Case Study III: Functional ANOVA

Markov Random Fields and Regional Climate Models

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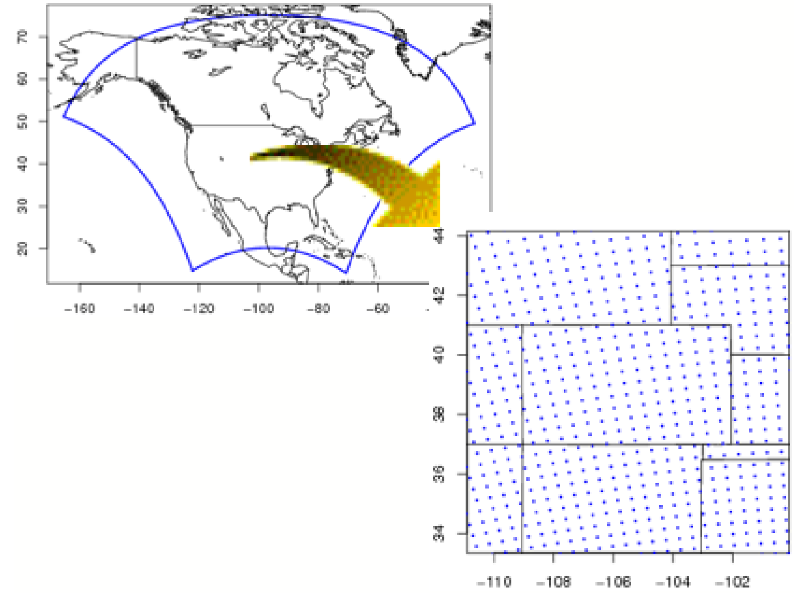
Goals

- Describe the distribution of (regional) climate model output.
- Understanding sources of variation.
  - NARCCAP/PRUDENCE: GCM, RCM, GCM$\times$RCM.
  - climateprediction.net: perturbed physics.
  - Others sources?
- Combining model output & weighting models.
- Recognizing model output represents spatial, temporal, or spatial-temporal fields $\Rightarrow$ *functional ANOVA*.

NARCCAP

- North American Regional Climate Change Assessment Program (NARCCAP)
  - NCAR, ISU, CCCma, OURANOS, LLNL, GFDL, Hadley, Scripps, PNNL, USSC, UCDHSC, etc.
  - NSF, NOAA, DOE, etc.
  - www.narccap.ucar.edu



- Systematically investigate the uncertainties in regional scale projections of future climate.

