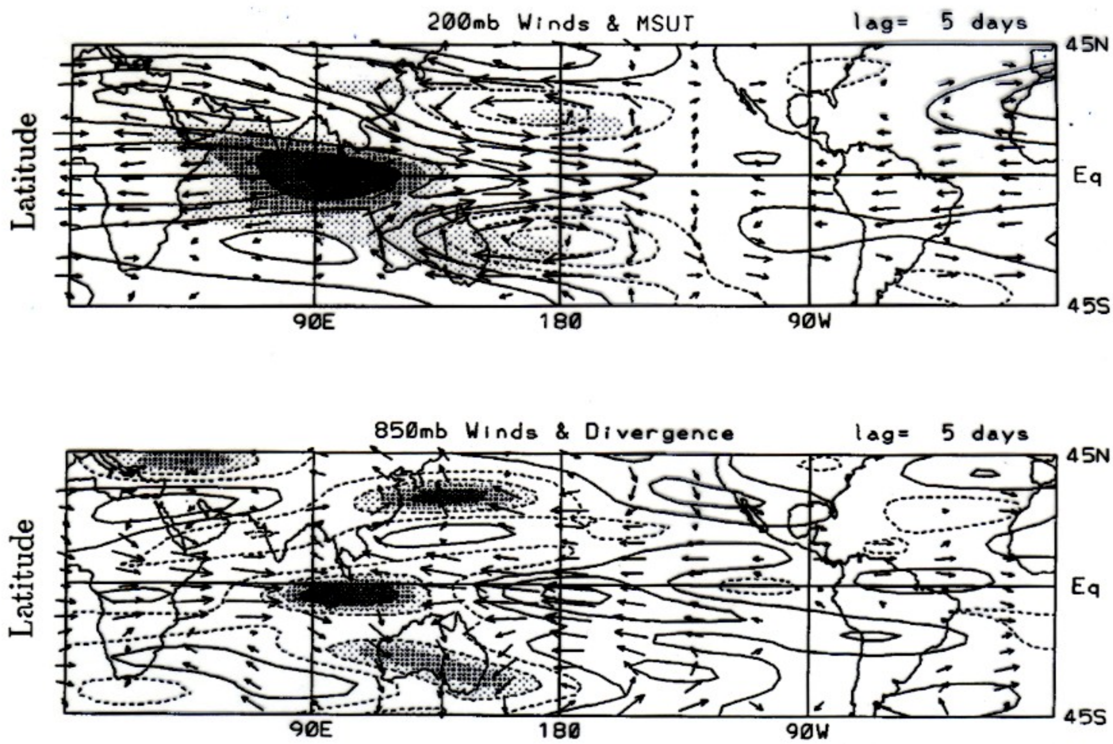
$$\Theta = \bar{\Theta} + \varepsilon\,\Theta_1$$

$$[\Theta_1] \cong 3°K$$

See M, JAS 2003

#1), #2) Obs. consistent!!

