Vortical Hot Towers

Hot towers :

intense deep convection cores with small horizontal scales (of order 10 km) and short convective lifetimes (of order 1 hour).

Next:

• Build elementary models which exhibit basic characteristics of hot towers to study the evolution of radial eddies (which represent "vortical hot towers") in various radial preconditionings.

• How heat (mass) sources can generate vortices?

• Explore the role of heat sources in cyclogenesis through a reduced form of the asymptotic system (1.3).

Although the terminology of hot towers (cloud scales) is used here, the model is also good for larger (meso) scale systems e.g. mesovortices under synoptic-scale preconditionings.

System in Axisymmetric Case

$$\mathbf{u} = u^r \mathbf{e}_r + u^\theta \mathbf{e}_\theta + w\mathbf{e}_z,$$

The system (1.3) is reduced to

$$\begin{aligned} \frac{\partial u^\theta}{\partial t} + u^r \frac{\partial u^\theta}{\partial r} + u^r \frac{u^\theta}{r} + w \frac{\partial u^\theta}{\partial z} + f u^r &= 0, && (4.2a) \\ \frac{\partial (r u^r)}{\partial r} + \frac{\partial (r w)}{\partial z} &= 0, && (4.2b) \\ w N^2(z) &= S_\theta. && (4.2c) \end{aligned}$$

Hence the radial velocity u^r is directly specified by the heat source

$$u^r = -\frac{1}{r} \int_0^r s \frac{\partial w}{\partial z} ds.$$

Note: no vertical shear due to axisymmetry

System in Axisymmetric Case

We decompose the heat source into a large scale mean and small scale perturbation as

$$S_\theta(t,r,z) = \overline{S}_\theta(t,z) + S'_\theta(t,r,z)$$

Hence, the flow quantities are decomposed as

$$\begin{aligned} w(t,r,z) &= \overline{w}(t,z) + w'(t,r,z), \\ \omega(t,r,z) &= \overline{\omega}(t,z) + \omega'(t,r,z), \\ \overline{u}^\theta(t,r,z) &= \overline{u}^\theta(t,r,z) + \left(u^\theta\right)'(t,r,z), \\ \overline{u}^r(t,r,z) &= \overline{u}^r(t,r,z) + \left(u^r\right)'(t,r,z), \end{aligned}$$

where $\overline{u}^r$ and $\overline{u}^\theta$ are respectively obtained from (4.3) and (4.4)

$$\begin{aligned} \overline{u}^r(t,r,z) &= -\frac{1}{2}\frac{\partial \overline{w}(t,z)}{\partial z} r, \\ \overline{u}^\theta(t,r,z) &= \frac{1}{2}\overline{\omega}(t,z) r. \end{aligned}$$

System in Axisymmetric Case

The equation for the evolution of vorticity (4.6) is simplified for $\overline{\omega}(t,z)$ as

$$\frac{\partial\overline{\omega}(t,z)}{\partial t}+\overline{w}(t,z)\frac{\partial\overline{\omega}(t,z)}{\partial z}=\frac{\partial\overline{w}(t,z)}{\partial z}\left(\overline{\omega}(t,z)+f\right).$$

Preconditioned Large Scale Flows

The mean flow satisfies (4.5) by

$$\frac{\partial\overline{u}^{\theta}}{\partial t}+\overline{u}^{r}\overline{\omega}+\overline{w}\frac{\partial\overline{u}^{\theta}}{\partial z}+f\overline{u}^{r}=0.$$