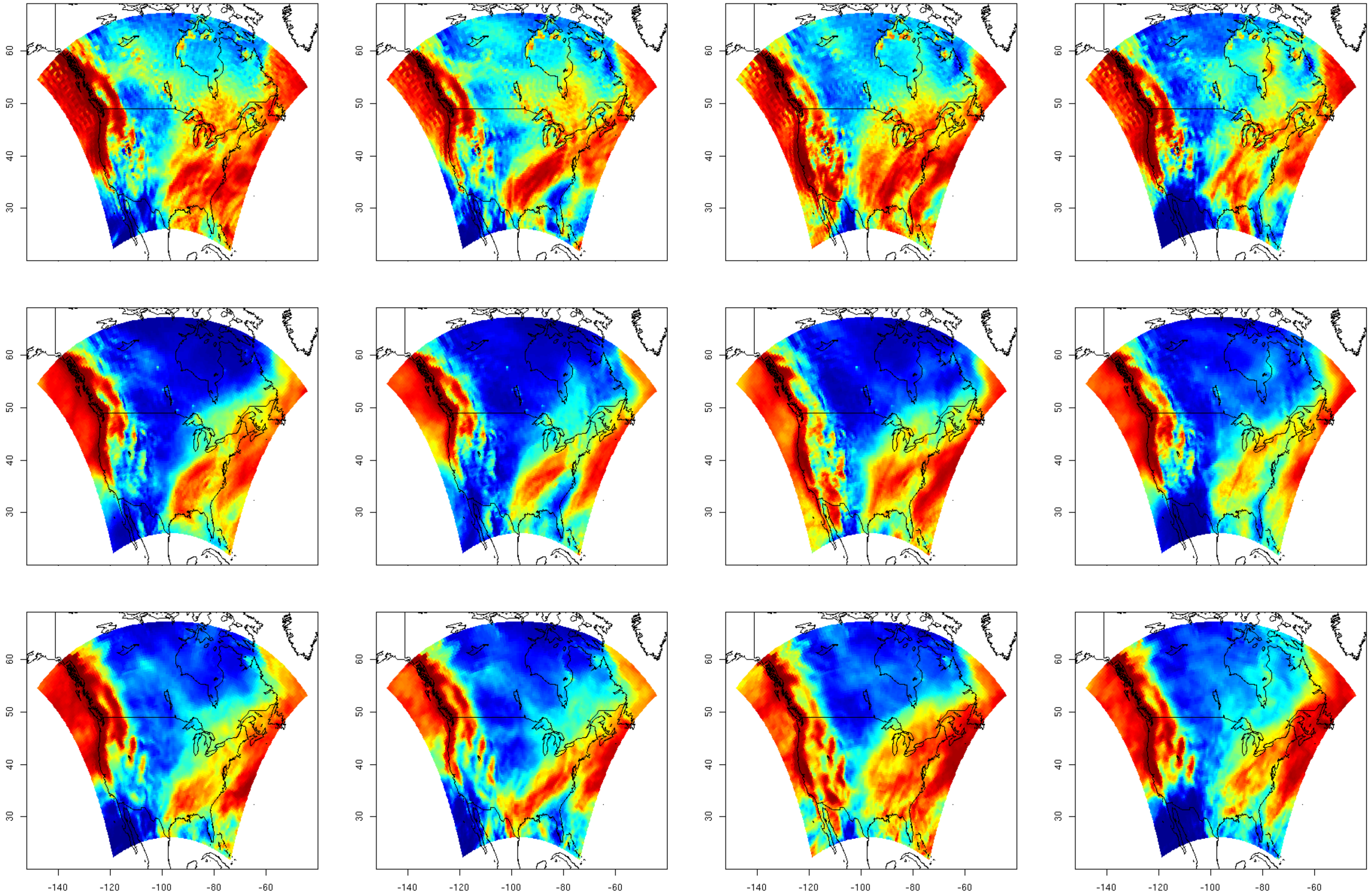
NARCCAP Design

- 4 GCMs provide boundary conditions for 6 RCMs

		GCM			
		GFDL	CGCM3	HADCM3	CCSM
	MM5			X	X
	RegCM3	X	X		
RCM	CRCM		X	X	
	PRECIS	X	X	X	X
	RSM	X			X
	WRF	X	X		X

A Work in Progress

- Three regional models – ECPC, MRCC, and RCM3
- Boundary conditions supplied by reanalysis.
- 1980-1999 (20 years)
- Total seasonal precipitation – winter (DJF) and summer (JJA)
- Common grid: $123 \times 101 = 12,423$ grid boxes


60
50
40
30
-140
-120
-100
-80
-60

A Statistical Model

- A hierarchical construction:

  Data model: $\mathbf{Y}_{ij} \sim \mathrm{N}\left(\boldsymbol{\mu}_i, \sigma_1^2 \mathbf{V}(\theta_1)\right), \ i = 1, 2, 3, \ j = 1, \ldots, 20$

  Process model: $\boldsymbol{\mu}_i \sim \mathrm{N}\left(\boldsymbol{\mu}, \sigma_2^2 \mathbf{V}(\theta_2)\right)$

  Prior model: non-informative.

- An alternative formulation:

$$\begin{aligned} \mathbf{Y}_{ij} &= \boldsymbol{\mu} + \boldsymbol{\alpha}_i + \boldsymbol{\epsilon}_{ij} \\ &= \text{Common} + \text{RCM} + \text{Error} \end{aligned}$$

A Statistical Model

- Spatial correlation matrix $\mathbf{V}(\theta) = \mathbf{R}(\theta) \otimes \mathbf{C}(\theta)$ where $\mathbf{R}$ and $\mathbf{C}$ are parameterized through 1-D "stationary" Markov random fields:

$$\mathbf{R}(\theta) = \begin{bmatrix} 1 & -\theta & & & & & \\ -\theta & 1+\theta^2 & -\theta & & & & \\ & -\theta & 1+\theta^2 & -\theta & & & \\ & & & \ddots & & & \\ & & & -\theta & 1+\theta^2 & -\theta & \\ & & & & -\theta & 1+\theta^2 & -\theta \\ & & & & & -\theta & 1 \end{bmatrix}^{-1}$$

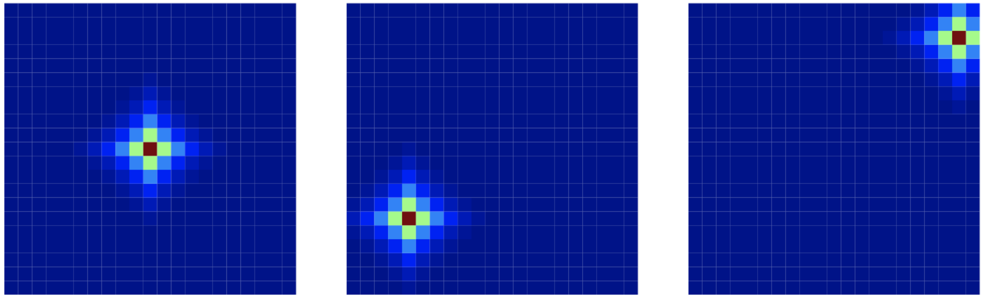
  - Computationally efficient: lower-dimensional + sparse precision matrices.
  - Other choices: tapering, reduced-rank kriging, etc.

MRF Formulation

- The conditional weight structure for an interior point:

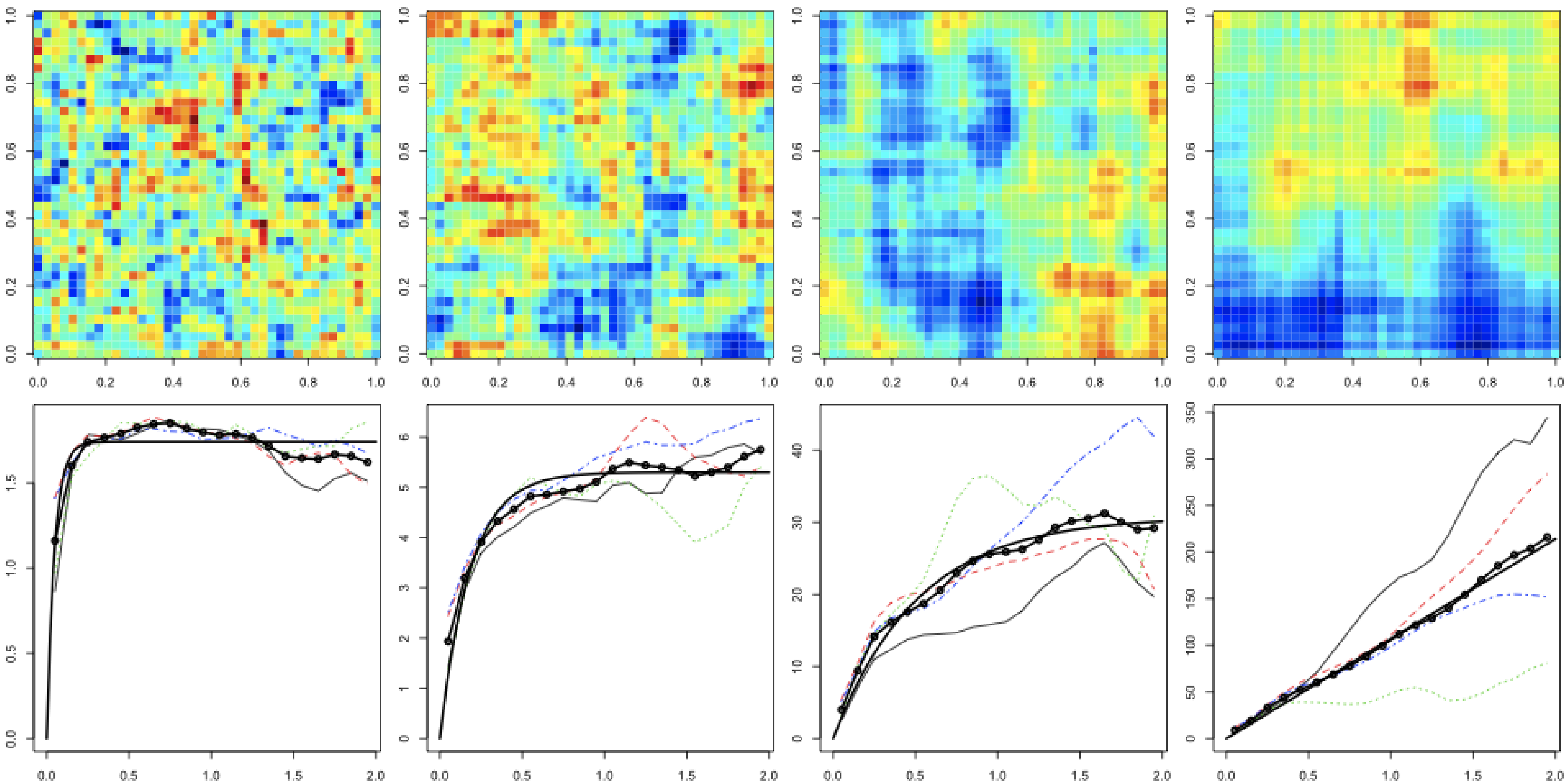
$$\frac{1}{(1+\theta^2)^2}\left\{\begin{array}{ccccccc} & & & \vdots & & & \\ & 0 & 0 & 0 & 0 & 0 & \\ & 0 & \theta^2 & -\theta(1+\theta^2) & \theta^2 & 0 & \\ \cdots & 0 & -\theta(1+\theta^2) & 0 & -\theta(1+\theta^2) & 0 & \cdots \\ & 0 & \theta^2 & -\theta(1+\theta^2) & \theta^2 & 0 & \\ & 0 & 0 & 0 & 0 & 0 & \\ & & & \vdots & & & \end{array}\right\}$$

- And the resulting correlation functions:



Another View

- Exponential-like behavior?