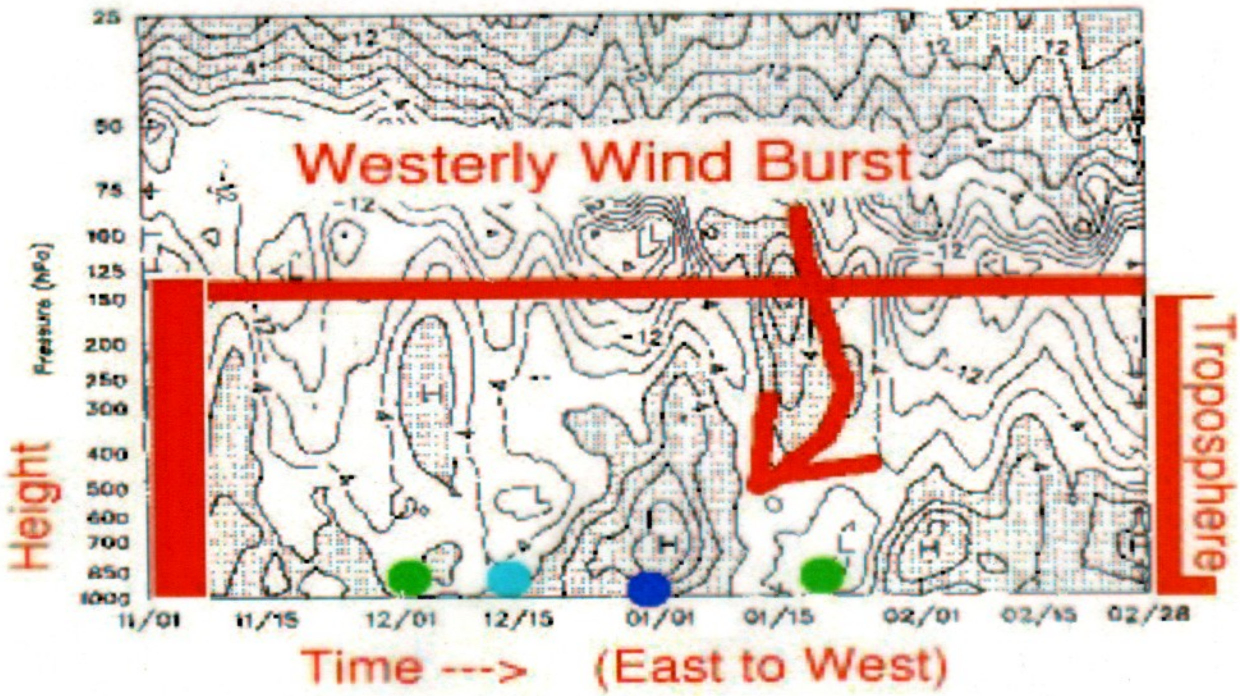
MJO: Large scale wind pattern.



- Top: 200 mb winds and precipitation.
- Bottom: 850 mb winds and divergence.
- Filtered at large scales.

From Hendon & Salby *J. Atmos. Sci.*, **51**, p 2230, fig. 3.

MJO: Vertical Shear



Lin & Johnson *J. Atmos. Sci.*, **53**, p 701, fig. 3 (a).

- Westerly wind burst zonal/vertical profile over a fixed position near the equator.
- Time goes from left to right and can be interpreted as left = east, right = west.
- Background easterlies.
- Westerly onset region.
- Strong westerly region.

MJO: Central Issue

Which aspects of the planetary scale dynamics are due to

1. Planetary scale mean heating as in traditional Matsuno-Gill models?

2. Upscale transport of potential temperature from synoptic to planetary scales through eddy fluxes.

3. Upscale transport of zonal momentum from synoptic to planetary scales through eddy fluxes.

IPESD framework can isolate these causes and their effects on planetary scale organized flows.

The IPESD Multiscale Model for the MJO

Joseph A. Biello - University of California, Davis
Andrew Majda - Courant Institute, New York University

- FRAMEWORK: IPESD multiscale models (Majda/Klein 2003)
- STRUCTURE: MJO structure given by specified heating profiles (PNAS 2004, JAS 2005, DAO 2006, BM+ Moncrieff JAS 2007)
  - planetary scale direct heating
  - upscale fluxes of momentum and heat from synoptic scales
- DYNAMICS: Khouider/Majda multi-cloud model. (Khouider/Majda JAS 2005, 2006, 2007)
  - active moisture through cloud model
  - nonlinear feedback from planetary to synoptic scales
  - organized embedded structures in a traveling envelope (Majda, Stechmann, Khouider PNAS)

FRAMEWORK: IPESD Theory (Majda/Klein 2003)

Synoptic Scale (Balanced) Dynamics:

$$
\begin{aligned}
u'_\tau - y\,v' + p'_x &= S'_u \\
v'_\tau + y\,u' + p'_y &= S'_v \\
\theta'_\tau + w' &= S'_\theta \\
p'_z &= \theta' \\
u'_x + v'_y + w'_z &= 0 \\
\overline{S'_\theta} &= 0
\end{aligned}
$$

Planetary Scale Quasi-Linear Dynamics:

$$
\begin{aligned}
\overline{U}_t - y\overline{V} + \overline{P}_X &= F^U \quad -d_0\,\overline{U} \\
y\overline{U} + \overline{P}_y &= 0 \\
\overline{\Theta}_t + \overline{W} &= F^\theta \quad -d_\theta\,\overline{\Theta} + \overline{S}_\theta \\
\overline{P}_z &= \overline{\Theta} \\
\overline{U}_X + \overline{V}_y + \overline{W}_z &= 0
\end{aligned}
$$

