Geodynamo Simulation using Yin-Yang grid on Earth Simulator

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Visualization of Geomagnetic Field

Aurora taken from Space Shuttle, May 1991.

http://commons.wikimedia.org/wiki/Image:Aurora-SpaceShuttle-E0.jpg
Geomagnetic Field

Ring current in the core

I = $10^9$ A
Earth’s structure

- Mantle (rock)
- Outer Core (liquid iron)
- Inner Core (solid iron)
- Convection
## Outer Core as an MHD Fluid

<table>
<thead>
<tr>
<th>Outer core radius</th>
<th>$R_o = 3.5 \times 10^6 m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal diffusivity</td>
<td>$\kappa \sim 5 \times 10^{-6} m^2/sec$</td>
</tr>
<tr>
<td>Viscosity</td>
<td>$\nu \sim 1 \times 10^{-7} m^2/sec$</td>
</tr>
<tr>
<td>Electrical resistivity</td>
<td>$\eta \sim 2 m^2/sec$</td>
</tr>
<tr>
<td>Mass density</td>
<td>$\rho \sim 1 \times 10^4 kg/m^3$</td>
</tr>
<tr>
<td>Poloidal magnetic field</td>
<td>$B = O(10^{-4}) T$</td>
</tr>
<tr>
<td>Flow velocity</td>
<td>$V \sim O(10^{-4}) m/sec$</td>
</tr>
<tr>
<td>Sound velocity</td>
<td>$V_s \sim O(10^4) m/sec$</td>
</tr>
<tr>
<td>Alfvén velocity</td>
<td>$V_A \sim O(10^{-3}) m/sec$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Non dimensional parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rayleigh number</td>
<td>$Ra \sim 10^{30}$</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>$Re = VR_o/\nu \sim O(10^9)$</td>
</tr>
<tr>
<td>Magnetic Reynolds</td>
<td>$Rm = VR_o/\eta \sim O(10^3)$</td>
</tr>
<tr>
<td>Magnetic Prandtl</td>
<td>$Pm = 5 \times 10^{-6}$</td>
</tr>
<tr>
<td>Prandtl number</td>
<td>$Pr = 0.2$</td>
</tr>
<tr>
<td>Ekman number</td>
<td>$E = \nu/\Omega R_o^2 = O(10^{-15})$</td>
</tr>
<tr>
<td>Elsasser number</td>
<td>$\Lambda = B^2/2\Omega \mu_0 \rho \eta \sim O(10)$</td>
</tr>
</tbody>
</table>

Magnetic energy density / Flow energy density

$$= \left( \frac{B^2}{2 \mu_0} \right) / \left( \frac{\rho V^2}{2} \right) = (V_A/V)^2 \sim O(10^2)$$
Computer Simulation of Geodynamo

- MHD simulation of the outer core
- DNS with real parameters are impossible.
  - “Hyperdiffusivity” approach was popular, but not now.
  - DNS with *quite* higher viscosity.
- A barometer of the distance to the real core:
  - Ekman number $E$
    - 1995: $E = O(10^{-4})$
    - 1998: $E = O(10^{-5})$
    - 2005: $E = O(10^{-6})$
    - $\ldots$
    - $???: E = O(10^{-15})$

“Already” succeeded to reproduce:
- Dipole field generation
- Intermittent reversals
Marvel…
Ekman number

\[ E = \nu / \Omega R^2_o = O(10^{-15}) \]
Computer Simulation of Geodynamo

- Outer core (a spherical shell between two spheres).
- Rotation.
- MHD fluid
- Temperature difference between the spheres.
- Gravity
- Initial condition: random & weak “seed” of magnetic field and temperature as a perturbation.
- MHD eqs.