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$$
\begin{aligned}
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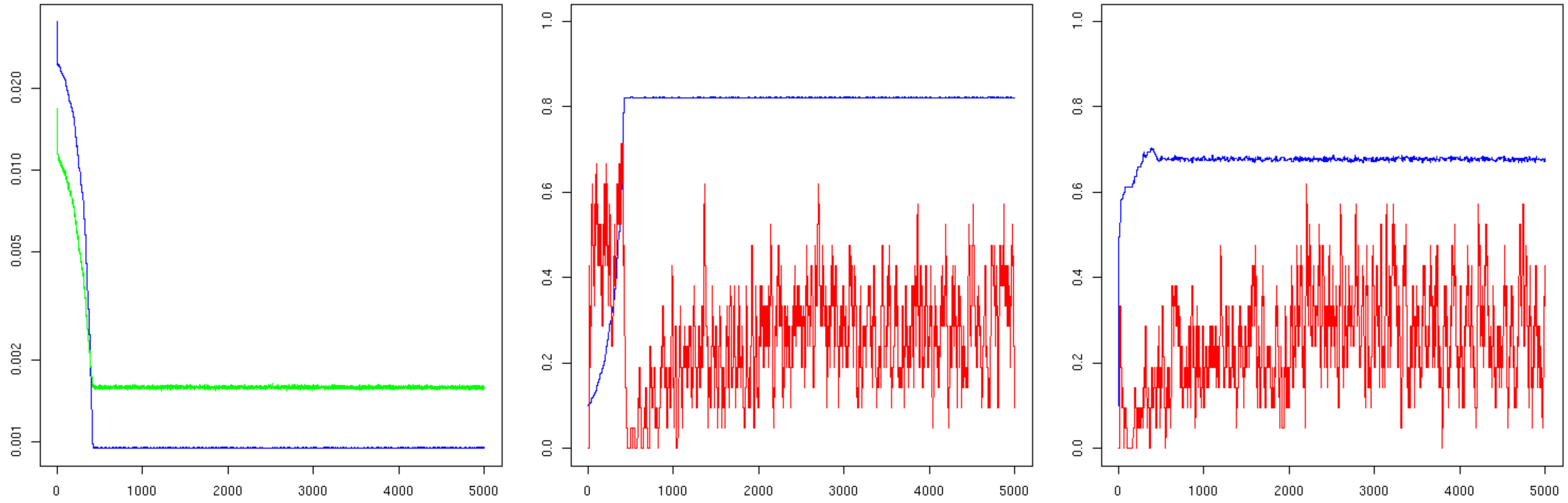
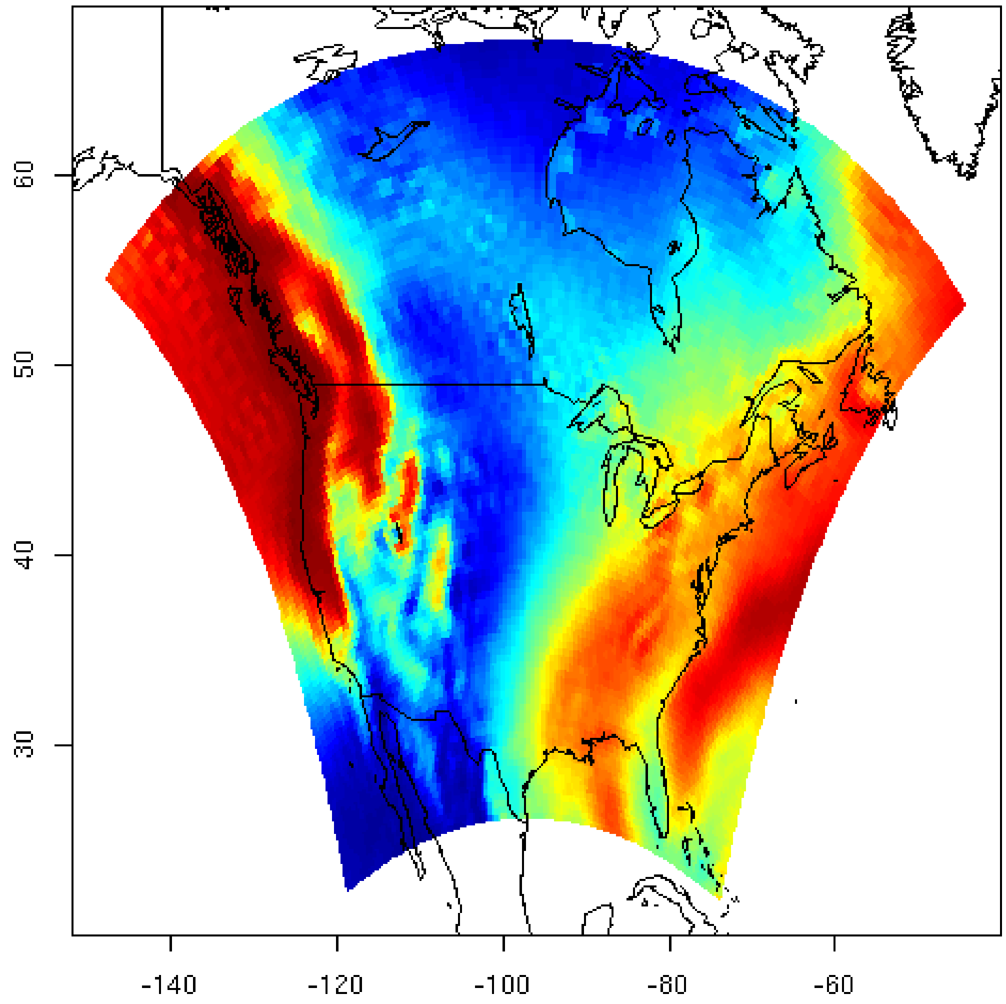
&= \text{Common} + \text{RCM} + \text{Error}
\end{aligned}
$$

Parameter Estimation

- MCMC to estimate parameters, posterior inference, etc.



Posterior Means



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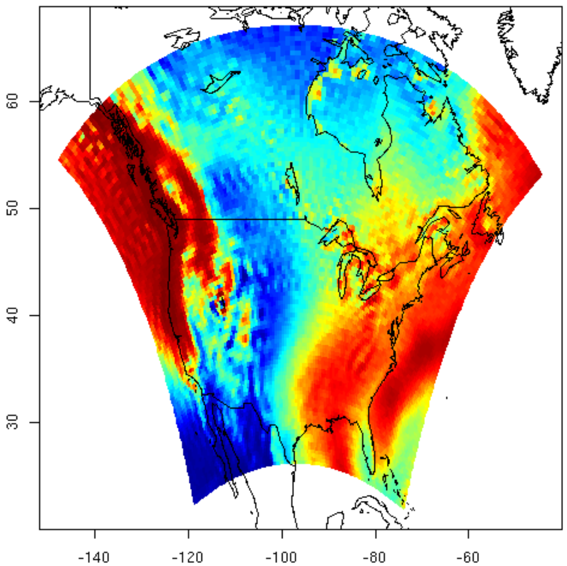