Equation for the evolution of small scale perturbation $(u^{\theta})'$ in a large-scale preconditioning:

$$\frac{\partial\left(u^{\theta}\right)'}{\partial t}+\left(\overline{u}^{r}+\left(u^{r}\right)'\right)\left(\frac{\partial\left(u^{\theta}\right)'}{\partial r}+\frac{\left(u^{\theta}\right)'}{r}\right)+\left(\overline{w}+w'\right)\frac{\partial\left(u^{\theta}\right)'}{\partial z}$$

$$=\ -f\left(u^{r}\right)'-\left(u^{r}\right)'\overline{\omega}-w'\frac{\partial\overline{u}^{\theta}}{\partial z}.$$

stretching

vertical advection of mean flow by perturbation

Mean Flow

Physical problems of interest for the mean flow

$f \neq 0 \Leftrightarrow$ preconditioned Barotropic Mean Flow

- Barotropic mean vorticity

  $\overline{w} = 0$ and hence, $\overline{u}' = 0$

  $\overline{\omega} = \overline{\omega}_0(z)$

- Deep convective mean flow

  $$\overline{w} = A(t)\sin(\pi z), \quad 0 \leq z \leq 1$$

  $\frac{d\bar{\omega}}{dt} = (\bar{\omega} + f)\overline{w}_z$ in the characteristic coordinates.

  This yields *cyclones in the lower troposphere and anti-cyclones in the upper troposphere.*

- Stratiform mean flow

  $$\overline{w} = -A(t)\sin(2\pi z), \quad 0 \leq z \leq 1$$

  This leads to *a mid-level cyclone and high and low level anti cyclone generation.*

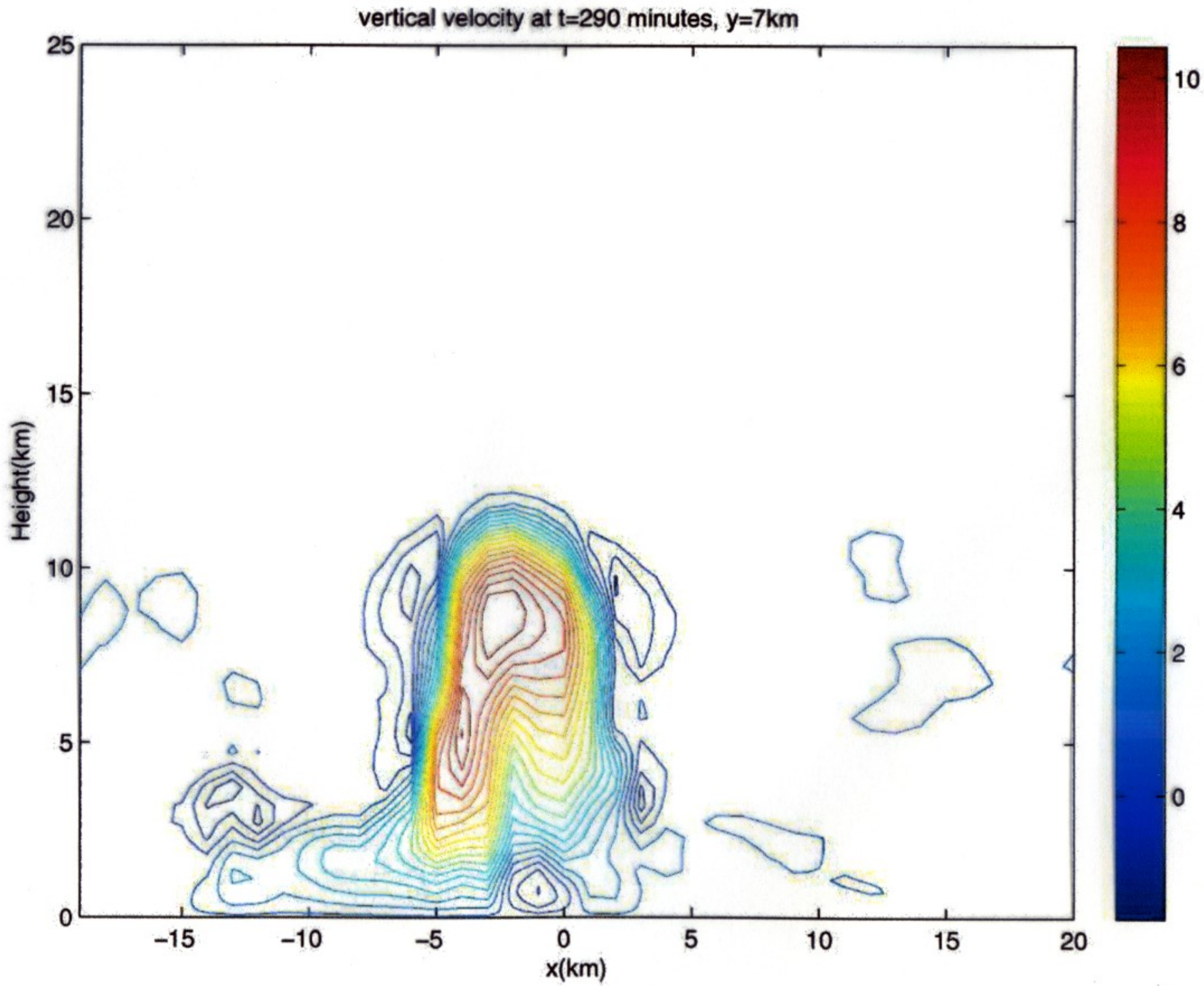
Elementary model for small-scale hot towers

