Energy Balance Models

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Summer Graduate School on Mathematics of Climate Change
NCAR - MSRI
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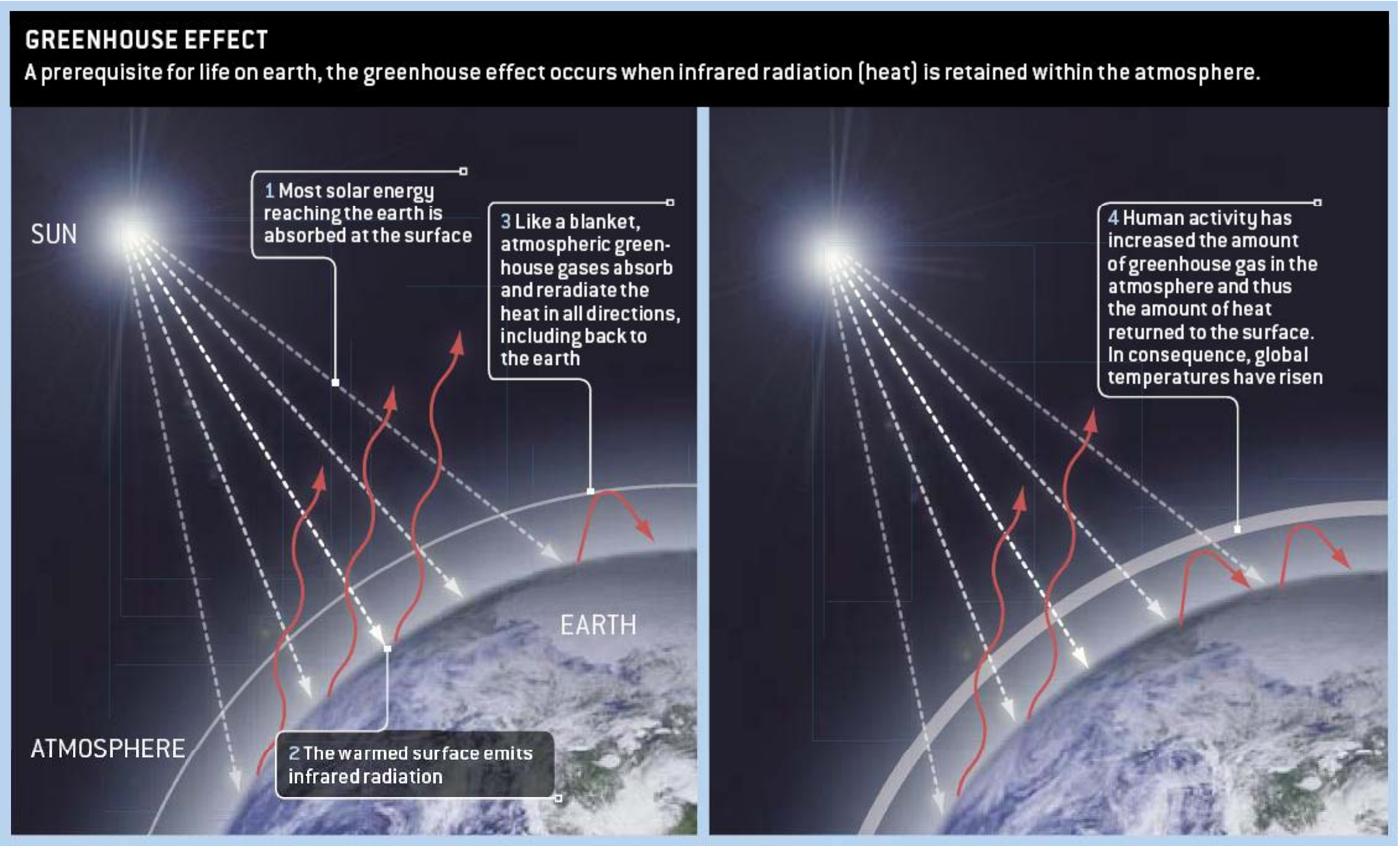
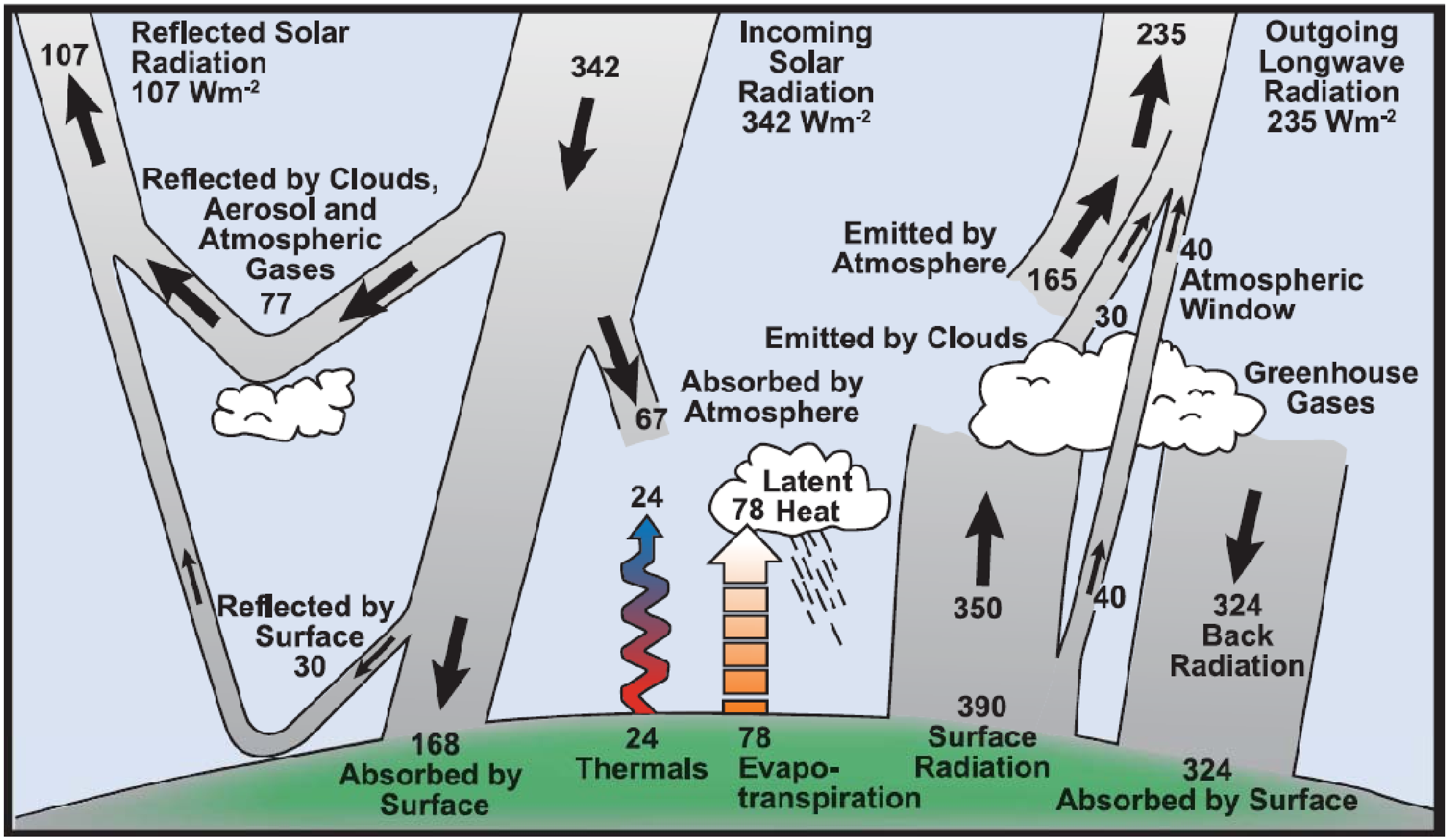
Earth's Energy Balance



Gary Stix, *Scientific American* September 2006, pp.46-49

Earth's Energy Balance



Historical Overview of Climate Change Science, IPCC AR4, p.96
http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1_Print_CH01.pdf

Insolation

Insolation = **In**coming **sol**ar radi**ation**

solar intensity at average distance from the sun: 1368 W/m^2

radius of the Earth: ρ meters
cross sectional area: $\pi\rho^2$ m^2
intercepted power: 1368 $\pi\rho^2$ Watts
surface area: $4\pi\rho^2$ m^2

average insolation: 1368/4 W/m^2 = 342 W/m^2

Simple Albedo Model

energy imbalance = insolation - reradiation

$$R\frac{dT}{dt} = Q(1-\alpha) - (A + BT)$$

T = global annual mean surface temperature (°C)

t = time (seconds)

Q = global annual mean insolation (342 W/m^2)

α = global mean surface albedo (reflectivity)

$A+BT$ = reradiation (W/m^2)

R = heat capacity of Earth's surface (J/m^2/°C)

Stable equilibrium at

$$T = \frac{Q(1-\alpha) - A}{B}$$

Simple Albedo Model

Stable equilibrium

$$T = \frac{Q\left(1-\alpha\right)-A}{B}$$

Q = 342 W/m^2 A = 202 W/m^2 B = 1.9 W/m^2/°C

albedo of land and water = 0.32
albedo of ice = 0.62

Snowball Earth: α = 0.62, T = -38 °C
Ice Free Earth : α = 0.32, T = 16 °C

Glaciers form if $T < T_c$ = -10 °C and melt if $T > T_c$.

Since 16 > -10, no glacier would form on an ice free Earth.
Since -38 < -10, no glacier would melt on a snowball Earth.

Less Simple Albedo Model

(Budyko, Sellers – Tung version)

energy imbalance = insolation – reradiation + transport

$$R\frac{\partial T}{\partial t} = Qs(y)\left(1-\alpha(y)\right) - \left(A + BT\right) + C\left(\bar{T} - T\right)$$

y = sine(latitude)

$T(y,t)$ = annual mean surface temperature at latitude $\arcsin(y)$

$Qs(y)$ = annual mean insolation at latitude $\arcsin(y)$

$\alpha(y)$ = surface albedo at latitude $\arcsin(y)$

$\bar{T}$ = global mean temperature

$C\left(\bar{T} - T\right)$: linear relaxation to mean

$s(y)$ = distribution of insolation across latitudes

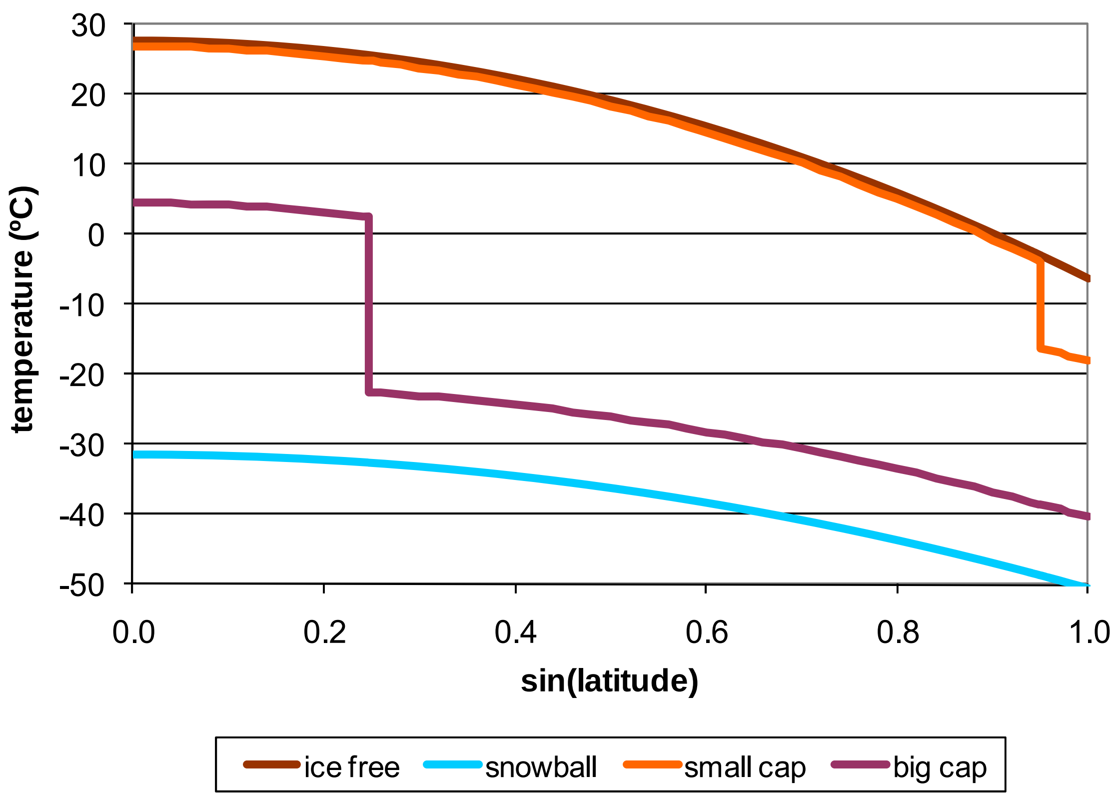
$$\int_0^1 s(y)\,dy = 1$$

Choice of y instead of latitude: $\bar{T} = \int_0^1 T(y)\,dy$

Budyko Model

Solve for equilibrium solutions:

$$Qs(y)\big(1-\alpha(y)\big)-\big(A+BT\big)+C\big(\bar{T}-T\big)=0$$



Budyko Model

$$R\frac{\partial T}{\partial t} = \underbrace{Qs(y)}_{\text{insolation}}\left(1-\alpha(y)\right)-\left(A+BT\right)+C\left(\bar{T}-T\right)$$

What determines Q and s ?

