Progress toward dynamical paleoclimate state estimation

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Plan

• Motivation & goals
  – paleo state estimation challenges
  – hypothesis: current weather DA sufficient

• Efficiently assimilating time-integrated obs
  – Results for a simple model
  – Results for a less simple model

• Optimal networks
  – where to site future obs conditional on previous
Motivation

- Reconstruct past climates from proxy data.
- Statistical methods (observations)
  - time-series analysis
  - multivariate regression
  - no link to dynamics
- Modeling methods
  - spatial and temporal consistency
  - no link to observations
- State estimation
  - This talk & workshop.
Long-term Goals

• Reconstruct last 1-2K years
  – Expected value and error covariance
  – Unique dataset for decadal variability
  – Basis for rational regional downscaling

• Test paleo network design ideas
  – Where to take highest impact new obs?
Challenges for paleo state estimation

No shortage of excuses for not trying!

1) proxy data are time integrated
   cf. weather assimilation of “instantaneous” obs

2) long-time periods
   computational expense
   predictability “horizon”

3) Proxies often chemical or biological
   forward model problem (tree rings)

4-n) nonlinearity; non-Gaussianity; bias; proxy timing; external forcing, etc.
   [similar problems in weather DA haven’t stopped decades of progress]
Is Precipitation Gaussian?

Aberdeen, WA

Blue Hill, MA

Annual Precipitation (100+ years)

Mt. Shasta, CA
Approach

- Develop a method as close to “classical” as possible
- Assume (until proven otherwise) that:
  - Errors are Gaussian distributed
  - Dynamics are ~linear in appropriate sense
  - I.e., Kalman filtering is a reasonable first approximation
- Why? Relax one key aspect of statistical reconstruction:
  - stationary statistics (leading EOFs; proxy regression)
- Key challenge topics addressed here:
  1. Efficient Kalman filtering for time-averaged observations
  2. Simplified models; assess predictability on proxy timescales
  3. Physical proxies only: ice core accumulation & isotopes
Paleoclimate

• Historical record of Earth’s climate.
• Benchmark for future climate change.
  – E.g., dynamics of decadal variability.
• “observations” are by proxy.
  – Examples:
    • Ice cores (accumulation, isotopes)
    • Tree rings.
    • Corals.
    • Sediments (pollen, isotopes).
  – Typically, related to climate variables, then analyzed.
Climate variability: a qualitative approach

Mayewski et al., 2004
The estimation problem

Observe & estimate a low-frequency signal in the presence of large amplitude high frequency noise.

Kalman filtering on high frequency timescale is problematic
Traditional Kalman Filtering

\[ x^a = x^b + K(y - Hx^b) \]

\[ K = BH^T[HBH^T + R]^{-1} \]

\[ Hx = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} \hat{H}x \, dt \equiv y^e \]

\[ BH^T = xy^e^T = \text{cov}(x, y^e) \quad \text{fast noise} \]

sequential filtering

Observations have little effect on the \textit{averaged} state.
Affecting the Time-Averaged State

To filter the $t_0 \rightarrow t_1$ time mean:

1. Perform $n$ assimilation steps over the interval.
   - Expensive: scales linearly with $n$.

2. Only update the time-mean (Dirren and Hakim 2005).
   - No more expensive than traditional KF.
Time-Averaged Assimilation

\[ x = \bar{x} + x' \quad \bar{x} = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} x \, dt \quad Hx = \hat{H}\bar{x} \]

\[ \bar{x}^a = \bar{x}^b + K_A (y - \hat{H}\bar{x}^b) \quad K_A = \bar{x}y e^T [y e y e^T + R]^{-1} \]

\[ x^{a'} = x^{b'} + K_P (y - \hat{H}\bar{x}^b) \quad K_P = x'y e^T [y e y e^T + R]^{-1} \]

\[ x'y e^T \approx 0 \rightarrow x^{a'} = x^{b'} \quad \text{Cost savings: just update time-mean} \]
EnKF Algorithm

1. Advance full ensemble from $t_0$ to $t_1$.
2. Compute time mean, perturbations.
   - observation estimate.
3. Update ensemble mean and perturbations.
   - Time-averaged fields only!
4. Add time perturbations to the updated mean.

- Time-mean can be accumulated while running the model
- Existing code requires only minor modification.
Testing on idealized models

- 1-D Lorenz (1996) system
- Idealized mountain--storm-track interaction
- QG model coupled to a slab ocean
- Analytical stochastic energy-balance model
Illustrative Example #1
Dirren & Hakim (2005)

Lorenz & Emanuel (1998): Linear combination of fast & slow processes

\[ X_m(j, t) = X_{hf}(j, t) + X_{lf}(j, t) \quad j = 1 : N_r \]

\[
\frac{dX_i}{dt}(j, t) = \frac{1}{\alpha_i}[(X_i(j + 1, t) - X_i(j - 2, t))]
\cdot X_i(j - 1, t) - X_i(j, t) + F] \quad i = \text{hf,lf}
\]

- LE \sim a scalar discretized around a latitude circle.