The fluxes from the synoptic scales are given by

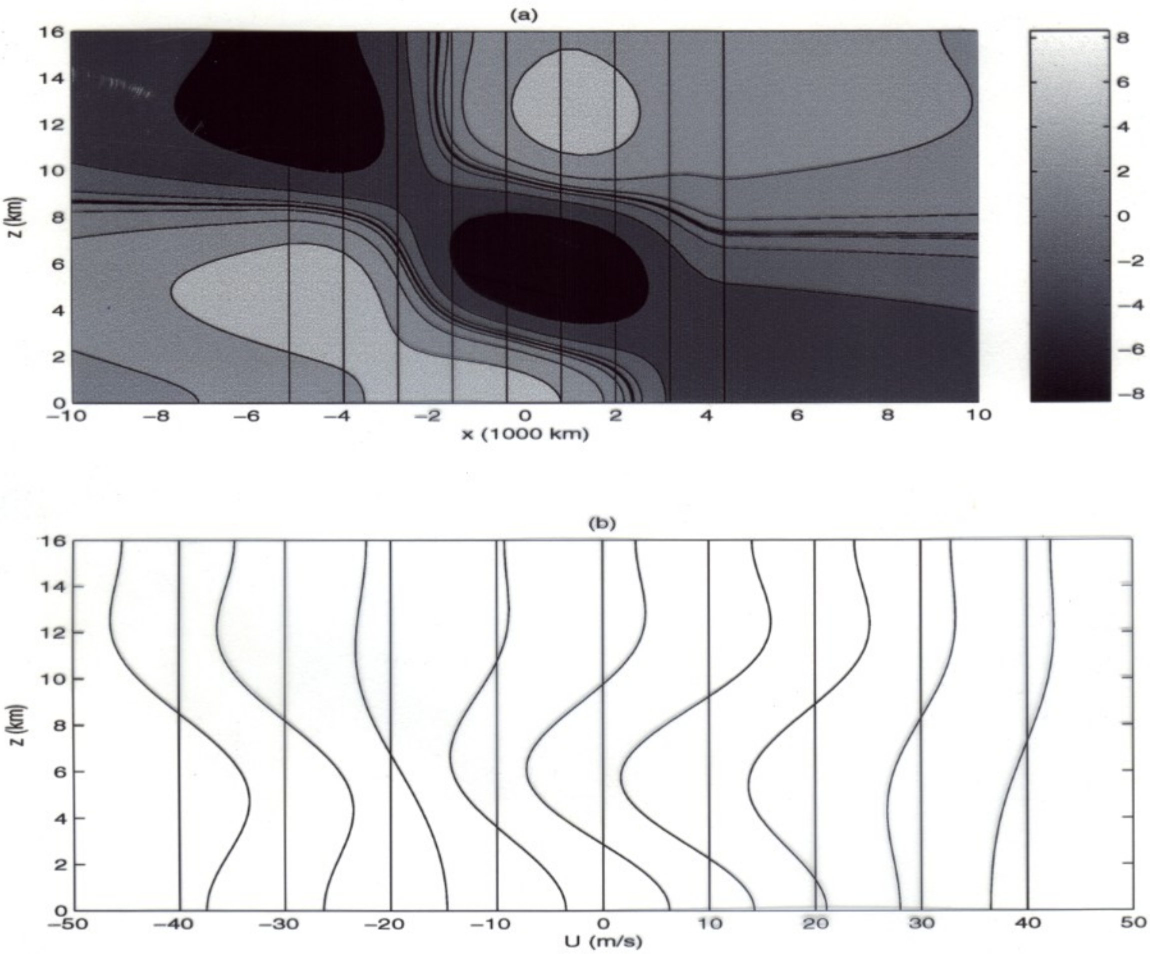
$$
\begin{aligned}
F^U &= -\overline{(v'\,u')_y} - \overline{(w'\,u')_z} \\
F^\theta &= -\overline{(v'\,\theta')_y} - \overline{(w'\,\theta')_z}
\end{aligned}
$$

Each forcing effect, i.e. upscale vertical and meridional momentum and temperature transport and planetary scale mean heating can be considered separately and superposed

Equatorial MJO model:

- Congestus heating in the east and westward tilted superclusters in the west of a moving warm pool.