Introduce a perturbation flow with a compact support which represents basic characteristics of hot towers to study the role of hot towers in the hurricane embryo.

Hot Towers exhibit the following basic features:

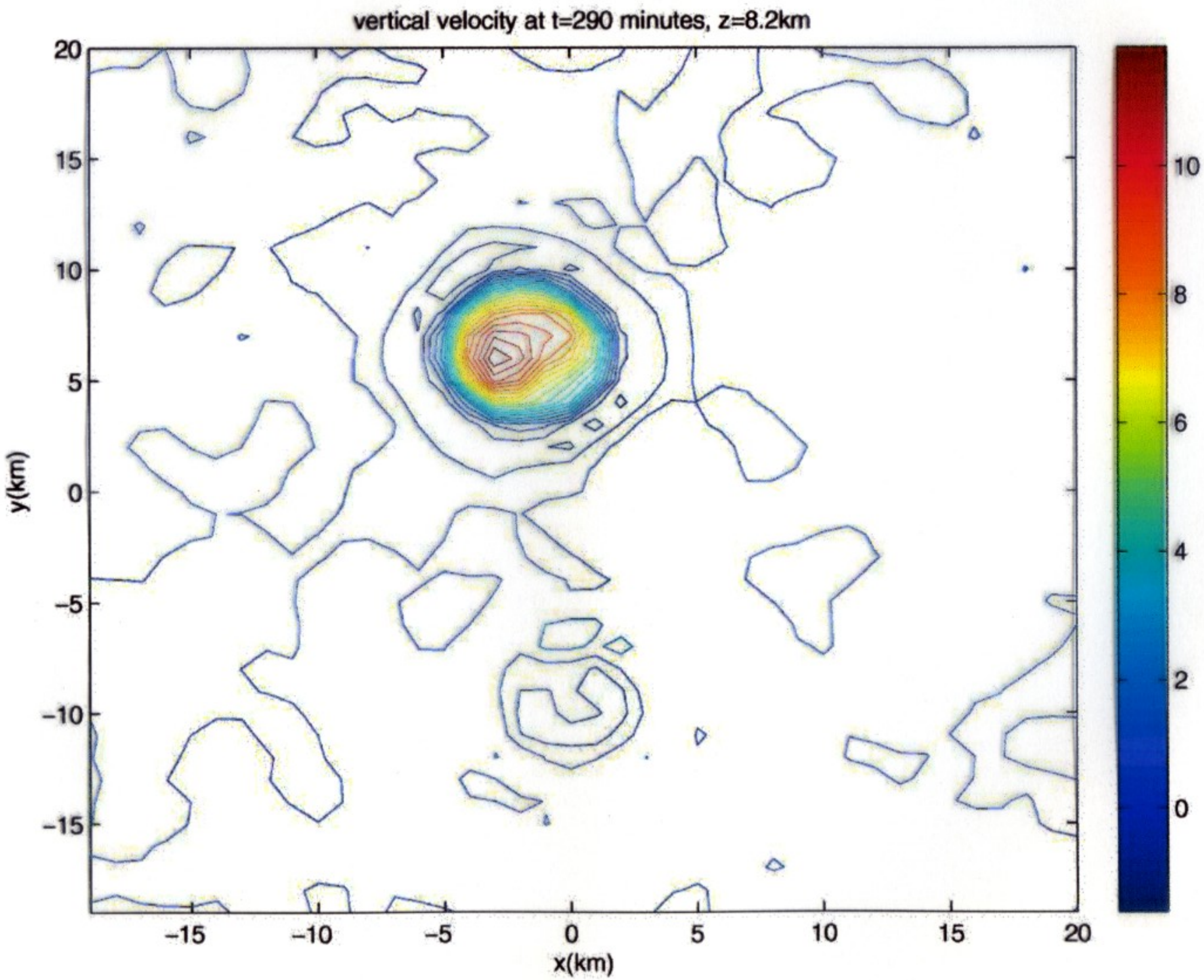
- they have a horizontally small-scale compact support ;
- their vertical structure resembles deep convective rising plumes;
- they consist of an intense updraft in their center and milder downdrafts around;
- they exhibit short convective lifetimes including generation, mature, and decaying stages.