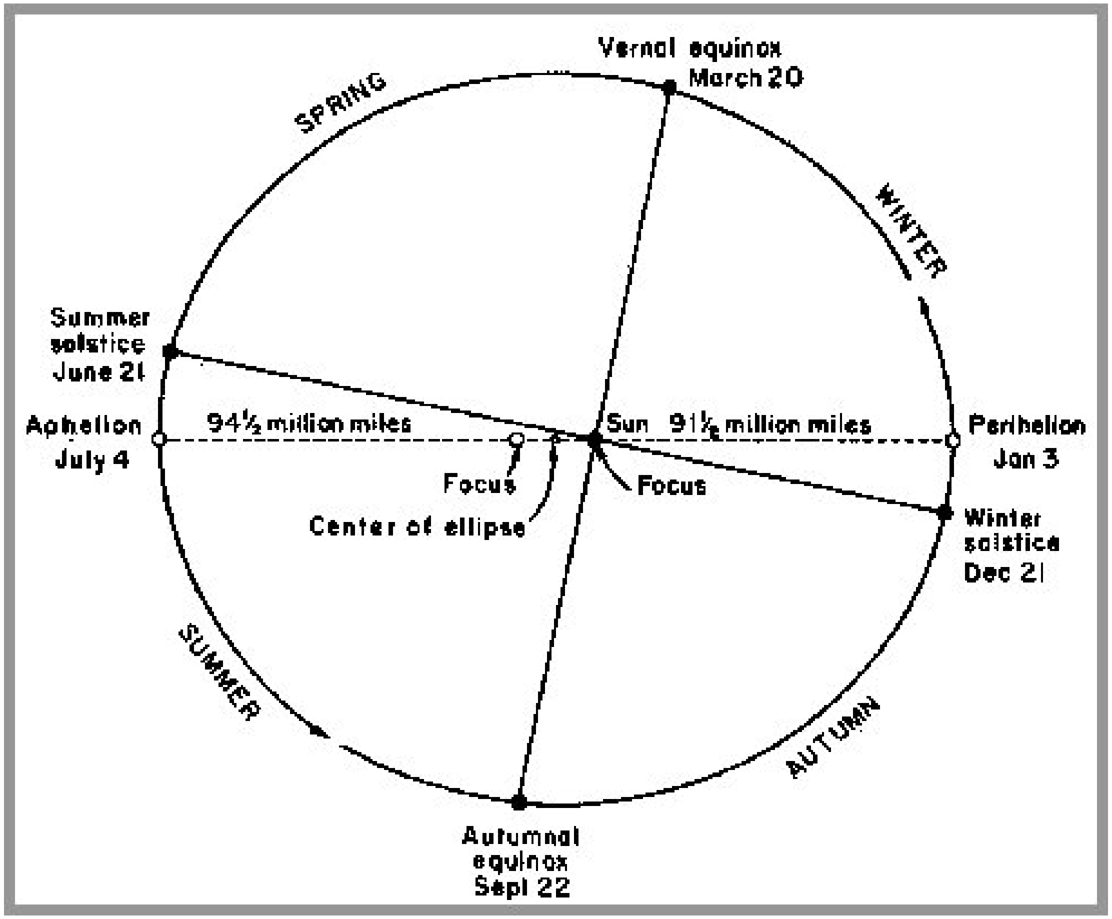
Milankovitch Cycles

Insolation is determined by the Earth’s orbit and axial tilt.

Milankovitch components:

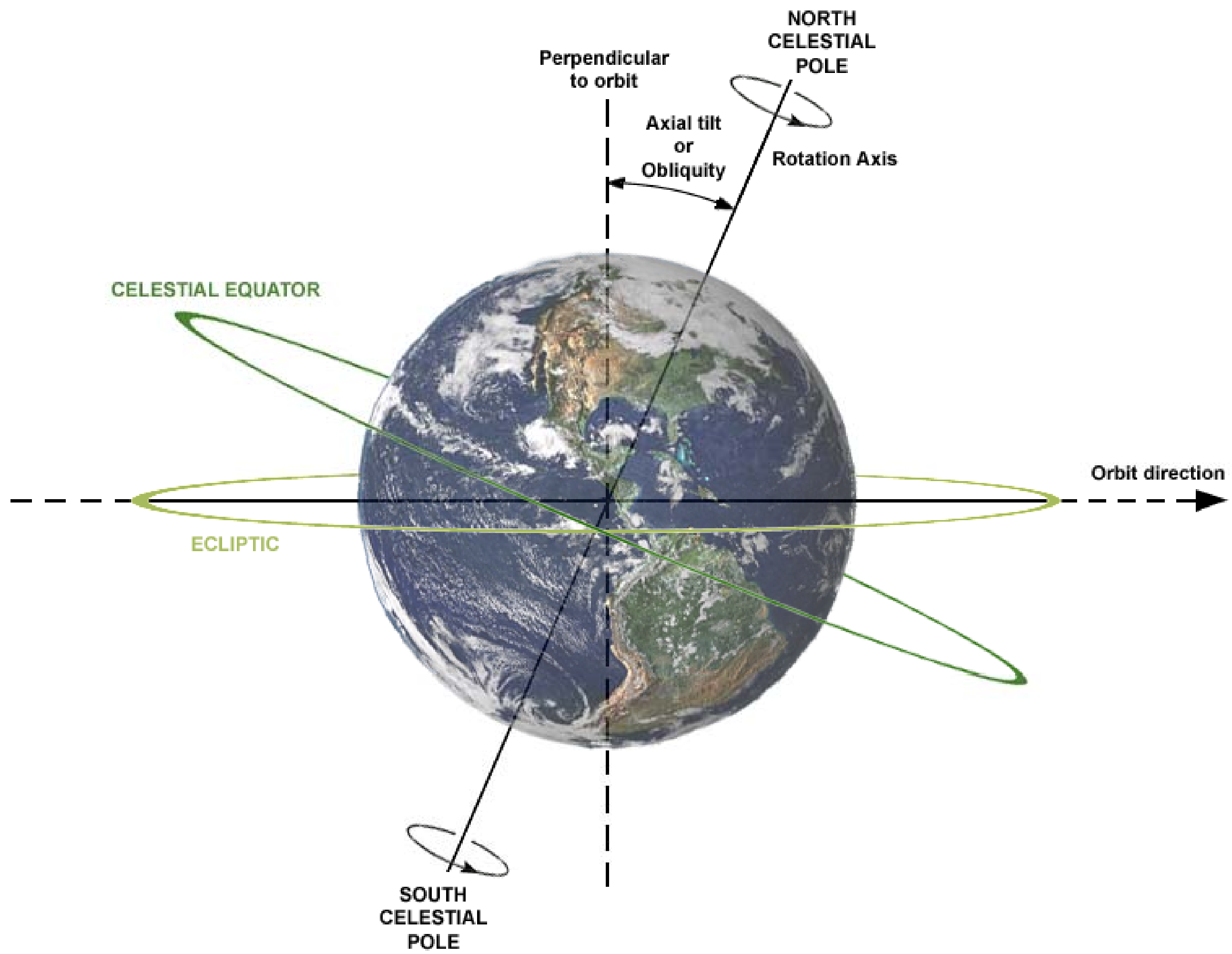
•**eccentricity** of Earth’s orbit
•**obliquitiy** (tilt) of the Earth’s rotation axis
•**precession** of the axis

Eccentricity



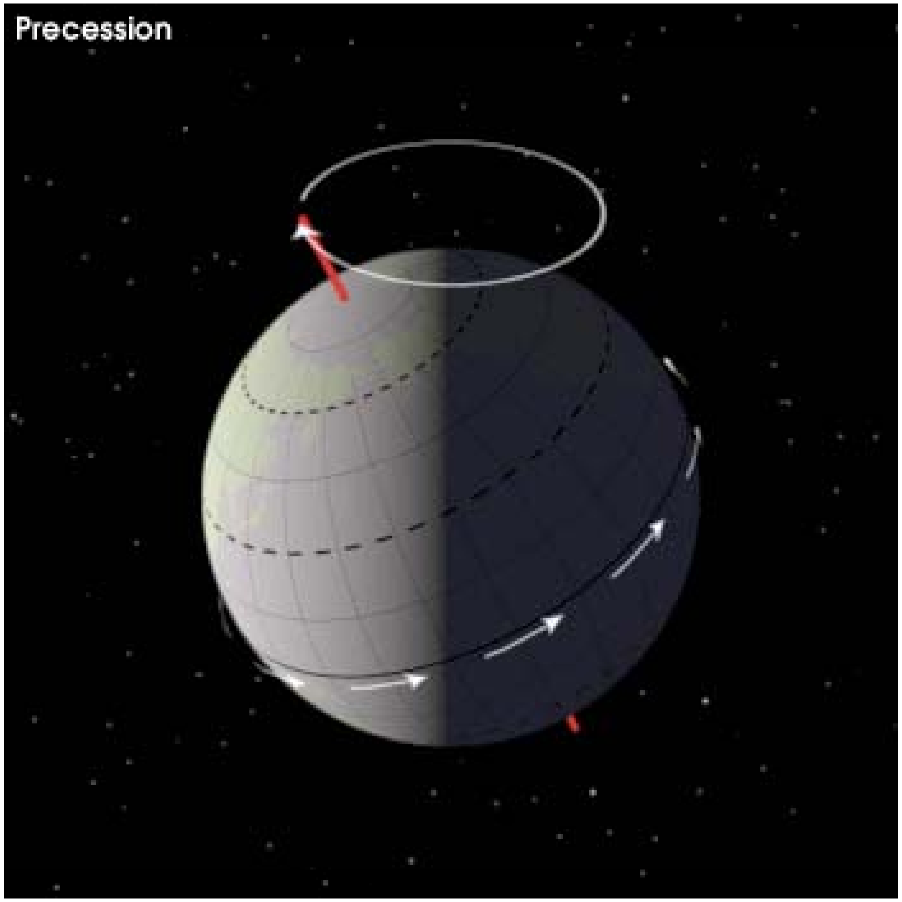
http://www.crrel.usace.army.mil/permafrosttunnel/Ice_Age_Earth_Orbit.jpg