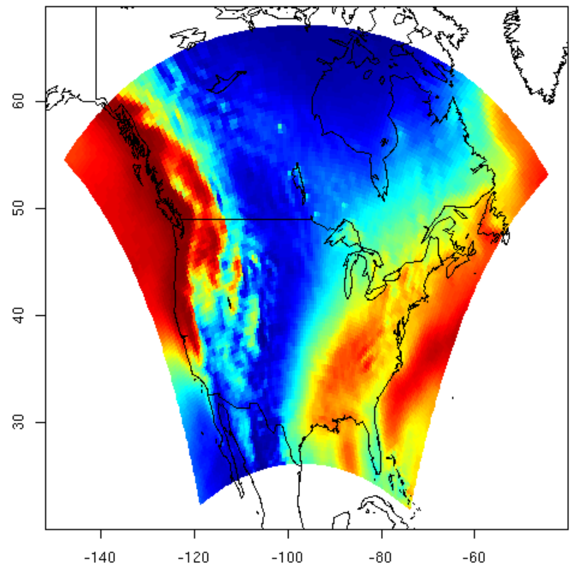

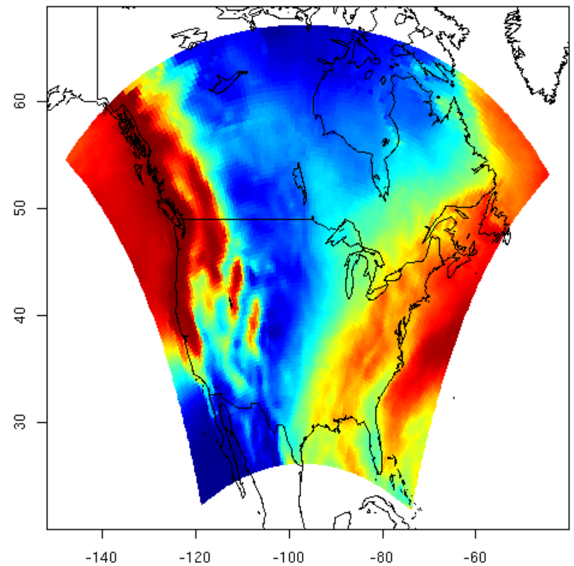

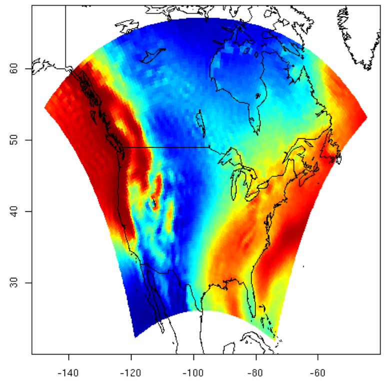

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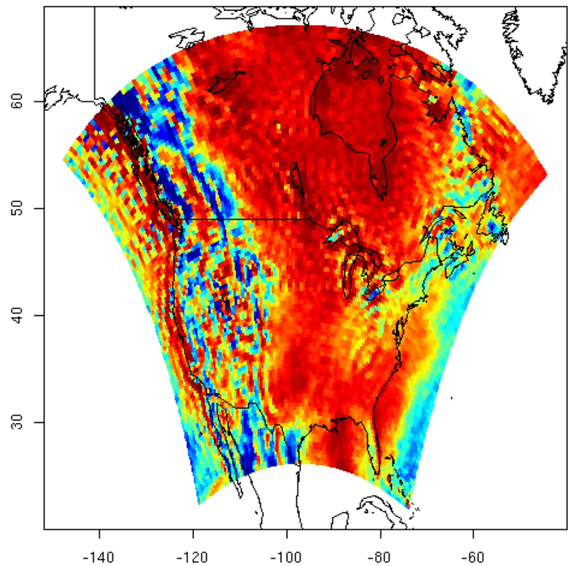
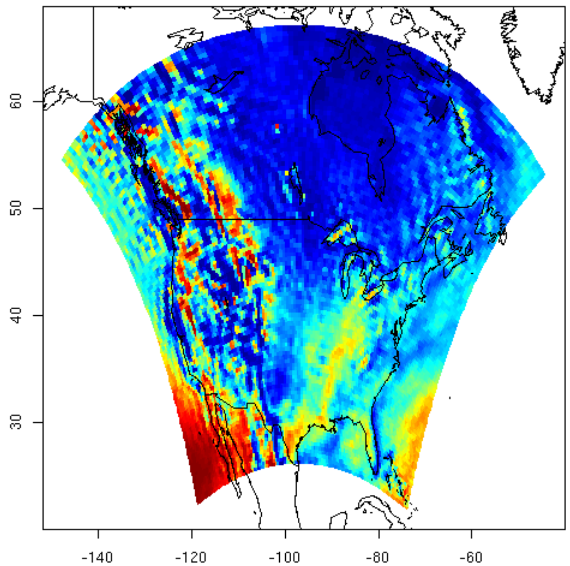
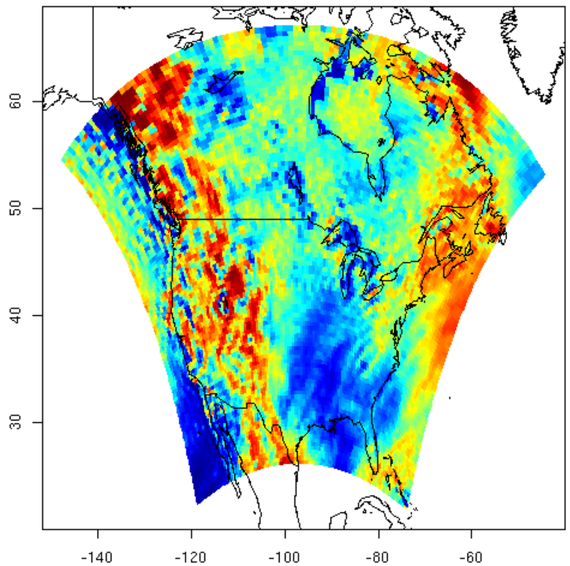
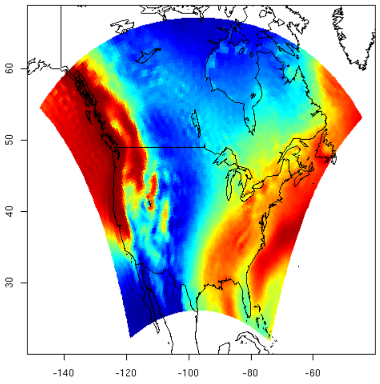