Elementary model for small-scale hot towers



horizontal cross-section

Elementary model for small-scale hot towers



Vertical View from top

Elementary model for small-scale hot towers

Motivated by the above typical features of hot towers, we consider the following profile of vertical velocity as an elementary hot tower model

$$\widehat{w} = \begin{cases} z^4(z-1)^4[850r(r-1)^6 + \frac{255}{2}(r-1)^6 + (1700r(r-1)^6 \\ \quad +5100r(r-1)^5 + \frac{255}{2r}(r-1)^6 + 765(r-1)^5] & 0 \le r, z \le 1, \\ 0 & otherwise. \end{cases} \tag{4.28}$$

Using the continuity equation, we obtain

$$\widehat{u^r} = \begin{cases} [-4z^3(z-1)^4 - 4z^4(z-1)^3]\left(850r^2(r-1)^6 + \frac{255}{2}r(r-1)^6\right), & 0 \le r, z \le 1, \\ 0 & otherwise. \end{cases} \tag{4.29}$$

The life cycle of a hot tower may be modeled by the $\sin^+$ function

$$\sin^+(\theta) = \begin{cases} \sin(\theta), & if \ \sin(\theta) > 0, \\ 0 & otherwise. \end{cases}$$

, T_{max} – lifetime

Hence,

$$w' = \widehat{w}\sin^+(\pi t/T_{max}),$$

$$(u^r)' = \widehat{u^r}\sin^+(\pi t/T_{max}).$$

What's needed in general to make a hot tower of compact support

w = "Heat Source"

(1) Start with $w_+(r) \geq 0$, comp. supp.

& $\int_0^\infty r\, w_+(r)\, dr > 0$

(2) Let

$$w = A\left(\lambda_+^{-2} w_+\left(\frac{r}{\lambda_+}\right) - \lambda_-^{-2} w_+\left(\frac{r}{\lambda_-}\right)\right)$$

$$\lambda_+ >> \lambda_-$$

First term is updraft - stronger

Second term is downdraft

(3) The term in (2) is convective heating

Elementary model for small-scale hot towers

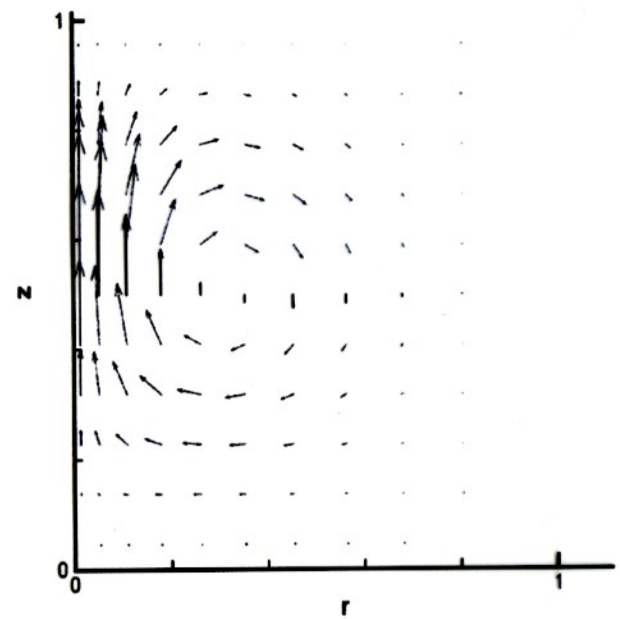


Figure 4.14: The perturbation flow field generated by the hot tower given by (4.28) and (4.29).

a deep convective rising plume, with an intense updraft in its center and a mild downdraft around.