Obliquity



http://upload.wikimedia.org/wikipedia/commons/6/61/AxialTiltObliquity.png

Precession



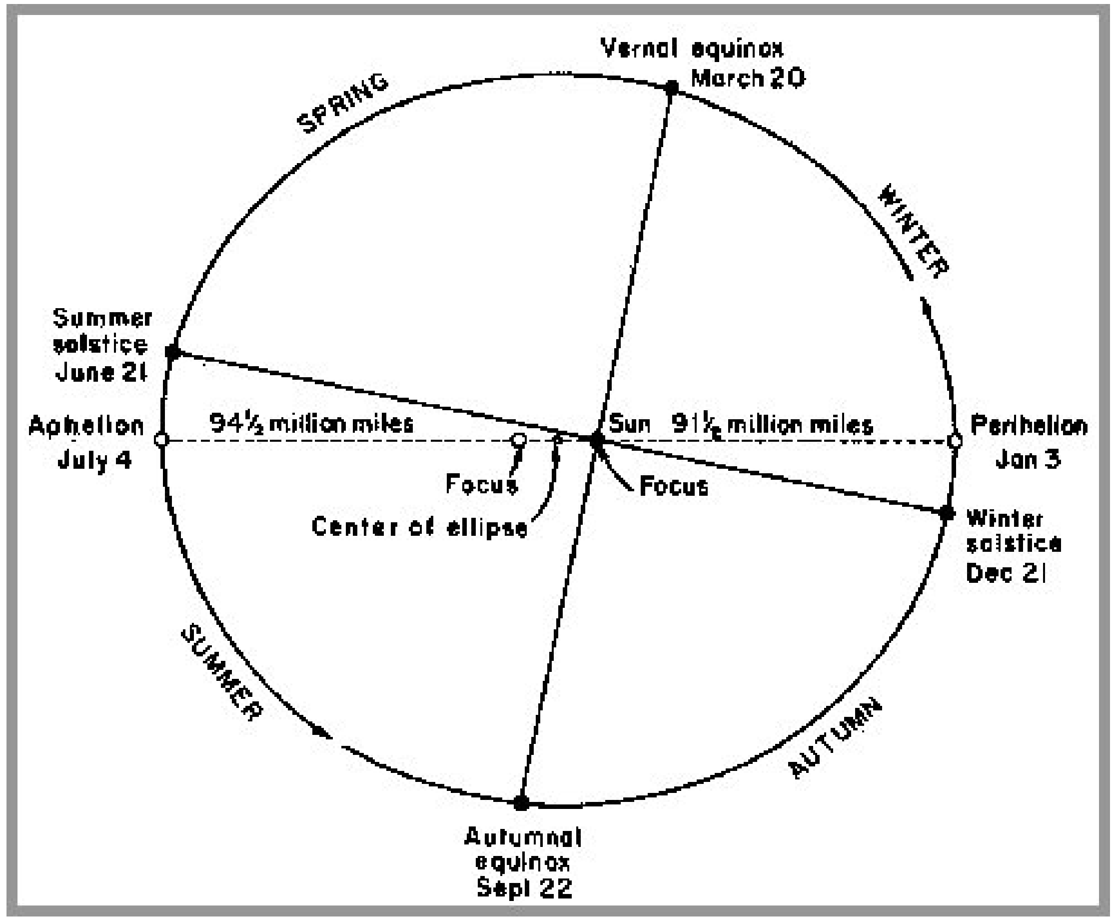
http://earthobservatory.nasa.gov/Library/Giants/Milankovitch/milankovitch_2.html

Current Eccentricity

Perihelion: 91.5×10^6 mi
Aphelion: 94.5×10^6 mi

Semimajor axis: 93×10^6 mi

Eccentricity: $1.5/93 = 0.016$



Eccentricity

Perihelion: 91.5
Aphelion: 94.5

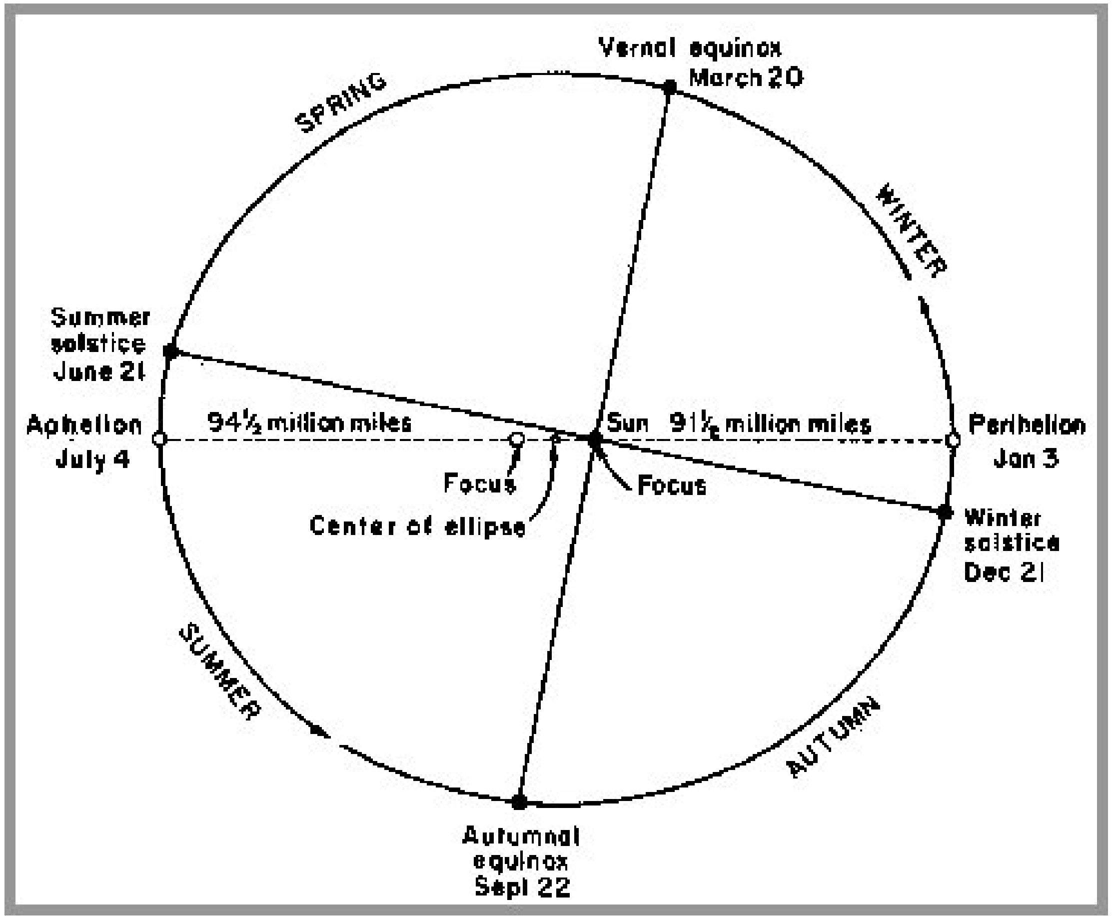
Change in distance from sun: 3/93 = 3.2%

Change in insolation: **6.4%**

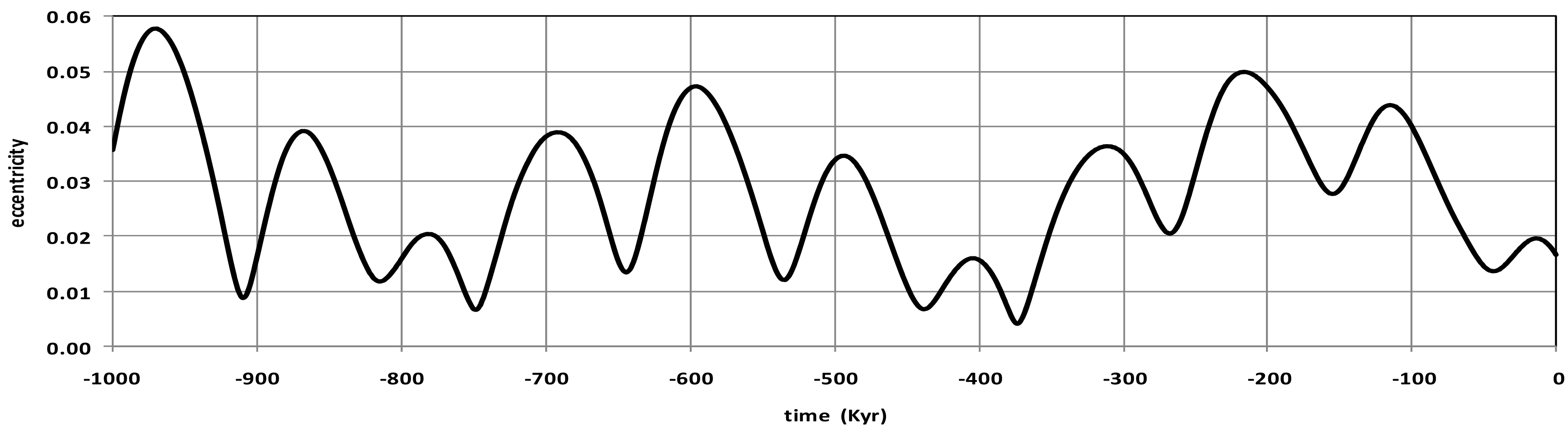
Six percent less insolation in the southern winter than the northern winter.

6.4% of 342 Wm^2 =

22 Wm^{-2}



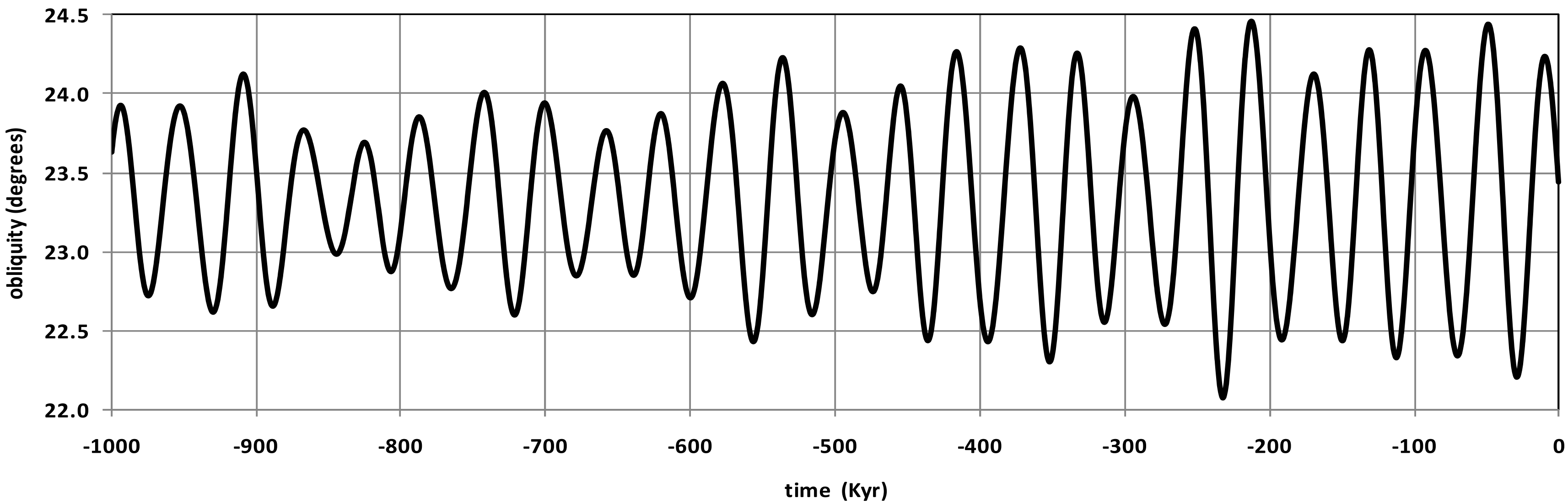
Eccentricity



Note periods of about 100 Kyr and 400 Kyr.

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

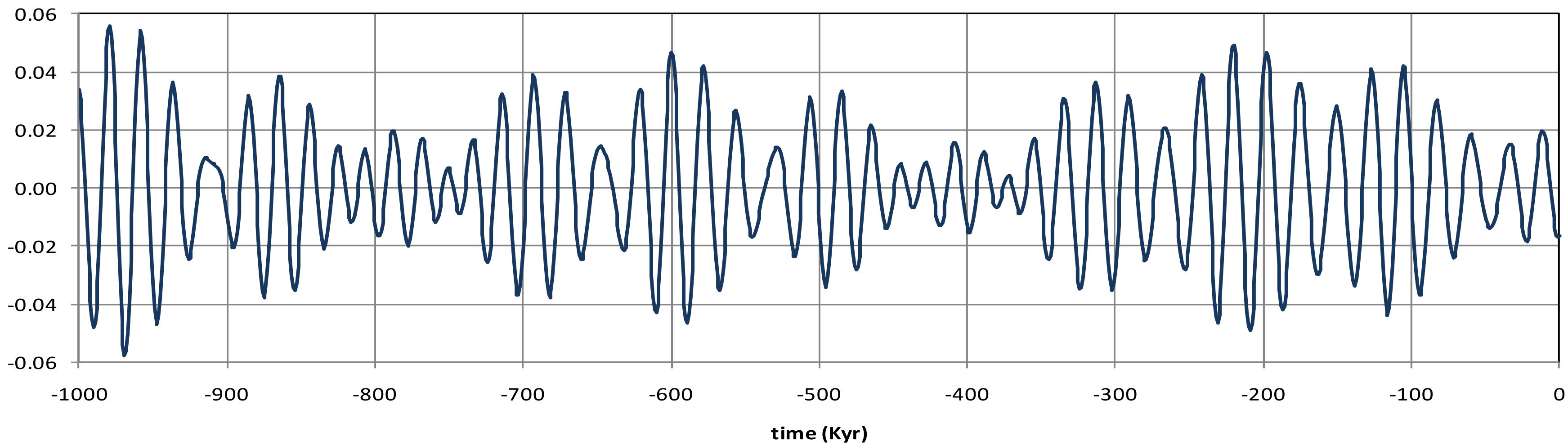
Obliquity



Note period of about 41 Kyr.