→ Thermal convection of the MHD fluid
→ MHD dynamo
Compressible MHD equations

\[ \frac{\partial \rho}{\partial t} = -\nabla \cdot f, \]
\[ \frac{\partial f}{\partial t} = -\nabla \cdot (vf) - \nabla p + j \times B + \rho g + 2\rho v \times \Omega + \mu (\nabla^2 v + \frac{1}{3} \nabla (\nabla \cdot v)), \]
\[ \frac{\partial p}{\partial t} = -v \cdot \nabla p - \gamma p \nabla \cdot v + (\gamma - 1) K \nabla^2 T + (\gamma - 1) \eta j^2 + (\gamma - 1) \Phi, \]
\[ \frac{\partial A}{\partial t} = -E, \]

with

\[ B = \nabla \times A, j = \nabla \times B, E = -v \times B + \eta j, \]
\[ p = \rho T, g = -g_0/r^2 \hat{r}, \Phi = 2\mu (\epsilon \cdot \epsilon - (\nabla \cdot v)^2 / 3). \]
Our Old Simulations in 1995 -- 1997

- On the latitude-longitude (lat-lon) grid
- Finite Difference Method
- NEC SX-4 (vector processor)
- Typical grid size and params:

\[ N_r \times N_\theta \times N_\phi = 50 \times 38 \times 128 \]

\[ E_k = 2 \times 10^{-4} \]

\[ Re = O(10^2) \]
Simulation Results
(moderate convection dynamo)

Vorticity isosurfaces (cyclones and anti-cyclones)
Generated magnetic field in our simulation

Magnetic field lines starting from the Earth’s surface.

Magnetic field lines starting from the core surface.
Dipole Field Reversals
Generation Mechanism of Dipole Field
Generation Mechanism of Dipole Field

Colored line: A magnetic field line

White arrows: Convection flow vectors

Generation Mechanism of Dipole Field

current

current
Generation Mechanism of Dipole Field

My plan was:
(1) Port this code to Earth Simulator
(2) Use maximum nodes of ES to
   - increase the resolution and
   - decrease Ekman number

Earth Simulator

- Peak performance/AP: 8Gflops
- Peak performance/PN: 64Gflops
- Shared memory/PN: 16GB

- Total number of APs: 5120
- Total number of PNs: 640
- Max usable PNs: 512
- Total peak performance: 40TFLOPS
- Total main memory: 10TB

Interconnection Network (full crossbar switch)

Processor Node #0

Processor Node #1

Processor Node #639
But...

- Encountered difficulty to achieve high performance on ES.

- The difficulty comes from the base grid system (lat-lon grid).
Numerical Problems of Lat-Lon Grid

- On the poles
  - Coordinate singularity
  - No problem
  - L’Hospital’s theorem

- Near the poles
  - Severe CFL condition
  - Serious problem
  - Needs spherical filter
Spherical Filter

Retain the grids, but drop useless information.

→ Filtering
Inefficiency of Lat-Lon Grid

- Needed optimized parallel spherical filter (with FFT).
  -- bottleneck
- Even if you could make highly optimized spherical filter, the lat-lon grid + spherical filter method is computationally inefficient.
  (1) Place many grid points near the poles, spoiling the low-latitude’s resolution.
  (2) Work hard to calculate data on all the grids.
  (3) Throw away most of the data!

This is true for other spherical discretization methods:
  - Double FFT spectral method (FFT both in latitude & longitude).
  - Single FFT, hybrid method (FD in latitude & FFT in longitude).
Grid Convergence Near the Poles

84% of grid points are located in high-latitude part (>45° N and S).

Only 16% grids cover the low-latitude part (between 45° N and S)
Quest for new spherical grid

Let’s re-view the lat-lon grid

It is almost ideal grid in the low latitude region.
- It is orthogonal coordinates (simple metrics)
- Nearly uniform grid spacing

This picture reminds us a baseball!
Baseball

A spherical surface is covered by a pair of two identical parts (patches).