Elementary model for small-scale hot towers

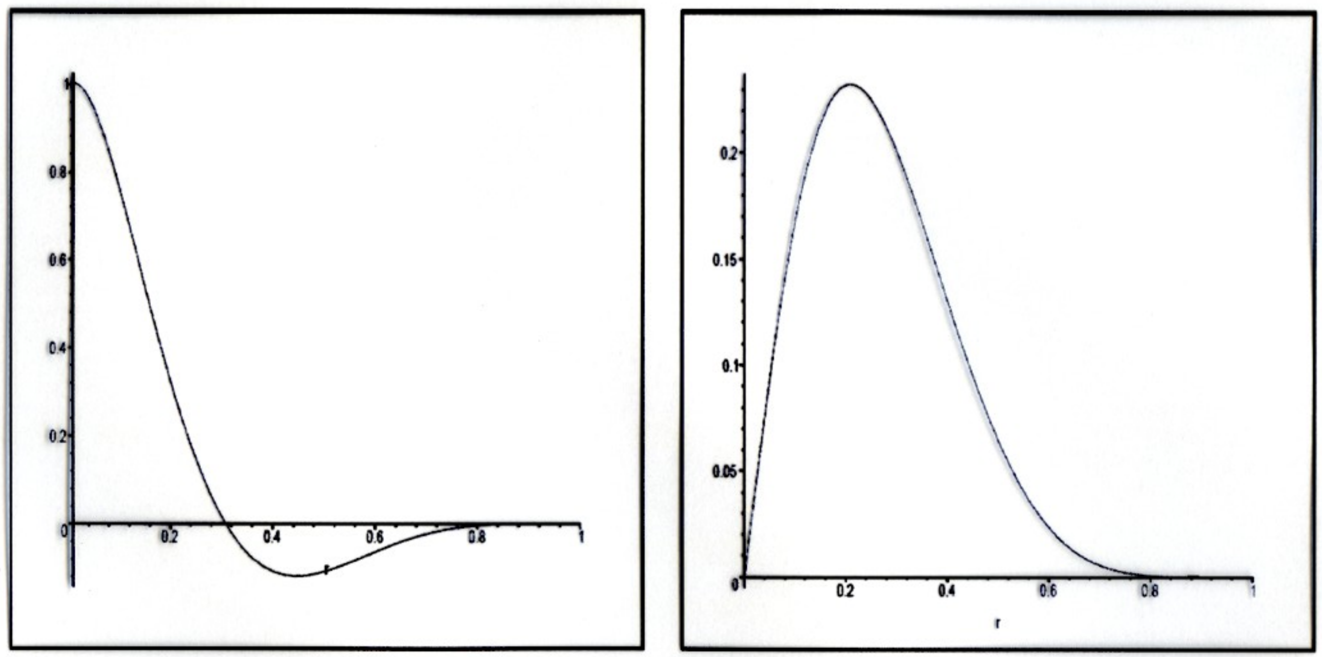


Figure 4.13: $\widehat{w}$ at $z = \frac{1}{2}$ (left) and $\widehat{u}^r$ at $z = \frac{1}{2} - \frac{\sqrt{7}}{14}$ where it is maximum (right) given by (4.28) and (4.29).

Evolution of hot towers in the absence of mean flows

Here we assume no background rotation ($\overline{\omega} = 0$ and $\overline{w} = 0$).
Show how initial conditions and the Coriolis parameter can affect the evolution of a hot tower in the absence of mean flow.