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

Precession Index

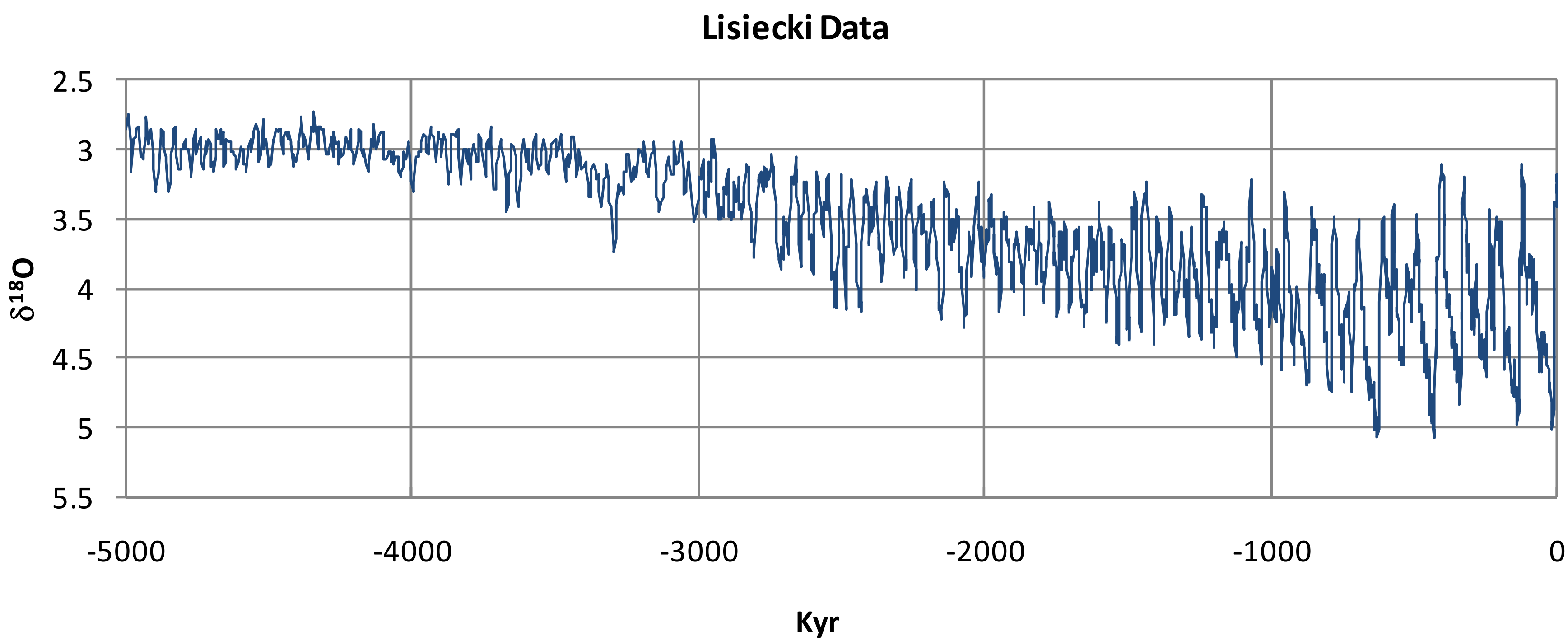


index = $e \sin\rho$, where e = eccentricity and ρ = precession angle (measured from spring equinox)

Note period of about 23 Kyr.

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

Lisiecki–Raymo $\delta^{18}O$ Stack



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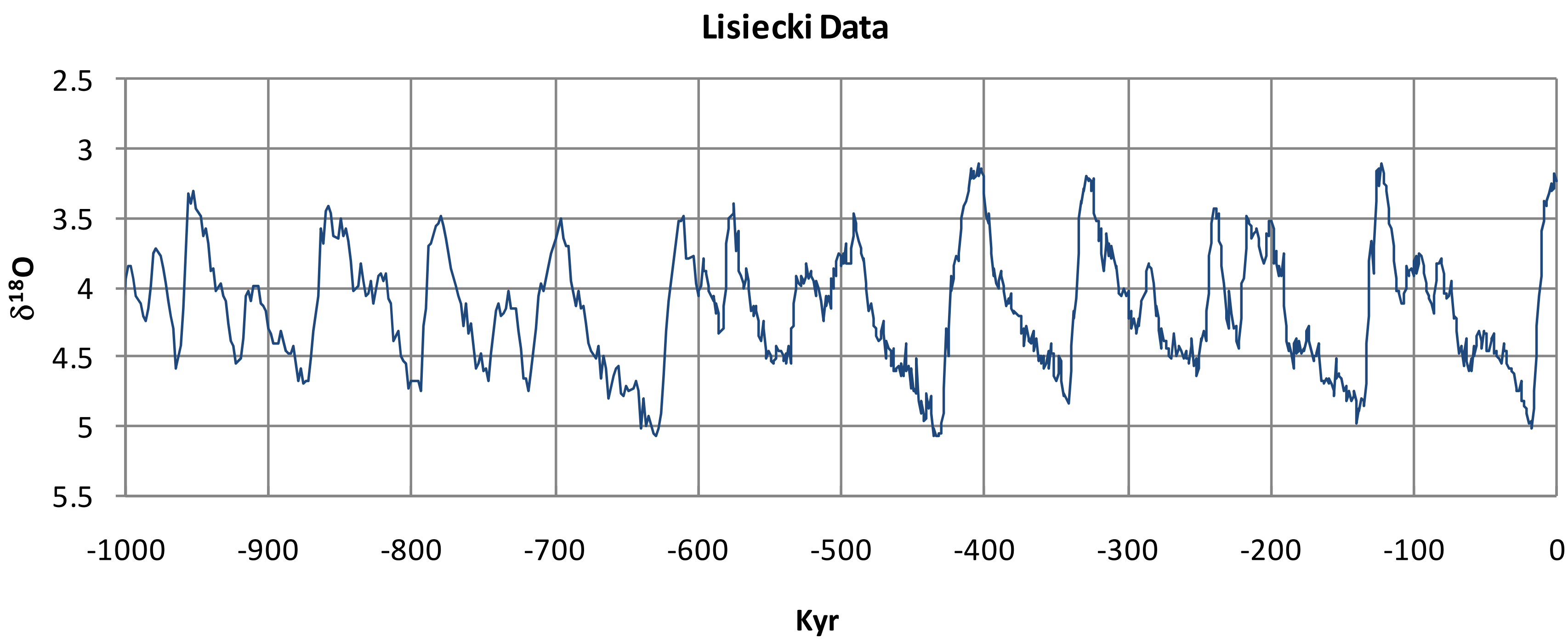

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Power Spectrum

Fourier Transform

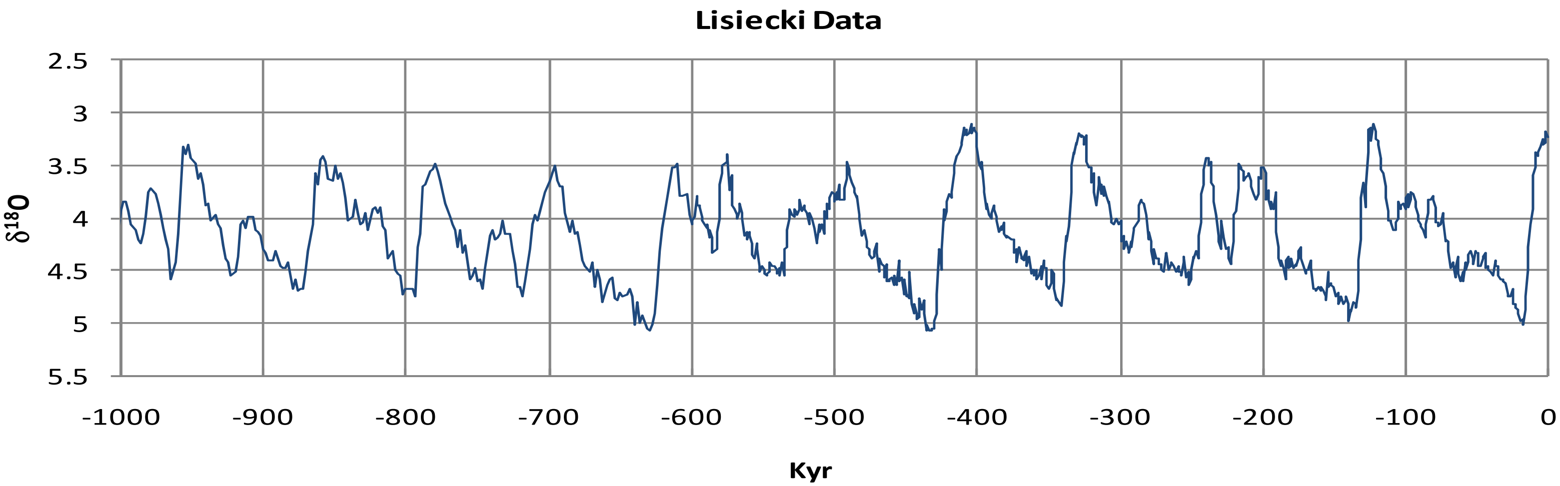
$$\hat{f}(\omega) = \int f(t) e^{-i2\pi\omega t} dt$$

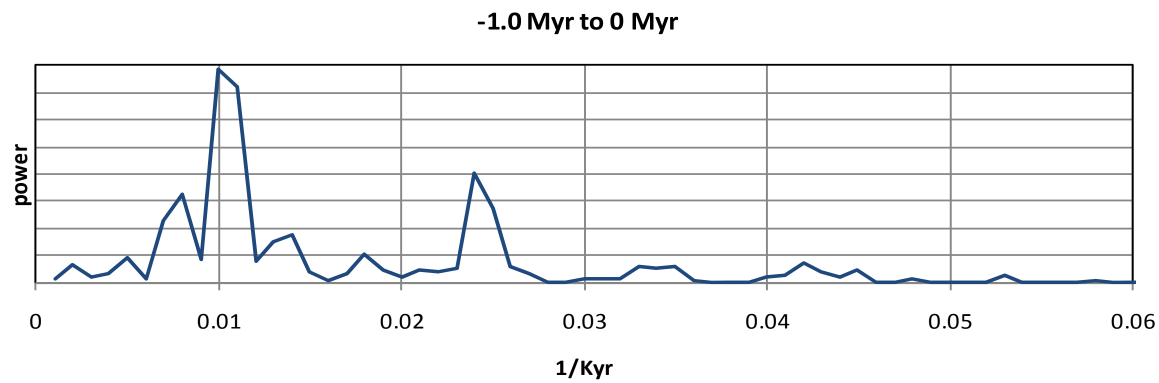
$$f(t) = \int \hat{f}(\omega) e^{i2\pi\omega t} d\omega$$