It has only one seam.
Can we combine two identical component grids to cover a spherical surface, like the baseball?
Can we combine two identical component grids to cover a spherical surface, like the baseball?
“Yin-Yang Grid”

The yin-yang symbol of complemental relation
Two Component Grids of Yin-Yang:
Yin grid & Yang grid
Suppose a point on the sphere with Yin (or n) coordinates given by
\[(x^n, y^n, z^n),\]
and with Yang (or e) coordinates given by
\[(x^e, y^e, z^e).\]
Their relation is given by
\[(x^n, y^n, z^n) = (-x^e, z^e, y^e),\]
or
\[(x^e, y^e, z^e) = (-x^n, z^n, y^n).\]

The coordinates relations are symmetric (complementary).
Concise Coding of Yin-Yang Grid

- Make one routine on the (partial) latitude-longitude grid.
- Recycle it for two times; one for Yin and one for Yang.

Routines for
- MHD solver
- boundary conditions
- interpolations
Yin-Yang grid is an Overset Grid in Spherical Geometry

Partial overlap between Yin and Yang grids.

Interpolation
boundary condition
Two Independent Component Grids
Two independent grids
Overset grid (Chimera grid) method

- “Divide and conquer” approach
- Partially overlapped meshes.
- Setting boundary values by mutual interpolations.
- Essentially parallel computation.

3D Yin-Yang Grid for Spherical Shell Geometry
Applications of Yin-Yang Grid at Earth Simulator Center

(a) Coupled GCM of atmosphere & ocean
(b) Fast spherical Poisson eq. solver by multigrid on Yin-Yang grid
(c) Mantle convection simulation
(d) Geodynamo
Yin-Yang Variations

Minimize the partial overlap region.

Dissection of a sphere into two identical parts.
Dissection of a Sphere into Two Identical Parts
Yin-Yang grids with minimum overlap
Vector-Parallel Processing on Yin-Yang Grid

2-dimensional domain decomposition in the horizontal directions.

Vectorization in the radial direction
2 MPI Communicators

(1) **Overall (world) communicator**
(2) **Yin/Yang communicator**
   - Yin’s communicator
   - Yang’s communicator
Performance of the Yin-Yang geodynamo code on ES

<table>
<thead>
<tr>
<th>processors</th>
<th>grid points</th>
<th>Tflops</th>
<th>efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>4096</td>
<td>$511 \times 514 \times 1538 \times 2$</td>
<td>15.2</td>
<td>46%</td>
</tr>
<tr>
<td>3888</td>
<td>$511 \times 514 \times 1538 \times 2$</td>
<td>13.8</td>
<td>44%</td>
</tr>
<tr>
<td>3888</td>
<td>$255 \times 514 \times 1538 \times 2$</td>
<td>12.1</td>
<td>39%</td>
</tr>
<tr>
<td>2560</td>
<td>$511 \times 514 \times 1538 \times 2$</td>
<td>10.3</td>
<td>50%</td>
</tr>
<tr>
<td>2560</td>
<td>$255 \times 514 \times 1538 \times 2$</td>
<td>9.17</td>
<td>45%</td>
</tr>
<tr>
<td>1200</td>
<td>$255 \times 514 \times 1538 \times 2$</td>
<td>5.40</td>
<td>56%</td>
</tr>
</tbody>
</table>