$f \neq 0 \Leftarrow$ Barotropic Mean Flow

The equation (4.16) for the evolution of $(u^\theta)'$

$$\frac{\partial (u^\theta)'}{\partial t} + (u^r)' \left(\frac{\partial (u^\theta)'}{\partial r} + \frac{(u^\theta)'}{r} \right) + w' \frac{\partial (u^\theta)'}{\partial z} = -f (u^r)' .$$

Due to the fact that $(u^r)' < 0$ for $z < ½$, and $(u^r)' > 0$ for $z > ½$, we know:

the source term $-f (u^r)'$ generates cyclones in the lower troposphere ($z < ½$) and anti-cyclones in the upper troposphere.

the term $(u^r)' (u^\theta)'/r$ is an amplification term when $z < ½$ and a dissipation term when $z > ½$. In the regions close to center, where r is very small, this term is dominant.

Evolution of hot towers in the absence of mean flows

No small scale initial vorticity $(\mathbf{u}^{\theta})'$ BAROTROPIC PRECONDITIONED Mean Flow

The initial vorticity is produced in the domain according to the term $\frac{\partial w'}{\partial z} f$ in the vorticity equation (4.6). As the time increases, a huge vorticity is produced close to the center. This is because the vorticity is given by $\frac{\partial (u^{\theta})'}{\partial r} + \frac{(u^{\theta})'}{r}$; in the regions close to the center, the term $\frac{(u^{\theta})'}{r}$ is dominant and this leads to a huge vorticity in the inner core.

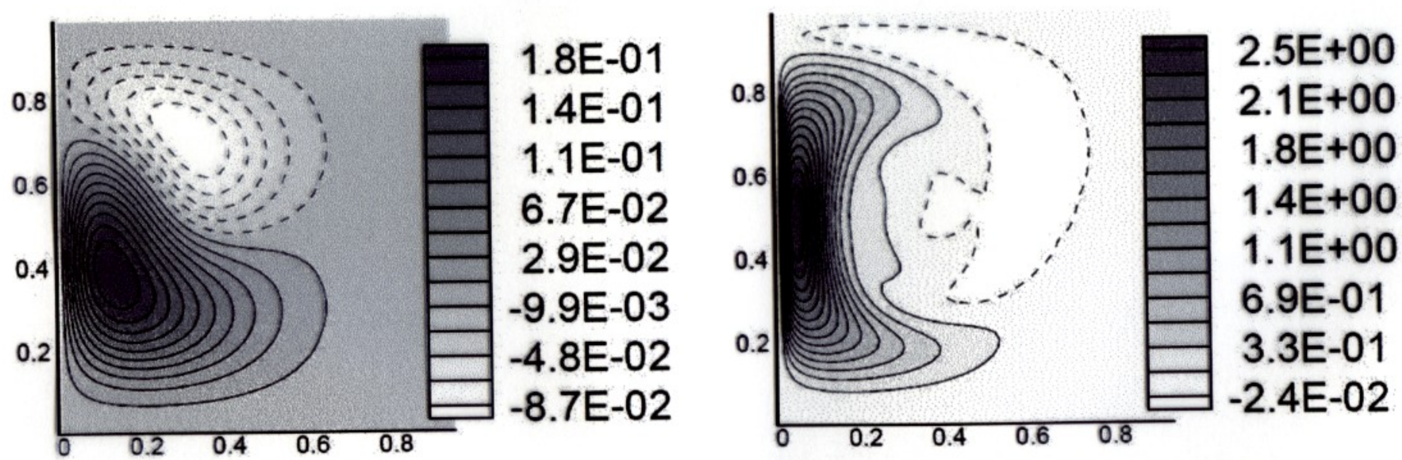


Figure 4.15: Case A1, contour plots of $(u^{\theta})'$ for $(u^{\theta})'_0 = 0$, $f = 1$, results at $t = T_{max}$ for $T_{max} = 1$ (left) and $T_{max} = 10$ (right). Dash lines show negative values (anti-cyclonic flow). Horizontal axis is r and vertical axis is z.

Effect of steady mean flows on hot towers

For simplicity, we assume zero initial small scale vorticity.