Power Spectrum

$$\left|\hat{f}(\omega)\right|^2$$

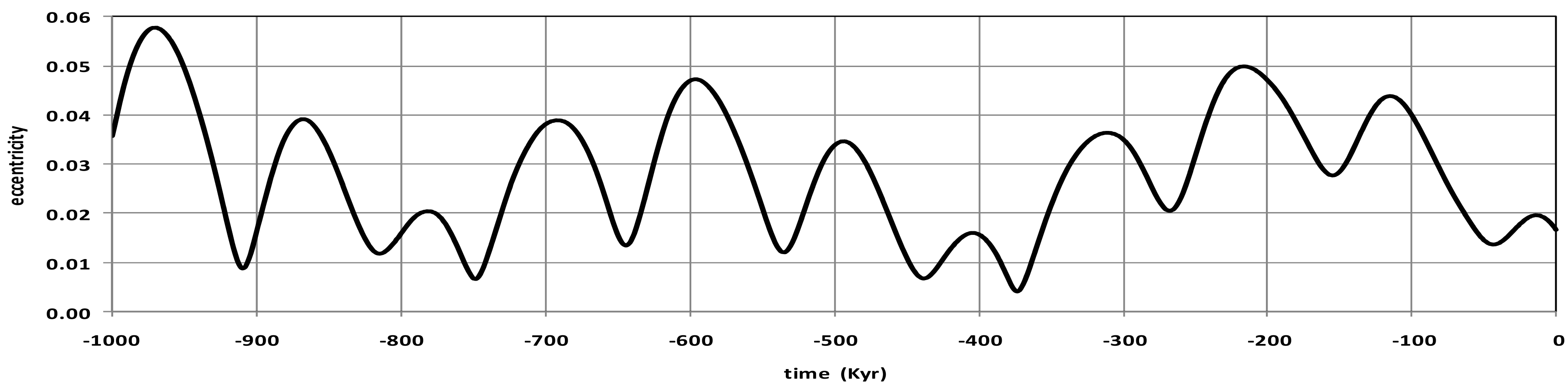
Lisiecki–Raymo $\delta^{18}O$ Stack

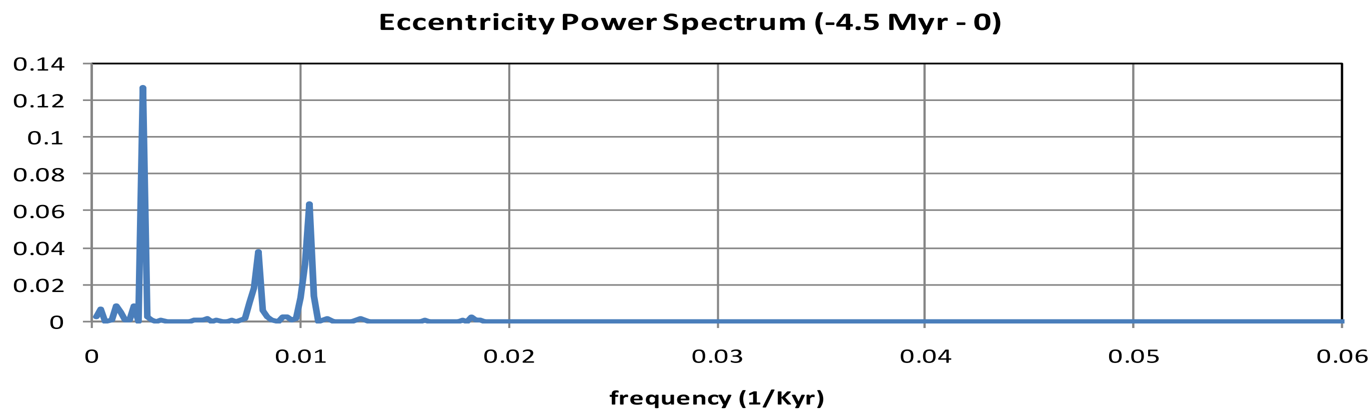




Strong peak at period 100 Kyr
Smaller peak at period 41 Kyr

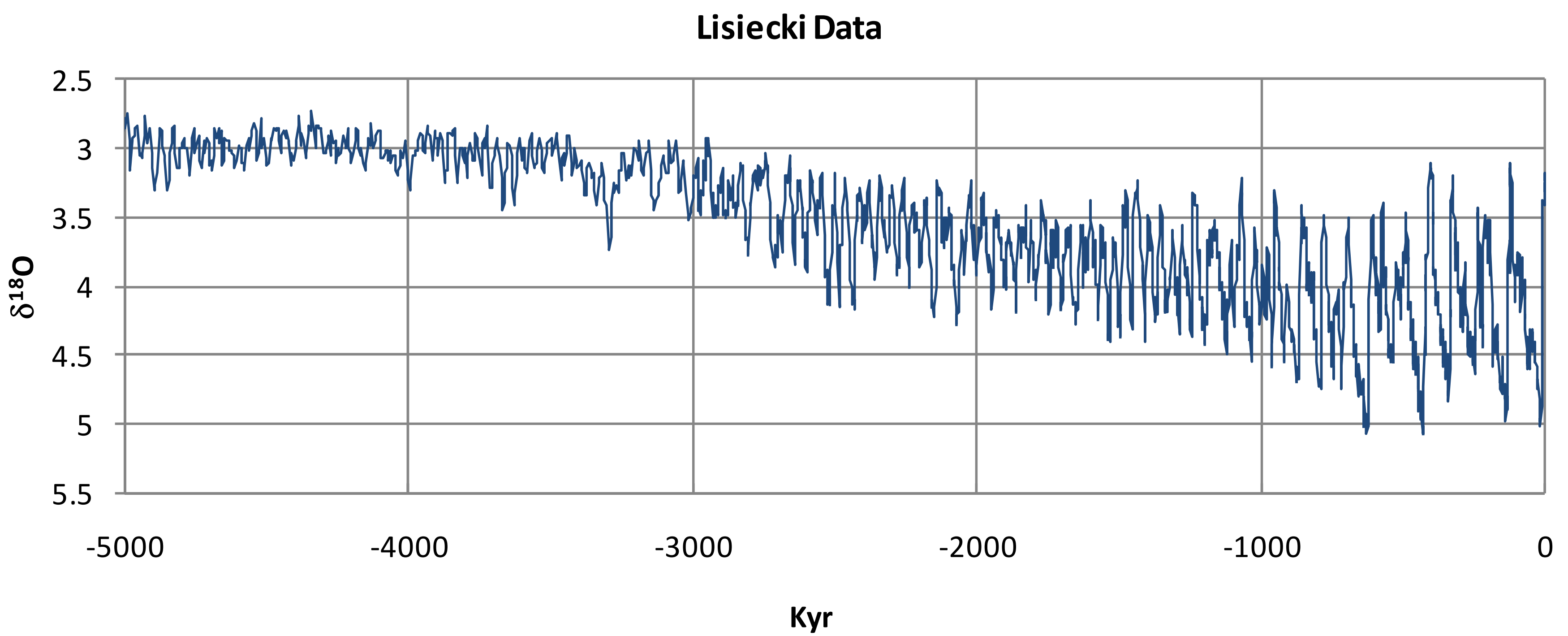
Eccentricity





Note periods of about 100 Kyr and 400 Kyr.

Lisiecki–Raymo δ^{18}O Stack



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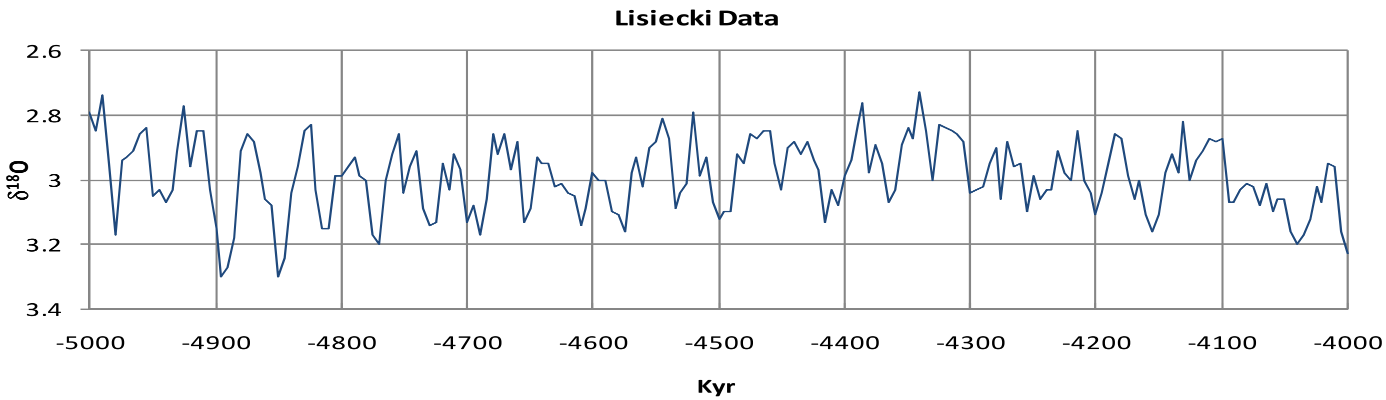
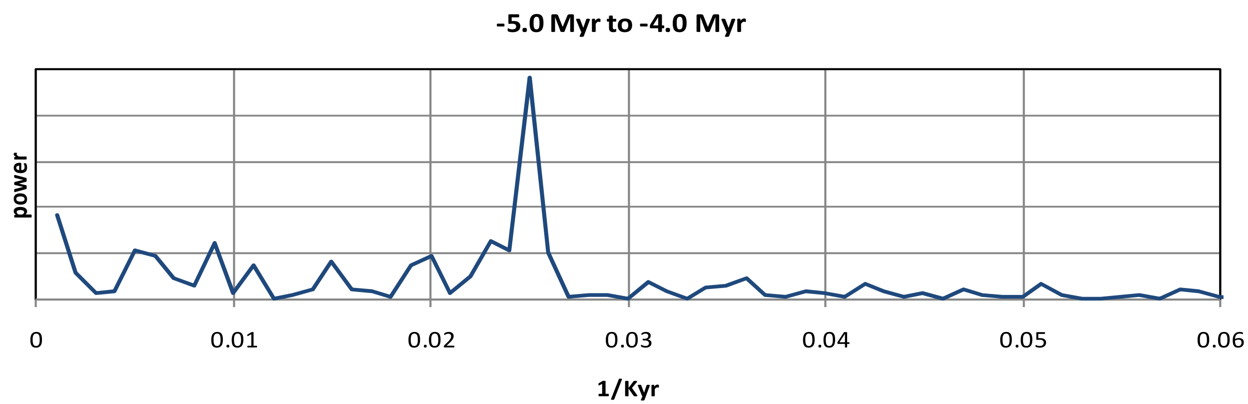
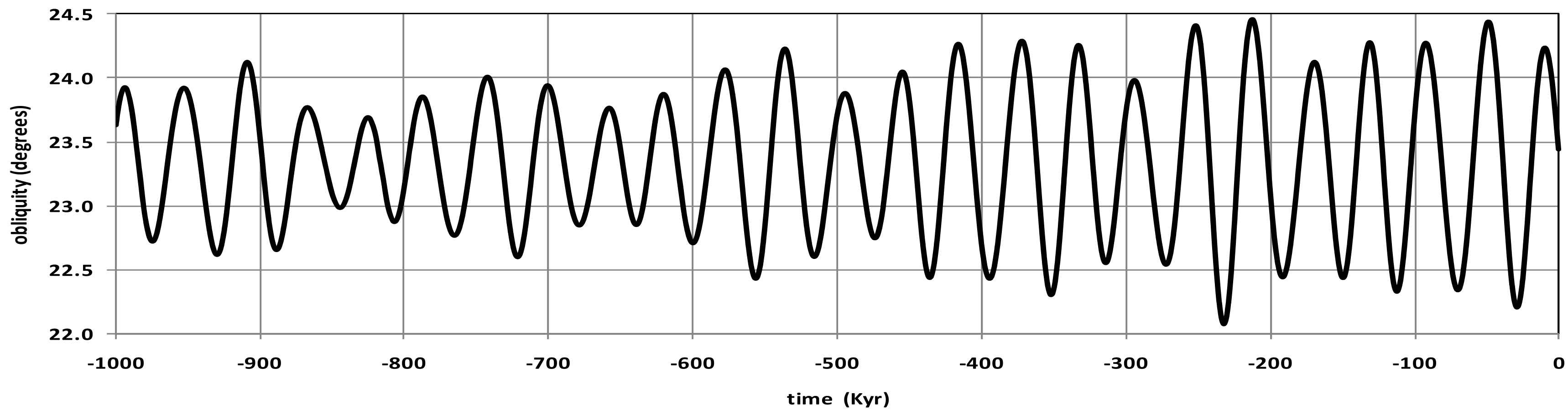
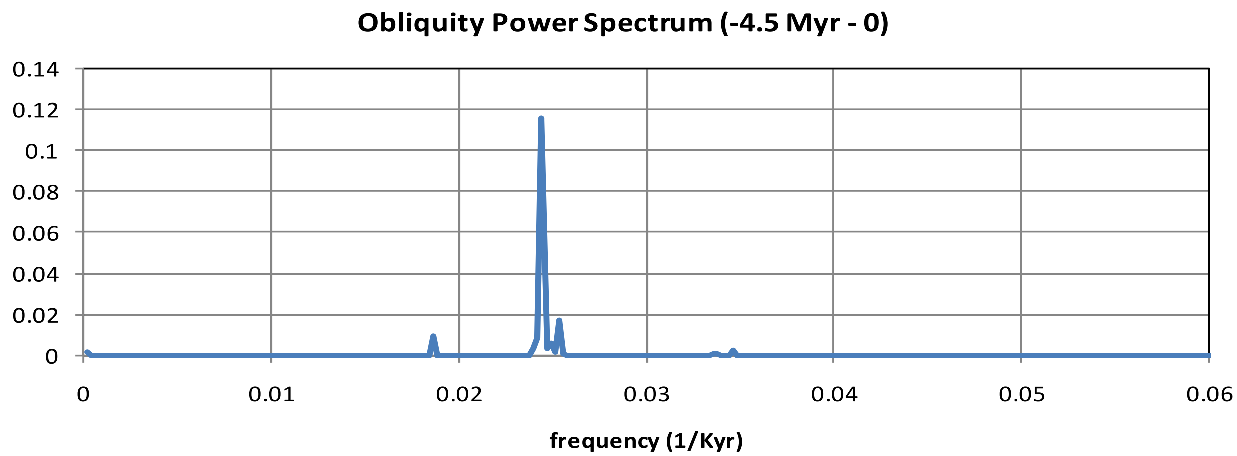