- LE has elements of atmos. dynamics:
  chaotic behavior, linear waves, damping, forcing

Observe all d.o.f.
- Low-frequency variable well constrained.
- Instantaneous states have large errors.
Improvement Percentage of RMS errors

\[ p(\tau, \tau_o) = \min \left( \frac{\sigma_o - RMS_{m}^{\tau}}{\sigma_o}, \frac{\sigma^{\tau_o} - RMS_{m}^{\tau}}{\sigma^{\tau_o}} \right) \times 100. \]

Obs uncertainty  Climatology uncertainty

Total state variable

Constrains signal at higher freq. than the obs themselves!
A less simple model
Helga Huntley (University of Delaware)

• QG “climate model”
  – Radiative relaxation to assumed temperature field
  – Mountain in center of domain

• Truth simulation
  – 100 observations (50 surface & 50 tropopause)
  – Gaussian errors
  – Range of time averages
Snapshot

Ground & Tropopause Potential Temperatures
Observation Locations

Observation Network, Nobs = 100, Surface

Observation Network, Nobs = 100, Tropopause
Average Spatial RMS Error

Ensemble compared against an ensemble control
Implications

• Mean state is well constrained by few, noisy, obs.
• Forecast error saturates at climatology for $\tau \sim 30$.
• For longer averaging times, model adds little.
  – Equally good results are obtained by assimilating with an ensemble drawn from climatology:
    • cheap (no model runs).
    • reduced sampling error (huge ensembles easy).
    • but, no flow-dependence to corrections.
    • subject to model error.
QG model coupled to a slab ocean
and it’s approximation by an energy balance model

With A. Pendergrass, G. Roe, & D. Battisti
Barsugli & Battisti (1998) energy balance model

\[
\frac{dT}{dt} = \begin{bmatrix} -a & b \\ \frac{c}{\beta} & -\frac{d}{\beta} \end{bmatrix} \vec{T} + \begin{bmatrix} N & 0 \\ 0 & 0 \end{bmatrix} \vec{W} = A\vec{T} + N\vec{W}
\]

- a, d : damping parameters (radiation)
- b, c : coupling coefficients
- \( \beta \) : ratio of heat capacities
- N : noise forcing
Eigenvectors

One fast mode and one slow mode
QG & BB spectral comparison

- Good agreement, particularly in phasing
Key to estimation: covariance propagation

\[
\langle \vec{e}^* \vec{e}^{*T} \rangle = \frac{1}{\tau^2} \int_0^T e^{At} \vec{e}^*(0) dt \left( \int_0^T e^{At} \vec{e}^*(0) dt \right)^T \\
+ \frac{1}{\tau^2} \int_0^T \left( \int_s^T e^{A(t-s)} \mathbf{N} dt \right) \left( \int_s^T e^{A(t-s)} \mathbf{N} dt \right)^T ds.
\]

- First term: initial condition error (damped)
- Second term: accumulation of noise.
Energy model DA spinup

Atmosphere temperature

Slab ocean temperature

Error variance ($K^2$)

Time (days)

Error of time averaged states - total error

Assimilation

Error variance ($K^2$)

Time (days)
One time-averaged forecast cycle
Sensitivity to Slab Depth

Increasing slab depth:
- Improves ocean
- **Degrades** atmosphere
Why does depth degrade atmosphere?

Noise “accumulates” in the atmosphere when the slab ocean is deeper.
Sensitivity to Coupling

Increasing coupling:

- Atmosphere: QG & BB opposite sensitivity
- Ocean: tighter coupling degrades the analysis
- BB: Noise “recycles”
Observing Network Design
with Helga Huntley (U. Delaware)
Optimal Observation Locations

• Rather than use random networks, how to optimally site new observations?
  – choose locations with largest change in a metric.
  – Here, metric = projection coefficient for first EOF
  – QG model with mountain
Ensemble Sensitivity

• Given metric $J$, find the observation that most reduces error variance.

• Find a second observation **conditional on first**.

• Let $\mathbf{x}$ denote the state (ensemble mean removed).
  