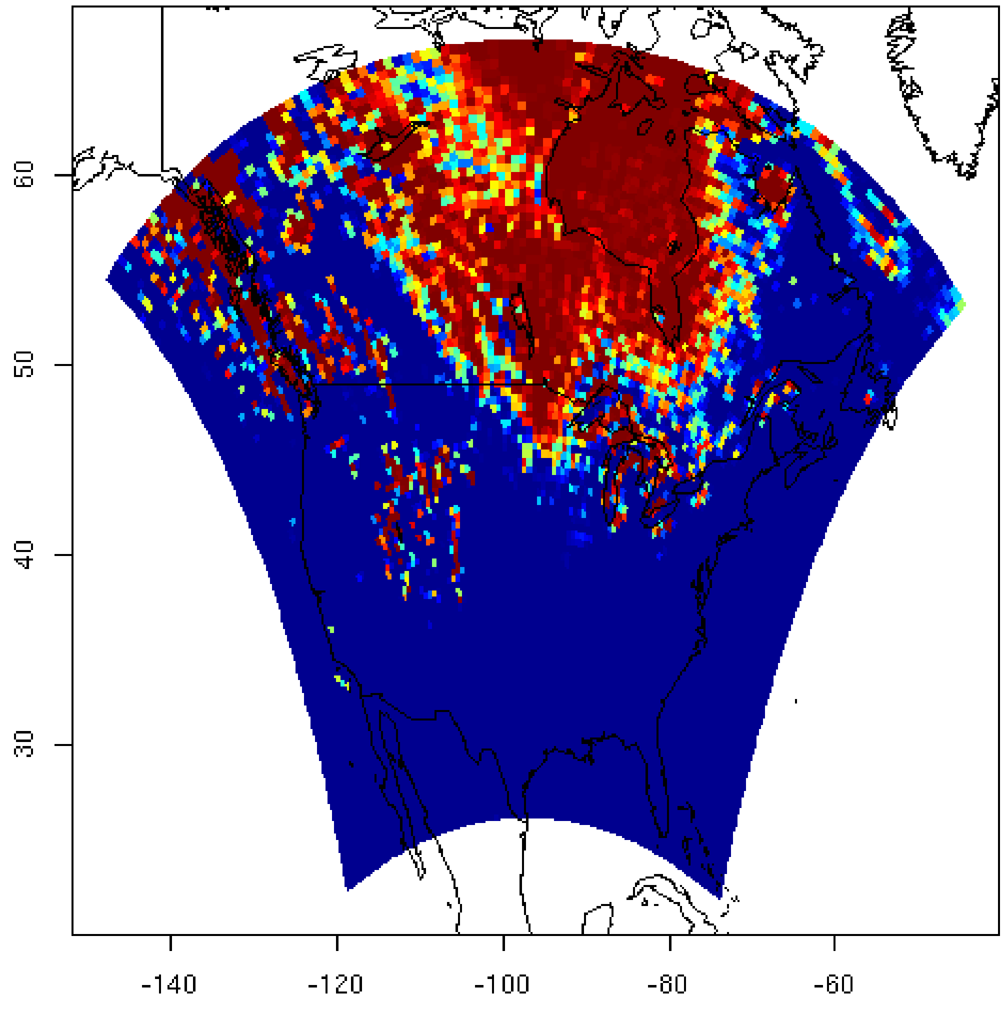




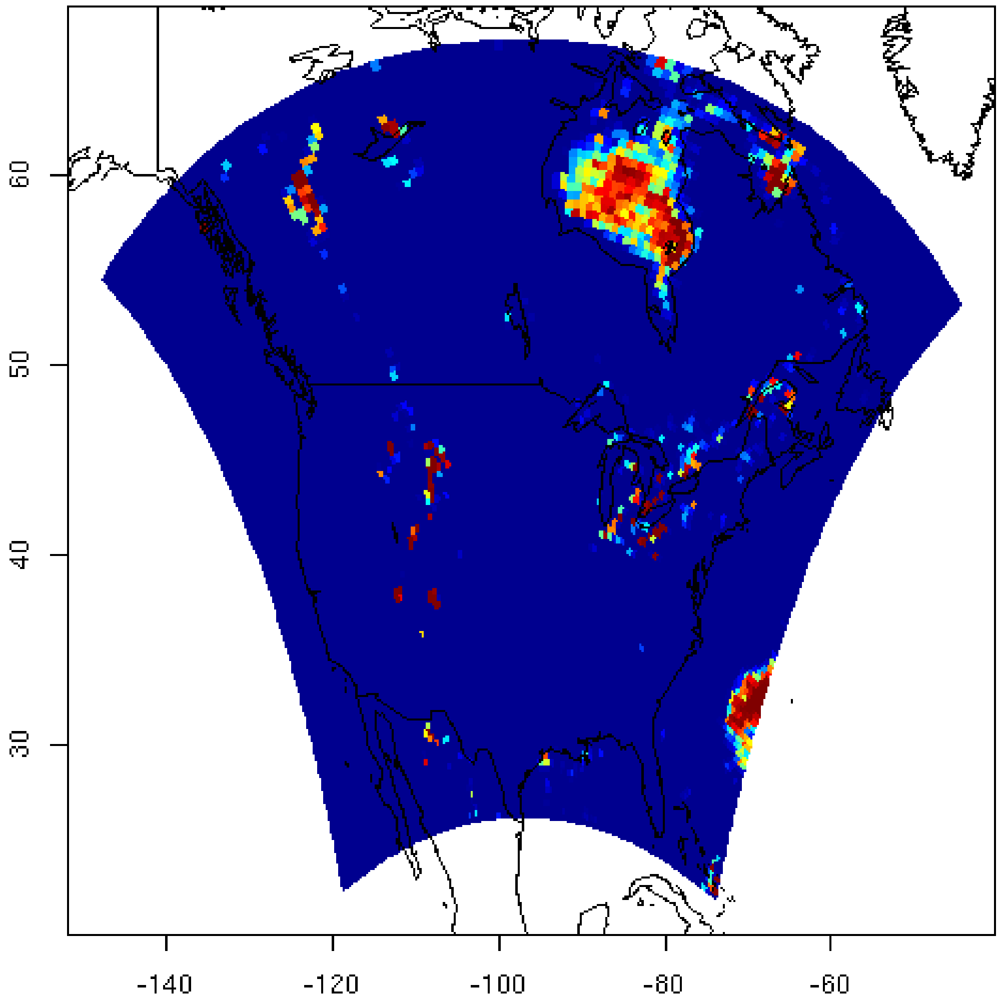
Inference

- for i in 1 to "a big number"...
  - sample $(\boldsymbol{\mu}^*, \boldsymbol{\mu}_1^*, \boldsymbol{\mu}_2^*, \boldsymbol{\mu}_3^*) \Rightarrow \boldsymbol{\alpha}^* = (\boldsymbol{\alpha}_1^*, \boldsymbol{\alpha}_2^*, \boldsymbol{\alpha}_3^*)$
  - construct (for each grid box):
    - $s_{\boldsymbol{\alpha}}^2$ (model-to-model variation)
    - s^2 (year-to-year variation)
  - identify and record grid boxes where $s_{\boldsymbol{\alpha}}^2$ is larger than s^2.
- compute $\widehat{P}[s_{\boldsymbol{\alpha}}^2 > s^2]$ for each grid box