Strong peak at period 41 Kyr

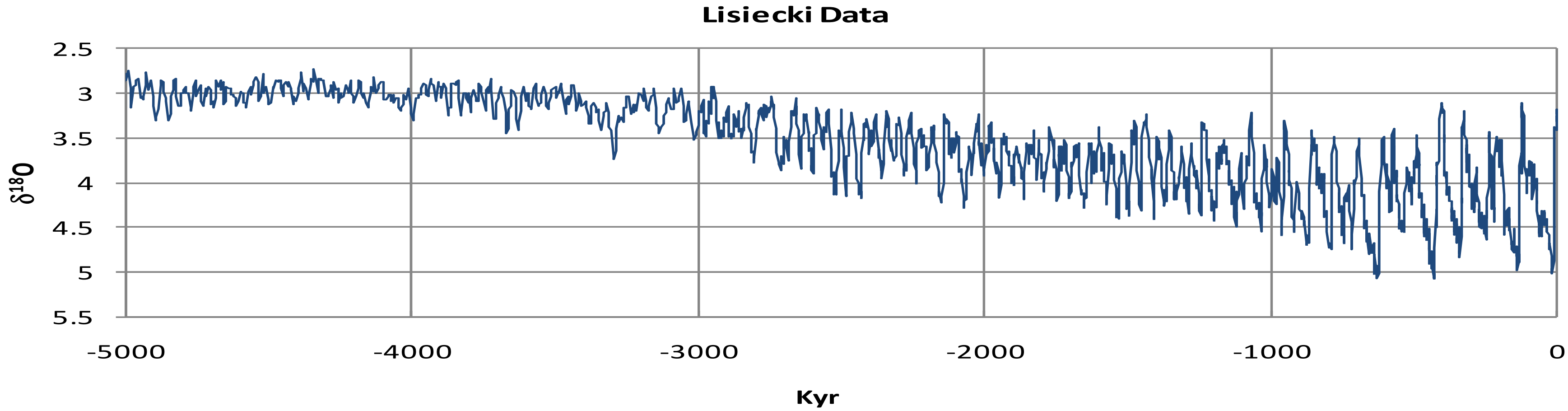
Obliquity

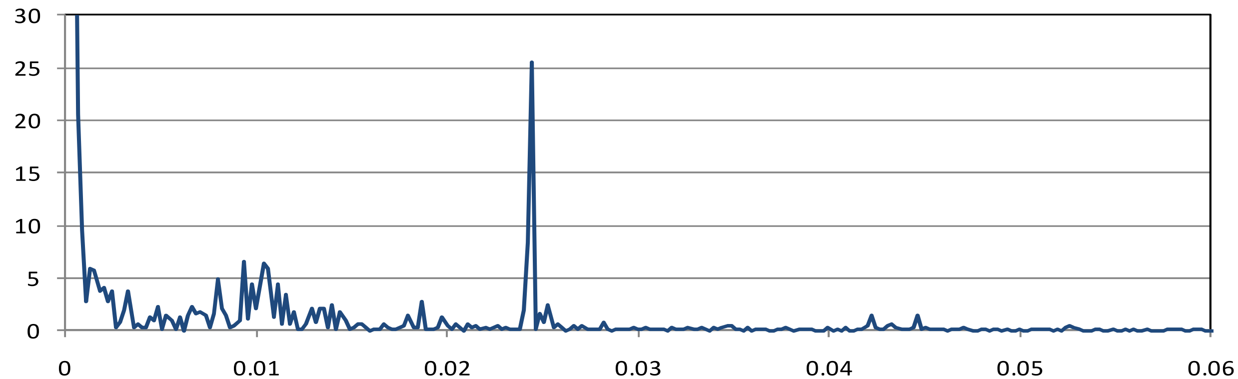




Note period of about 41 Kyr.

Lisiecki–Raymo $\delta^{18}O$ Stack





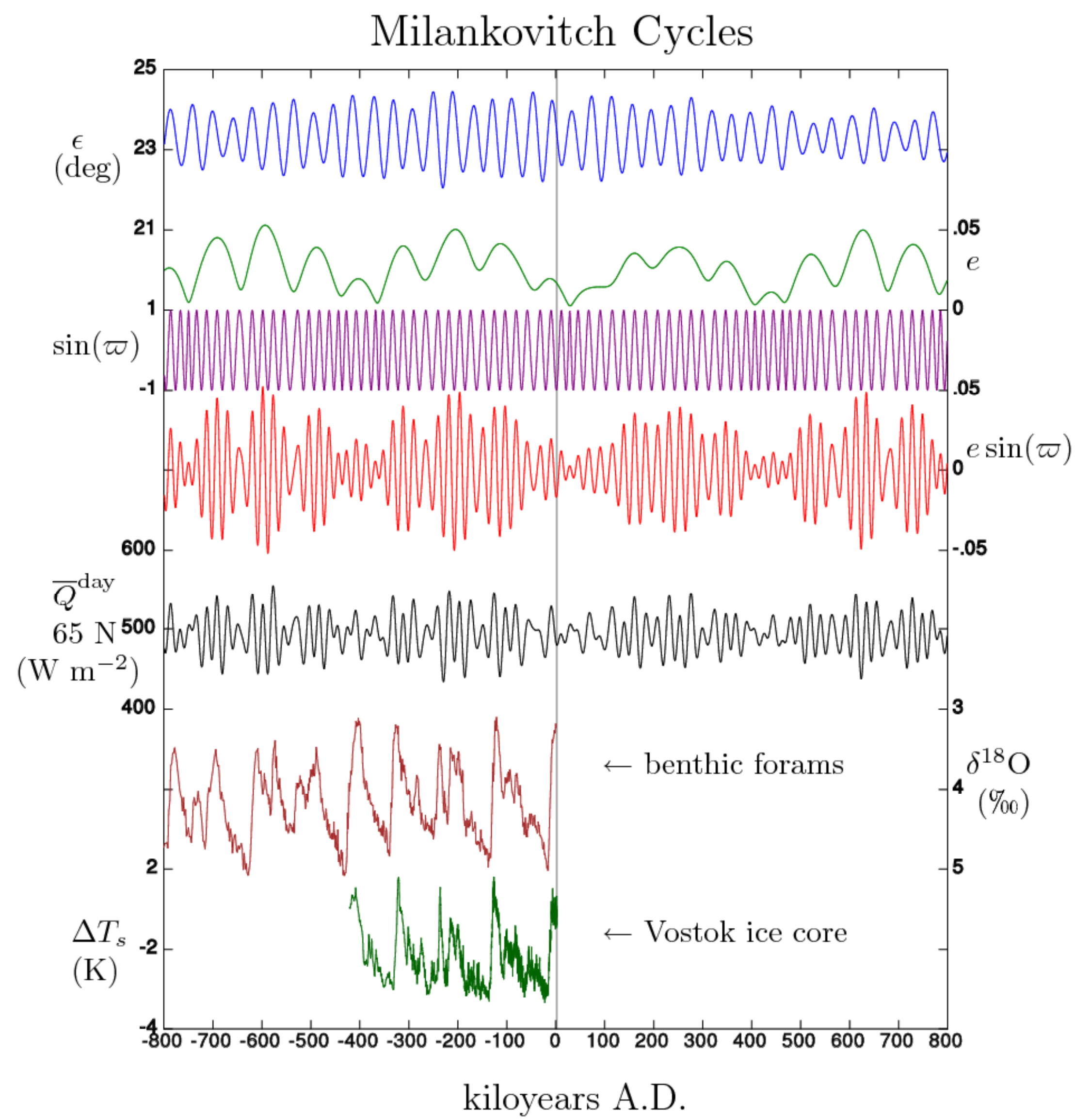
Note dominance of 41 Kyr period (obliquity)

Milankovitch Insolation Proxy

How do the Milankovitch cycles combine to drive the glacial cycles?

Glaciers melt when its hot.
Most glaciers are in the northern hemisphere.
It is hottest in the northern hemisphere at summer solstice.
65° N latitude is a good place for glaciers.

Check out insolation at 65° N on summer solstice.



http://en.wikipedia.org/wiki/Milankovitch_cycles

Daily Average Insolation at Summer Solstice at 65° N

Insolation at a point on the Earth's surface

$$I(\beta,\rho,r,\theta,\varphi,\gamma)=\frac{K(e)}{4\pi r^2}\left[-\cos\varphi\left(\cos\beta\cos(\theta-\rho)\cos\gamma+\sin(\theta-\rho)\sin\gamma\right)-\sin\varphi\sin\beta\cos(\theta-\rho)\right]^+$$

(φ,γ) = (latitude, longitude)

(r,θ) = position of Earth in orbital plane

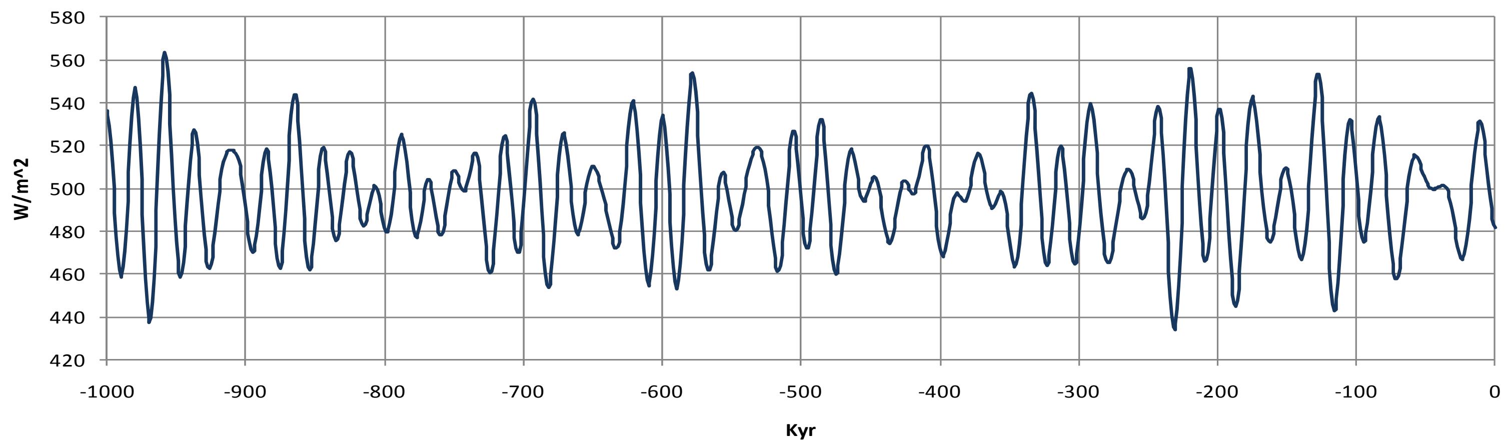
β = obliquity angle

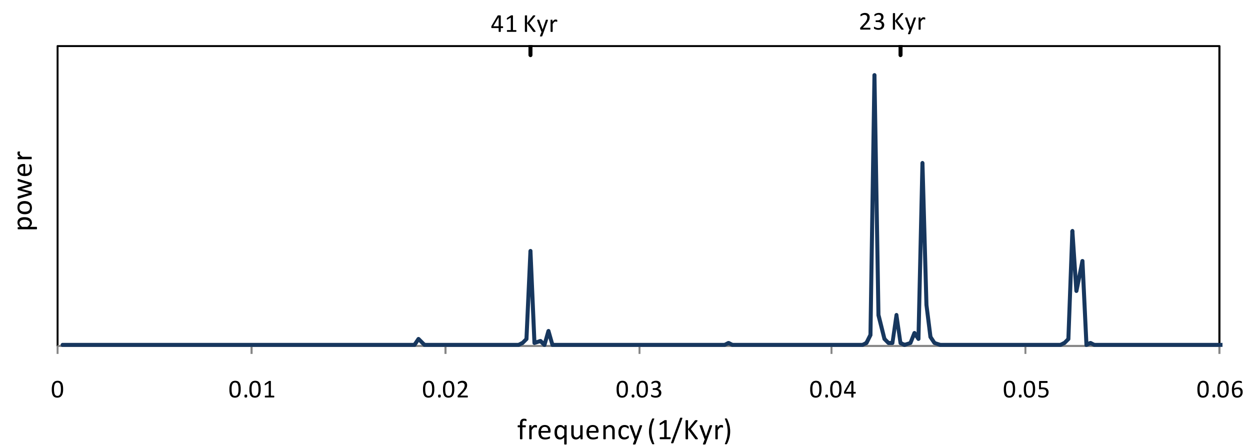
ρ = precession angle

Daily average insolation at latitude φ at summer solstice

$$\bar{I}(e,\beta,\rho',\varphi)=Q_0\frac{(1-e\sin\rho')^2}{(1-e^2)^2}\frac{1}{2\pi}\int_0^{2\pi}\left[\cos\varphi\cos\beta\cos\gamma+\sin\varphi\sin\beta\right]^+d\gamma$$