*Flat MPI*
# Performance Comparison of Simulations on ES

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Flops/PN</td>
<td>26.6T/640</td>
<td>16.4T/512</td>
<td>14.9T/512</td>
<td>5T/243</td>
<td>15.2T/512</td>
</tr>
<tr>
<td>efficiency</td>
<td>65%</td>
<td>50%</td>
<td>45%</td>
<td>32%</td>
<td>46%</td>
</tr>
<tr>
<td>grid points (g.p.)</td>
<td>$7.1 \times 10^8$</td>
<td>$8.6 \times 10^9$</td>
<td>$1.7 \times 10^{10}$</td>
<td>$5.5 \times 10^9$</td>
<td>$8.1 \times 10^8$</td>
</tr>
<tr>
<td>g.p./AP</td>
<td>$1.4 \times 10^5$</td>
<td>$2.1 \times 10^6$</td>
<td>$4.2 \times 10^6$</td>
<td>$2.8 \times 10^6$</td>
<td>$2.1 \times 10^5$</td>
</tr>
<tr>
<td>Flops/g.p.</td>
<td>38K</td>
<td>1.9K</td>
<td>0.87K</td>
<td>0.91K</td>
<td>19K</td>
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<tr>
<td>Simulation kind</td>
<td>fluid</td>
<td>fluid</td>
<td>fluid</td>
<td>wave propagation</td>
<td>fluid</td>
</tr>
<tr>
<td>Field</td>
<td>atmosphere</td>
<td>turbulence</td>
<td>inertial fusion</td>
<td>seismic wave</td>
<td>geodynamo</td>
</tr>
<tr>
<td>Method</td>
<td>spectral</td>
<td>spectral</td>
<td>finite volume</td>
<td>spectral element</td>
<td>finite difference</td>
</tr>
<tr>
<td>Parallelization</td>
<td>MPI-microtask</td>
<td>MPI-microtask</td>
<td>HPF (flat MPI)</td>
<td>flat MPI</td>
<td>flat MPI</td>
</tr>
</tbody>
</table>

TABLE III: Performances on the Earth Simulator reported at SC

Yin-yang geodyamo code
Rules I took in the Code Development

• Avoid global communication
• Keep the model simple and symmetric
  – Grid system ==> Yin-Yang grid
  – Parallelization ==> Flat MPI
    • Use MPI for both intra- and inter- nodes communications
    • If inter-node network speed is high enough, this is the simplest programming model for the programmer.
No Global Communication

- **Trade off**
  - Un(geo)physical model
    - Boundary condition of the magnetic field
      - Magnetic field has only radial component
    - Compressibility
      - Most in the community use the Boussinesq model
- **Effects of compressibility should be negligible when Mach number is small.**
- **We have compressible mode, but we effectively reduced the sound wave speed.**
Simulation Parameters

- Grid mesh (Yin-Yang grid)

\[ N_r \times N_\theta \times N_\phi \times 2 = 511 \times 514 \times 1538 \times 2 \]

For 360 degree equator ==> 2045 grid points
==> on the core surface: 1 grid = 10.75km

- Rayleigh number \[ Ra = 2 \times 10^8 \]

- Prandtl number \[ Pr = 1 \]

- Magnetic Prandtl number \[ Pm = 1 \]

- Ekman number \[ E = 4.6 \times 10^{-7} \]
Time Development of Energies

Convection energy

Magnetic energy
Distribution of Dynamo Source $D$

\[ D = -\mathbf{v} \cdot (\mathbf{j} \times \mathbf{B}) \]

Temperature in the equatorial plane
Distribution of $D$ and $j$

Dynamo source $D$ (green) and electric current lines (blue).

Current coils
Current Coil and Magnetic Field Lines

$B$  
j

helical spring
Virtual Reality Visualization
Dynamo due to “field line stretching” by upward flow parallel to the magnetic field lines.

Accelerating upward flow.
Another Dynamo by Downward Flow

Green: dynamo source $D$

Field lines are drawn by downward flow.
Recent Development of Yin-Yang Grid

- Inner core
  - Another coordinate singularity at the origin.
  - Chimera method for Yin-Yang and Cartesian.
Recent Development of Yin-Yang Grid
Summary and Lessons

- For a new computer of new architecture, re-design the simulation method from the beginning
  - Yin-Yang grid enables us to perform max node run on the Earth Simulator.
  - No other (spectral-based) geodynamo code cannot run on maximum nodes.
- Yin-Yang geodynamo code has led us to a new dynamo regime.
  - Lowest Ekman number ever achieved.
  - Current coils with straight flux tubes.
- For massively parallel computation... Avoid global communication.
To Avoid the Global Communication

- Re-formulate the model to reduce the maximum signal speed of the system.
- Nature has no global communication anyway. (Speed of light.)
- My idea: Go back or change the basic formulation to have no global communication, and reduce the “speed of the light” of the system.