  – Analysis covariance $A = \text{cov}(\mathbf{x}, \mathbf{x})$

  – Changes in metric given changes in state

  – Metric variance $\delta J = \left[ \frac{\partial J}{\partial \mathbf{x}} \right]^T \delta \mathbf{x} + O(\delta \mathbf{x}^2)$

$$\sigma = \frac{1}{M - 1} \delta J \delta J^T = \left[ \frac{\partial J}{\partial \mathbf{x}} \right]^T A \left[ \frac{\partial J}{\partial \mathbf{x}} \right]$$
Sensitivity + State Estimation

• Estimate variance change for the i’th observation
  \[ \delta \sigma = \left[ \frac{\partial J}{\partial x} \right]^T (A_{i-1} - A_i) \left[ \frac{\partial J}{\partial x} \right] \]

• Kalman filter theory gives \( A_i \):
  \[ A_i = (I - K_i H_i) A_{i-1} \]

where
  \[ K_i = A_{i-1} H_i^T [H_i A_{i-1} H_i^T + R_i]^{-1} \]

• Given \( \delta \sigma \) at each point, find largest value.
Results for $\tau = 20$

- The four most sensitive locations, conditional on previous point.
4 Optimal Observation Locations

Avg Error - Anal = 2.0545
- Fcst = 4.8808
Summary

• Time for paleo assimilation of select proxy data.
  – ensemble filters
  – ice-core accumulation & isotopes

• Modified Kalman filter approach
  – Update time mean
  – Easy, works well in existing EnKF.

• Filter corrects time scales shorter than proxy timescale.
  – Dynamics scatter information.

• Beyond predictability time scale, random samples drawn from model climate work well.
  – Model error problematic.
Ensemble Sensitivity (cont’d)

- For identity $H$, choose the point maximizing:

$$
\delta \sigma = \frac{[\text{cov}(\delta J, \delta x_i)]^2}{\text{var}(\delta x_i) + R}
$$

- Choose second point conditional on first:

$$
\delta \sigma = \left[ \frac{\text{cov}(\delta J, \delta x_i) - \frac{\text{cov}(\delta J, \delta x_1)\text{cov}(\delta x_1, \delta x_i)}{\text{var}(\delta x_1) + R}}{\text{var}(\delta x_i) - \frac{\text{cov}(\delta x_1, \delta x_i)}{\text{var}(\delta x_1) + R} + R} \right]^2
$$

- Etc.
Ensemble Sensitivity (cont’d)

• A recursive formula, which requires the evaluation of just \( k+3 \) lines (1 covariance vector + \( (k+6) \) entry-wise mults/divs/adds/subs) for the \( k' \)th point.
Results for $\tau = 20$

First EOF
Results for $\tau = 20$

- The ten most sensitive locations (unconditional)
- $\sigma_0 = 0.10$
Results for $\tau = 20; \sigma_o = 0.10$

Note the decreasing effect on the variance.
Control Case: No Assimilation

Avg error = 5.4484
100 Random Observation Locations

Avg Error - Anal = 1.0427
- Fcst = 3.6403
4 Random Observation Locations

Avg Error - Anal = 5.5644
- Fcst = 5.6279
Assimilating just the 4 chosen locations yields a significant portion of the gain in error reduction in $J$ achieved with 100 obs.
15 Chosen Observations

• For this experiment, take
  – 4 best obs to reduce variability in 1st EOF
  – 4 best obs to reduce variability in 2nd EOF
  – 2 best obs to reduce variability in 3rd EOF
  – 2 best obs to reduce variability in 4th EOF
  – 3 best obs to reduce variability in 5th EOF
• Number for each EOF chosen by $\delta \sigma < 0.7$
• All obs conditional on assimilation of previous obs.
### 15 Obs: Error in 1st EOF Coeff

<table>
<thead>
<tr>
<th>EOF1</th>
<th>Control</th>
<th>100R</th>
<th>4R</th>
<th>4O</th>
<th>8O</th>
<th>15 total</th>
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</thead>
<tbody>
<tr>
<td>Fest</td>
<td>5.4484</td>
<td>3.6403</td>
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<td>4.7586</td>
<td>3.5138</td>
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<tr>
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<td>1.9020</td>
<td>1.8819</td>
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15 Obs: Error in 2nd EOF Coeff

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<th>EOF2</th>
<th>Control</th>
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<th>4R</th>
<th>4O</th>
<th>8O</th>
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<td>4.2796</td>
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<td>1.4563</td>
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</table>
15 Observations: RMS Error

### Table

<table>
<thead>
<tr>
<th>RMS</th>
<th>Control</th>
<th>100 R</th>
<th>4 R</th>
<th>4 O</th>
<th>8 O</th>
<th>15 O</th>
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</thead>
<tbody>
<tr>
<td>Fcst</td>
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<td>0.2539</td>
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</tbody>
</table>
Current & Future Plans
Angie Pendergrass (UW)

• modeling on the sphere: SPEEDY
  – simplified physics
  – slab ocean

• ice-core assimilation
  – annual accumulation
  – oxygen isotopes