Daily Average Insolation at Summer Solstice at 65° N





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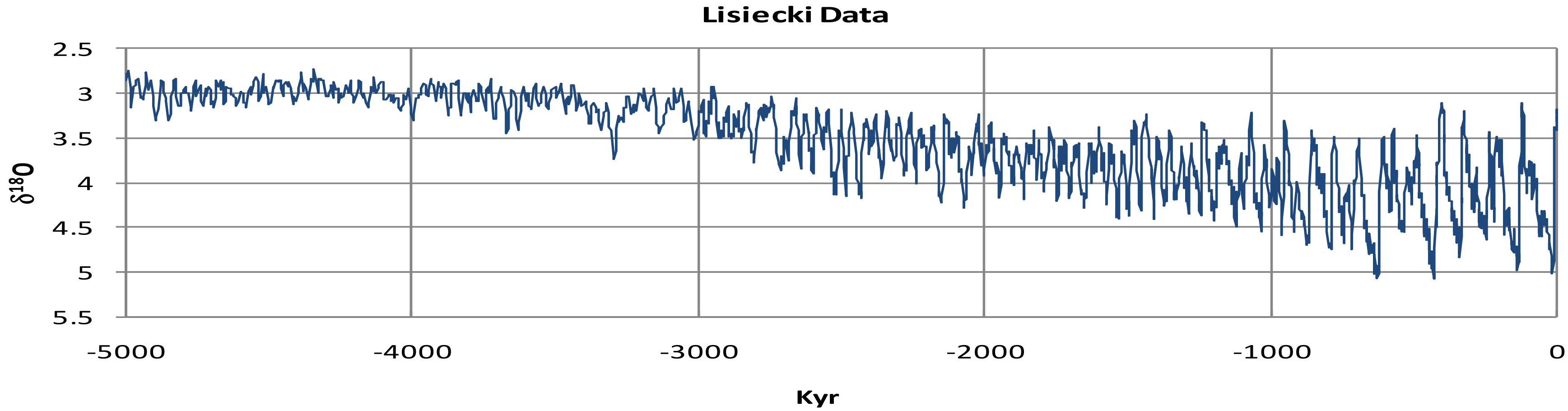
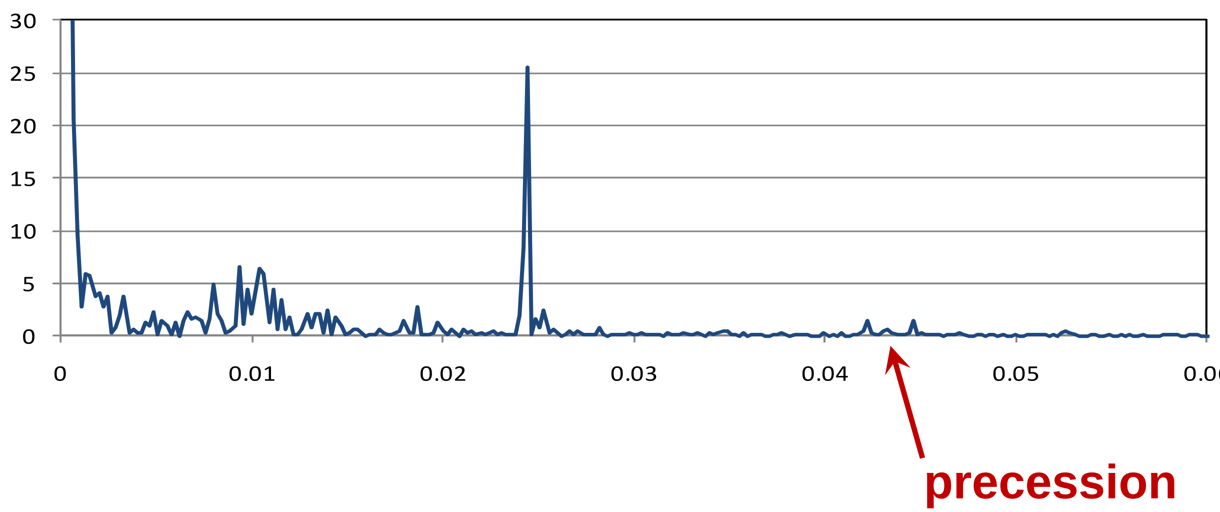
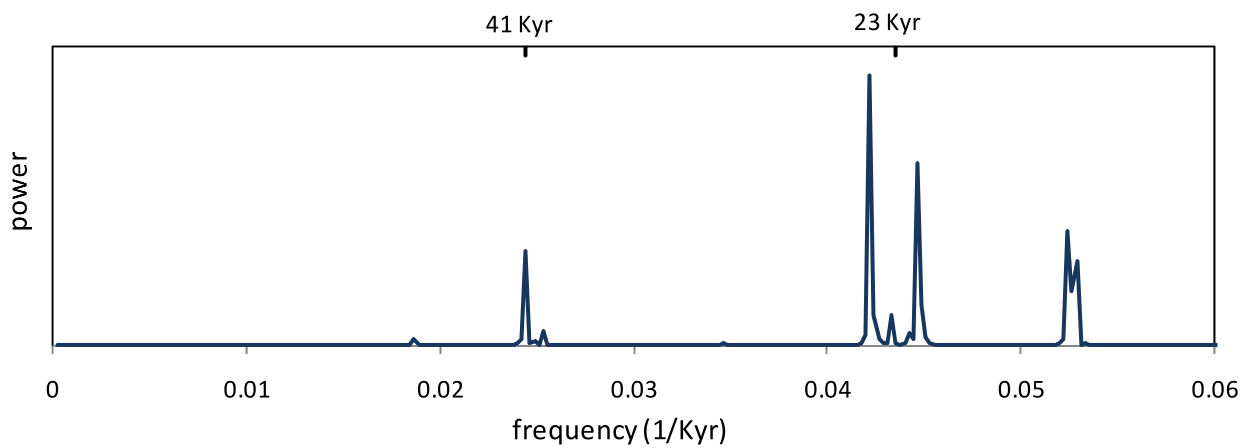
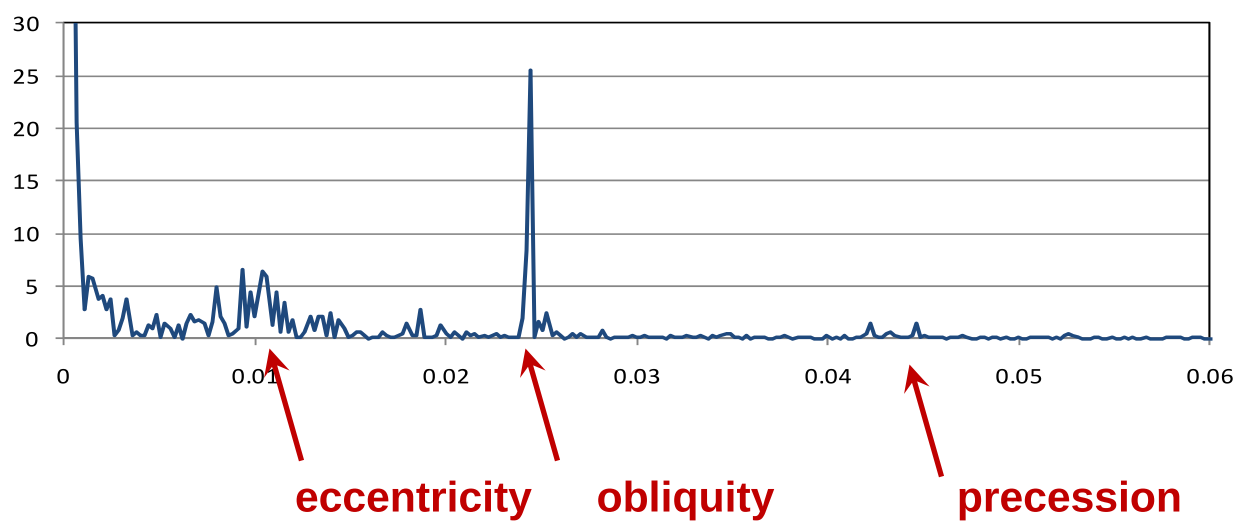






Q^{65}
(insolation proxy)



data

Back to Budyko

$$R\frac{\partial T}{\partial t} = Qs(y)\left(1-\alpha(y)\right) - \left(A+BT\right) + C\left(\bar{T}-T\right)$$

$$Q = Q(e) = \frac{Q_0}{\sqrt{1-e^2}}$$

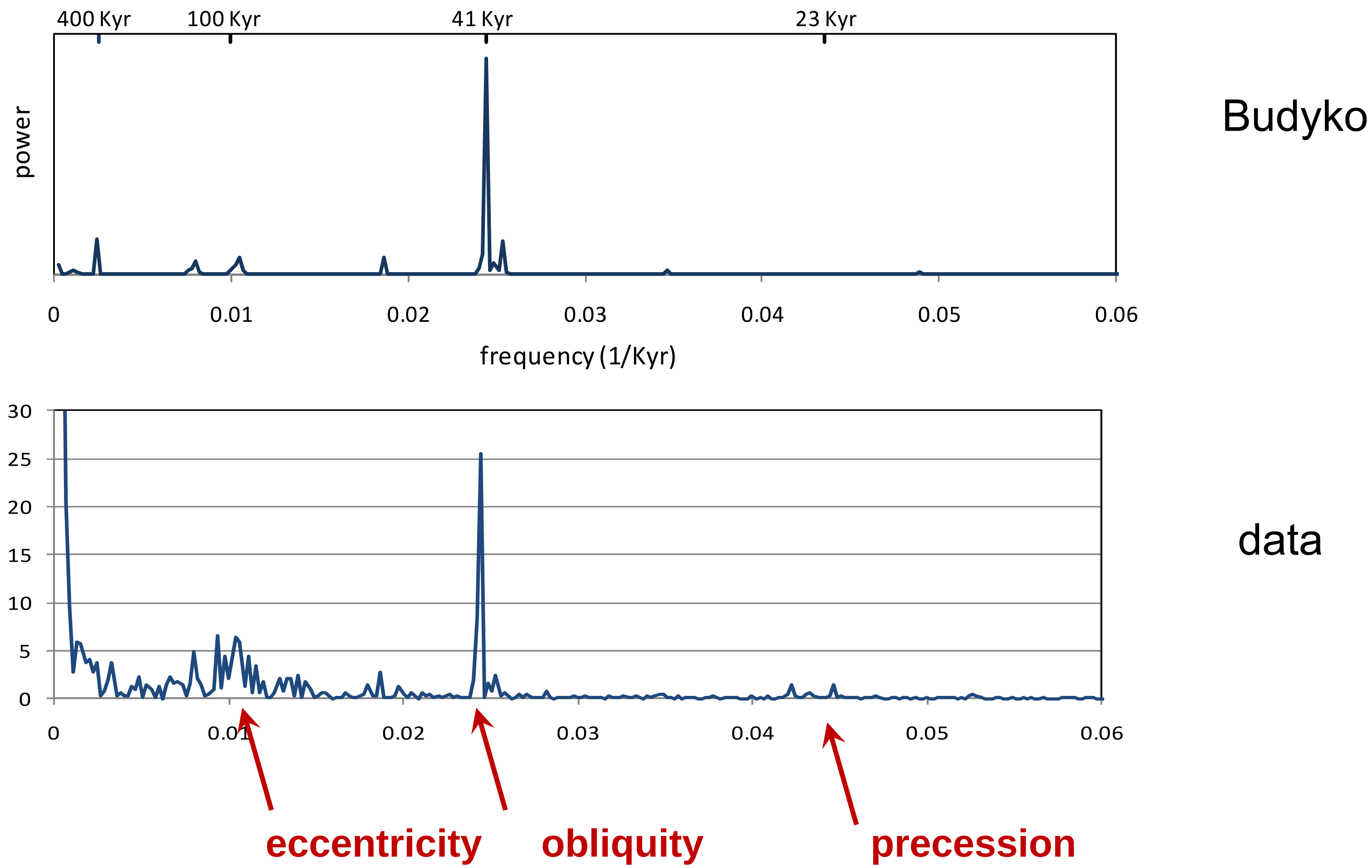
$$s(y) = s(y,\beta) = \frac{2}{\pi^2}\int_0^{2\pi}\sqrt{1-\left(\sqrt{1-y^2}\sin\beta\cos\gamma - y\cos\beta\right)^2}\,d\gamma$$