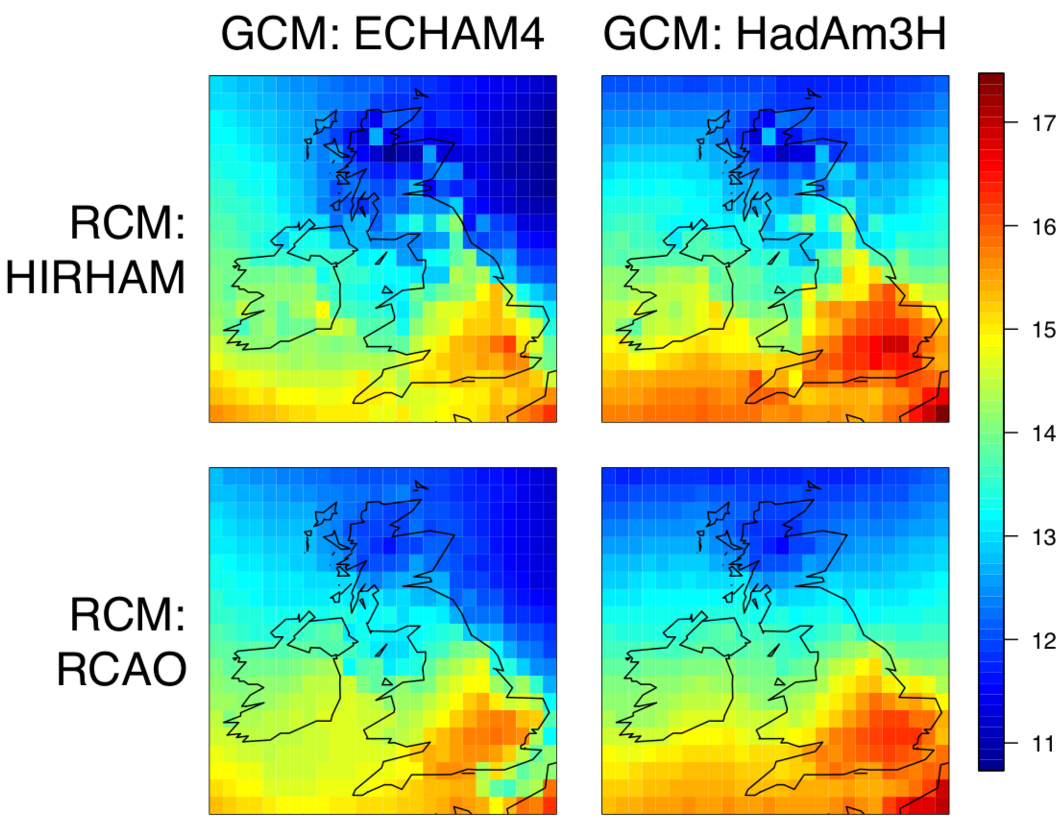
Winter Precipitation



Summer Precipitation

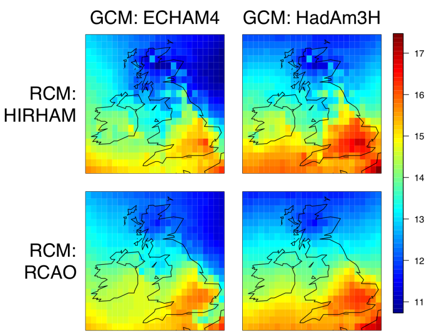


A PRUDENCE Example



- 2x2 "experiment"
  - 2 GCMs, 2 RCMs
  - PRUDENCE
- 1961-1990
- JJA average temp

A Two-Factor Model



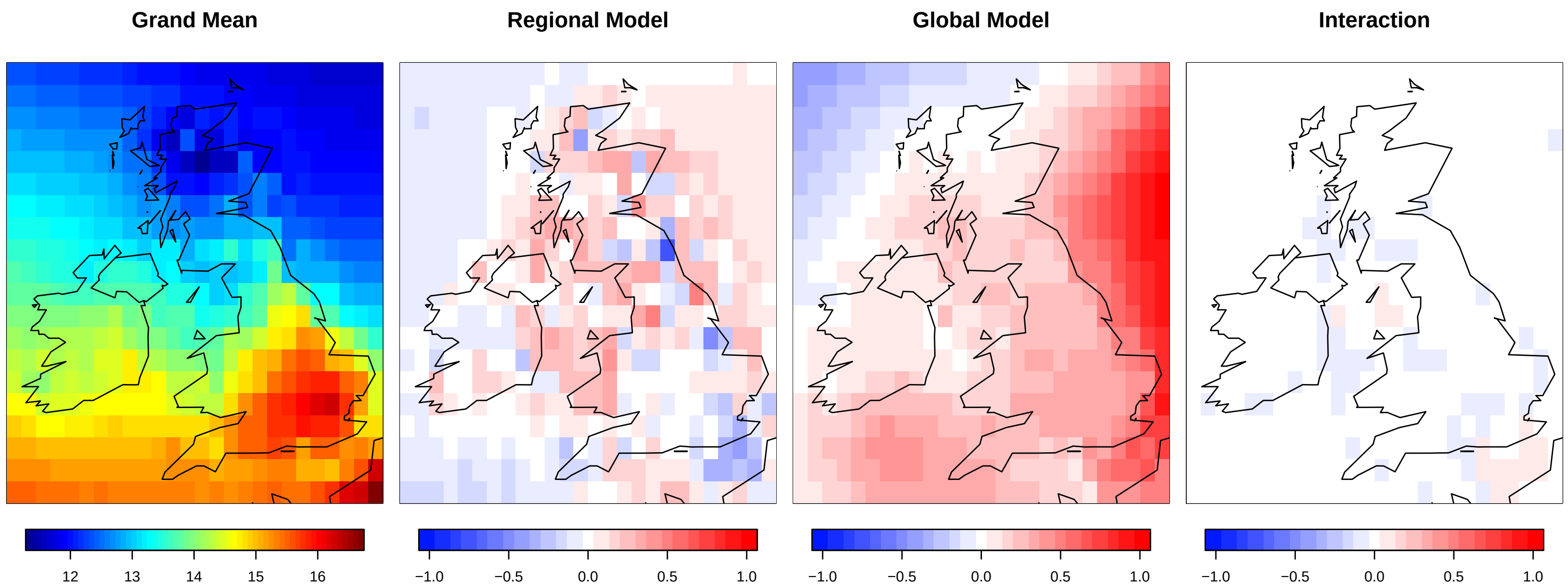
$$Z_{ijt}(s) = \mu_{ijt}(s) + \epsilon_{ijt}(s)$$