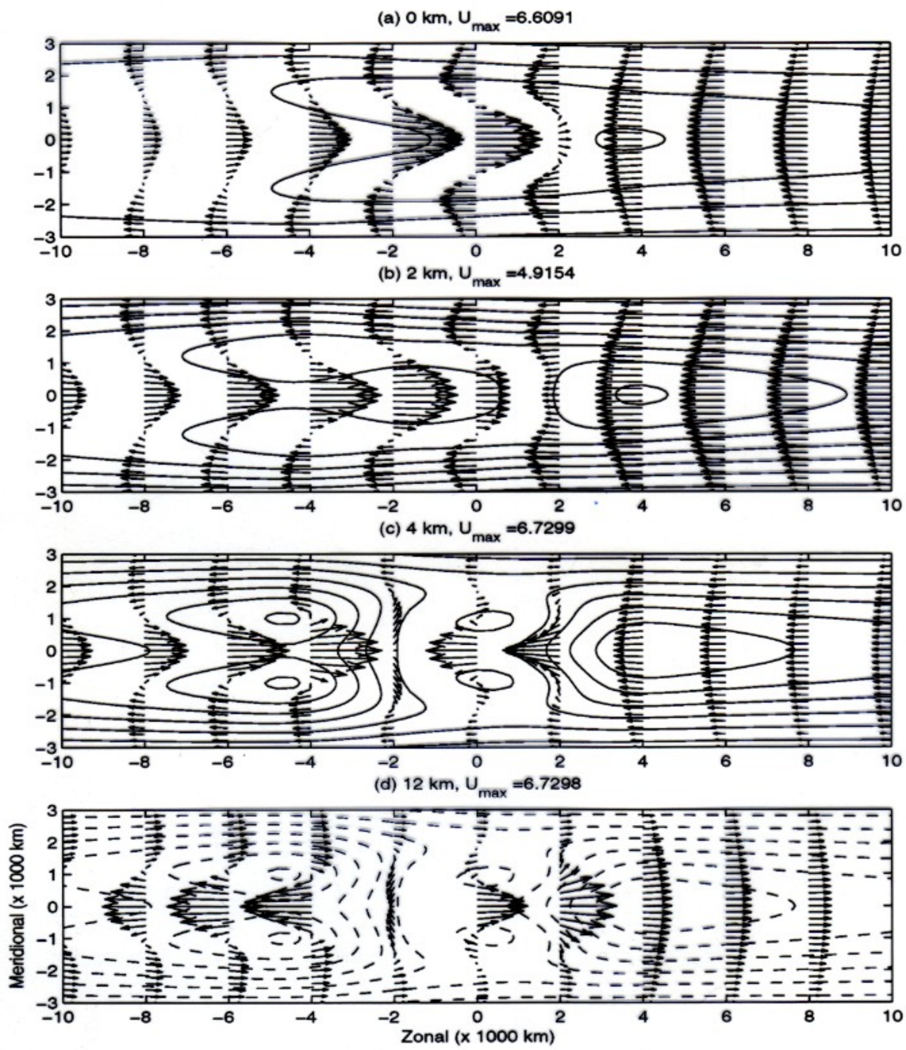
- Planetary mean heating is weaker, but has same structure of synoptic scale fluctuations.



- (a) Zonal velocity (westerly = light, easterly = dark) as a function of height and longitude above equator and (b) as a function of height above the cuts from (a).

Equatorial MJO model:

- Congestus heating in the east and westward tilted superclusters in the west of a moving warm pool.
- Planetary mean heating is weaker, but has same structure of synoptic scale fluctuations.



- Pressure and flow at $z = 0, 2, 4, 12$ km.

Summary: Which aspects of the MJO arise from upscale transport versus direct heating?

Lower/Upper Troposphere Direct Heating Alone

- Easterlies Leading Westerlies
- Maximum westerlies at base of troposphere

Addition of Congestus/Supercluster Upscale Transport

- Quadrupole structure
- Upward/Westward tilt of westerly wind burst
- Maximum westerlies at 4-6 km height
- Intensification of midlevel easterly jet leading the westerly onset region
- Cyclone pair trailing convective activity
- Upper troposphere outflow from convective region