e = eccentricity

β = obliquity

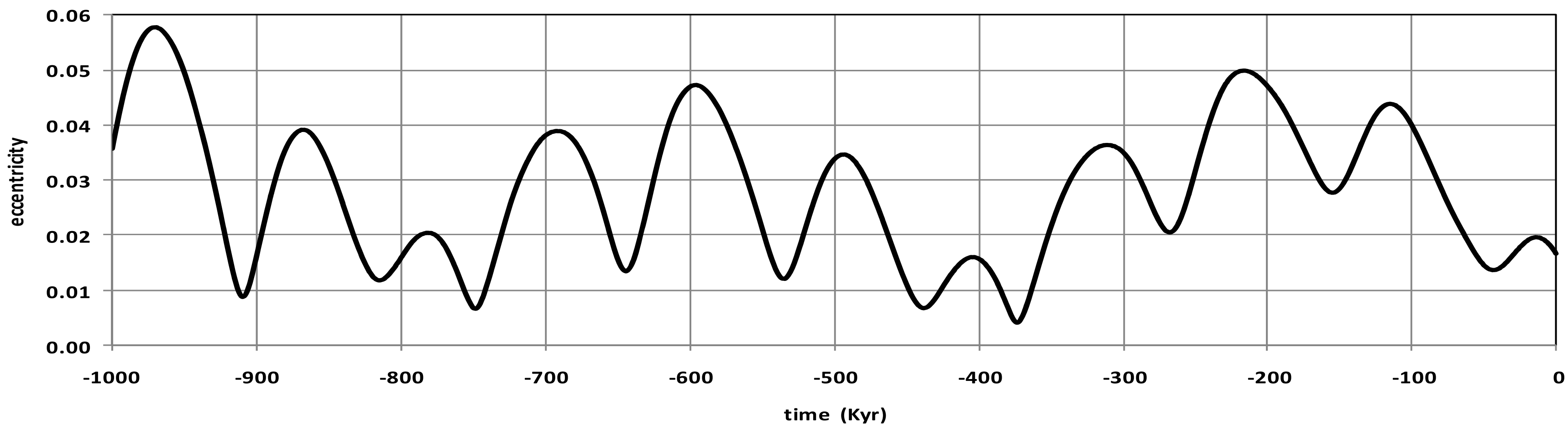
Note that Q depends only on eccentricity, $s(y)$ depends only on obliquity, and nothing depends on precession.

Budyko Forced by Milankovitch



Eccentricity

$$Q(e) = \frac{Q_0}{\sqrt{1-e^2}}$$



The annual average effect due to eccentricity is not that much:

As e varies between 0 and 0.06, $(1\text{-}e^2)^{-1/2}$ varies between 1 and 0.0018, or about 0.2%.

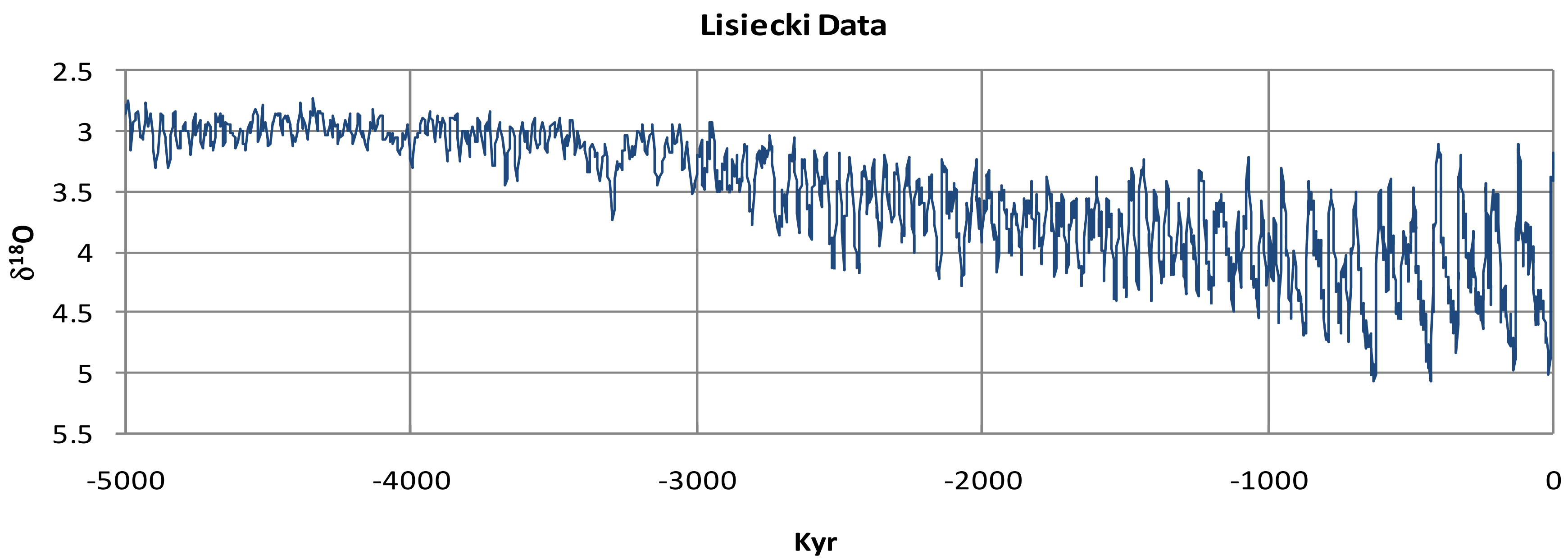
Interesting Open Questions

Why is the climate dominated by 41 Kyr cycles (obliquity) 5 Myr ago, but dominated by 100 Kyr cycles (eccentricity) during the last million years?

What changed? (The answer does not seem to be the Milankovitch cycles.)

If eccentricity has been forcing the climate for the last million years, what happened to 400 Kyr cycle?

Interesting Open Questions



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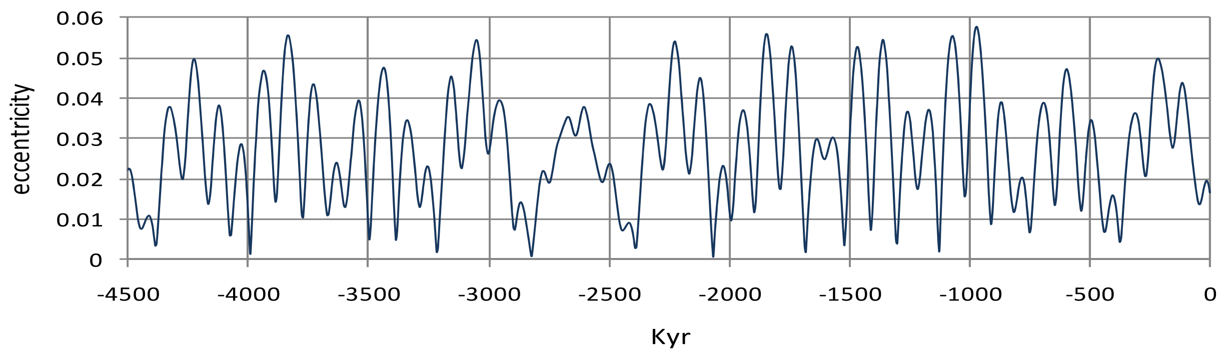

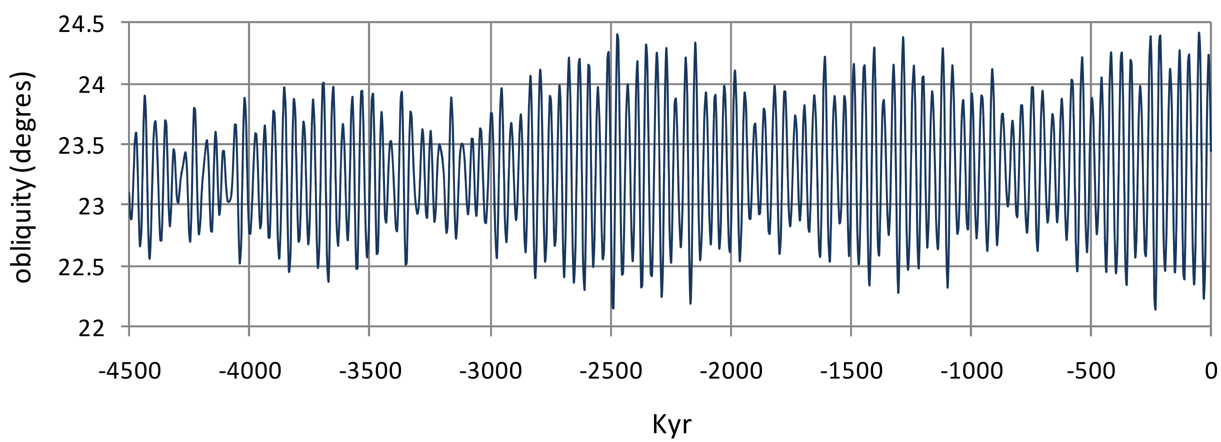

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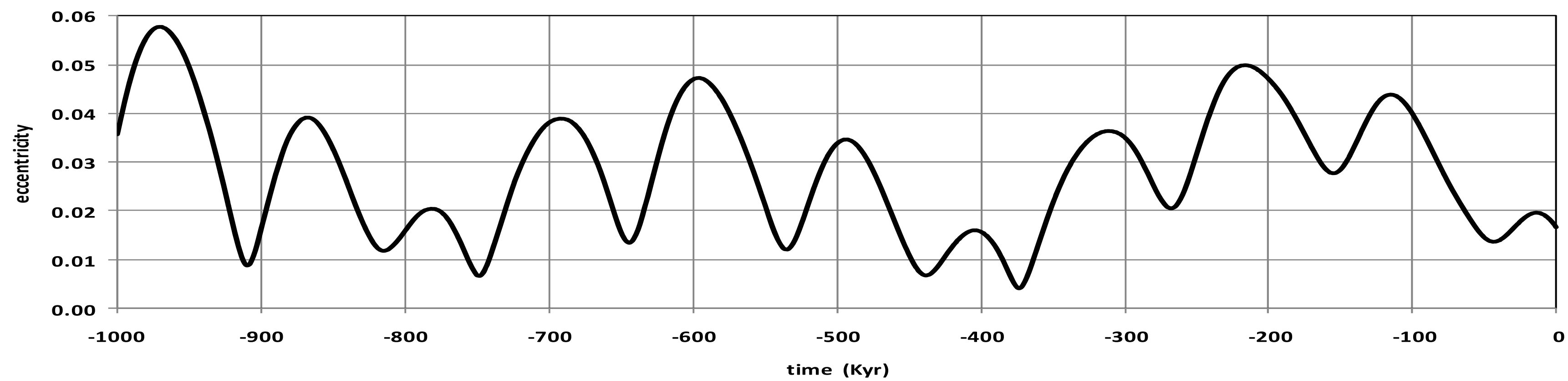

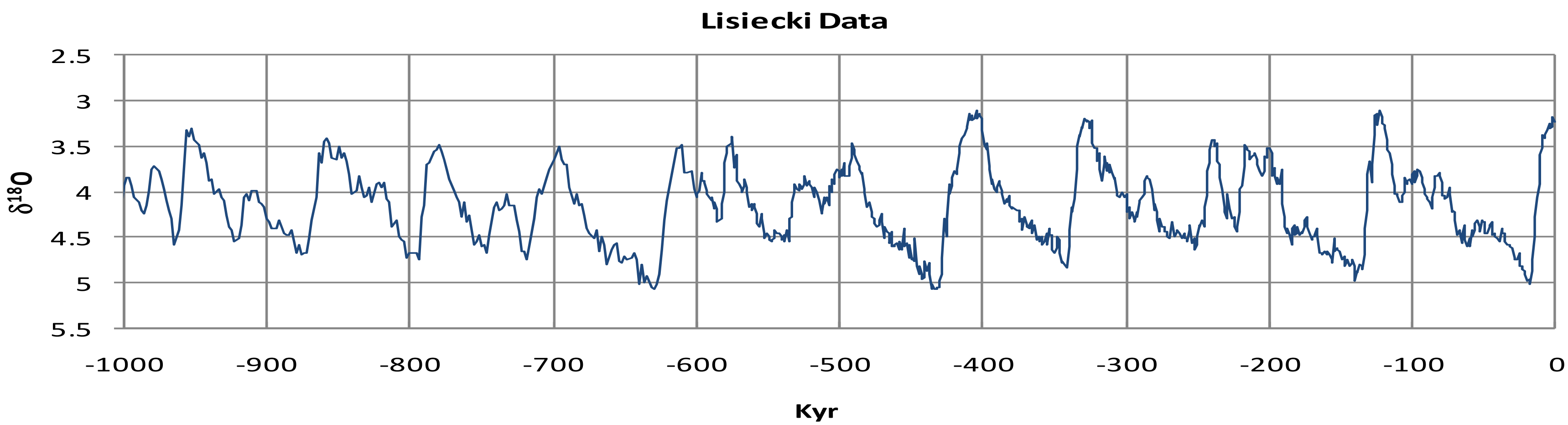

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