Output of RCM i, GCM j, at time t and location s = Expected/ "Climate" response + Spatially correlated residual/ "internal model variability"

$$\begin{aligned} \mu_{ijt}(s) &= \mu(s) + i\alpha(s) + j\beta(s) + ij(\alpha\beta)(s) + \gamma t, \\ &= \text{Common} + \text{RCM} + \text{GCM} + \text{Interaction} + \text{Time} \end{aligned}$$

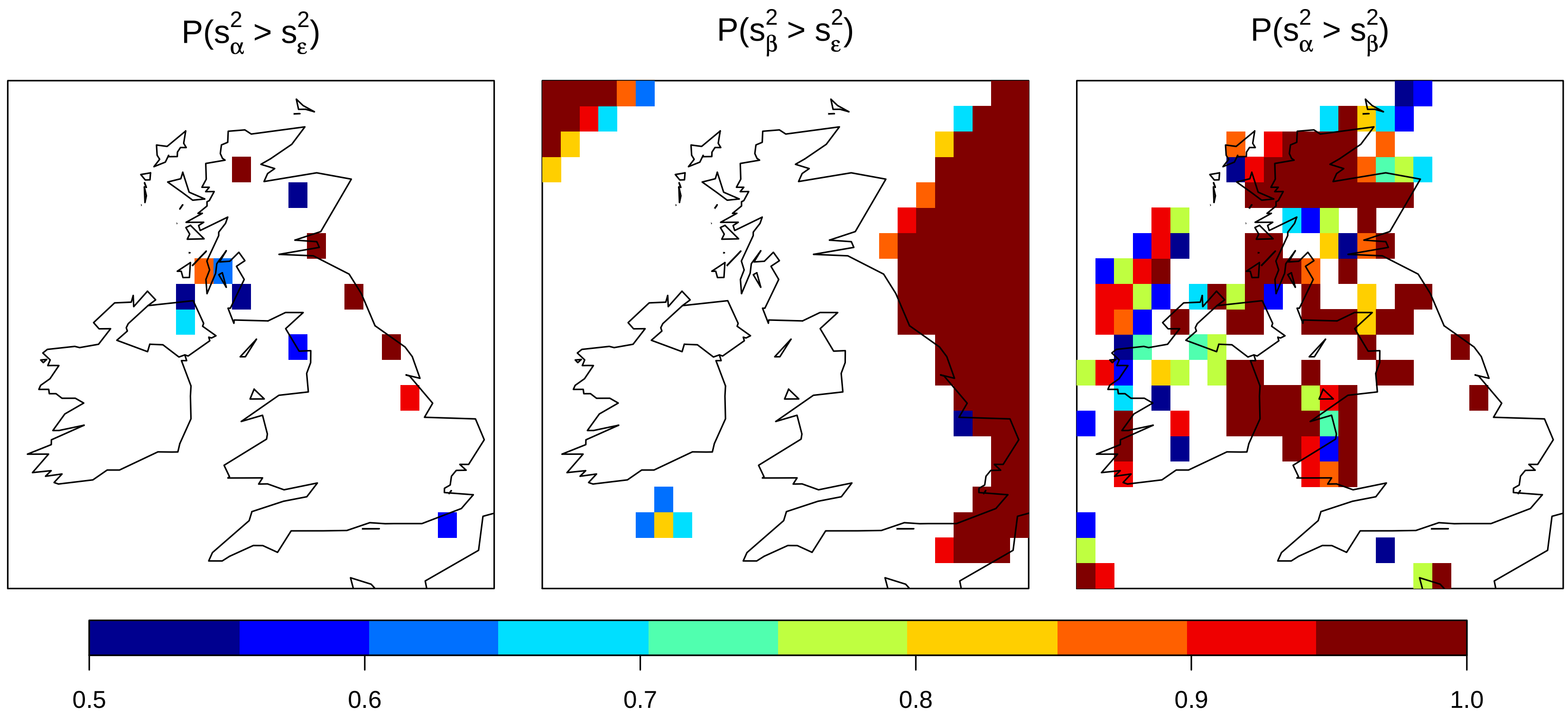
- $i, j = -1, 1$ (contrast coding)
- Hierarchical model with Gaussian process priors used for each effect.
- MCMC used to estimate parameters, posterior inference, etc.

Posterior Means



- Estimates of spatial effects.

Functional ANOVA



- Ratios of variances.