The equation (4.16) for the evolution of $(u^\theta)'$

$$\frac{\partial (u^\theta)'}{\partial t} + (u^r)'\left(\frac{\partial (u^\theta)'}{\partial r} + \frac{(u^\theta)'}{r}\right) + w'\frac{\partial (u^\theta)'}{\partial z} = -f\,(u^r)' - (u^r)'\overline{\omega} - w'\frac{\partial \overline{u^\theta}}{\partial z}$$

Effect of steady mean flows on hot towers

Low level cyclonic and high level anti-cyclonic mean flow

The mean vorticity here corresponds to a large-scale deep convective flow and it is defined by $\overline{\omega} = \sin(2\pi z)$.

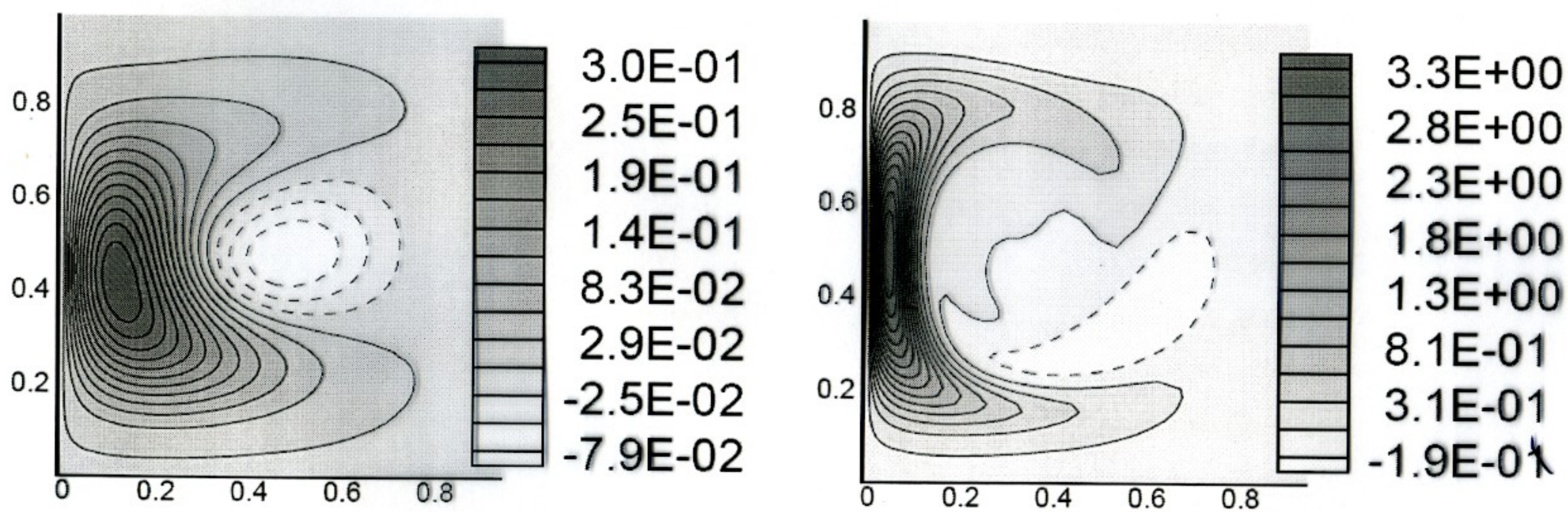


Figure 4.19: Same as Fig. 4.15 for Case B1, $(u^{\theta})'_0 = 0$, $f = 0.5$.

Summary

We have shown how a heat/mass source can generate large vorticity in a suitable preconditioning and useful elementary insight into the role of hot towers in cyclogenesis has been obtained through combination of exact solutions and simple numerics.

Although the terminology of hot towers and cloud scales are used here, the canonical model studied in this paper, is also relevant for larger scales, and the elementary model study also gives insight into how mesovortices may be generated due to the heat sources by mesoscale convective systems under various synoptic scale preconditionings.

Future work:

The insights obtained in this study, are useful as elementary structures in multi-scale models for the hurricane embryo.