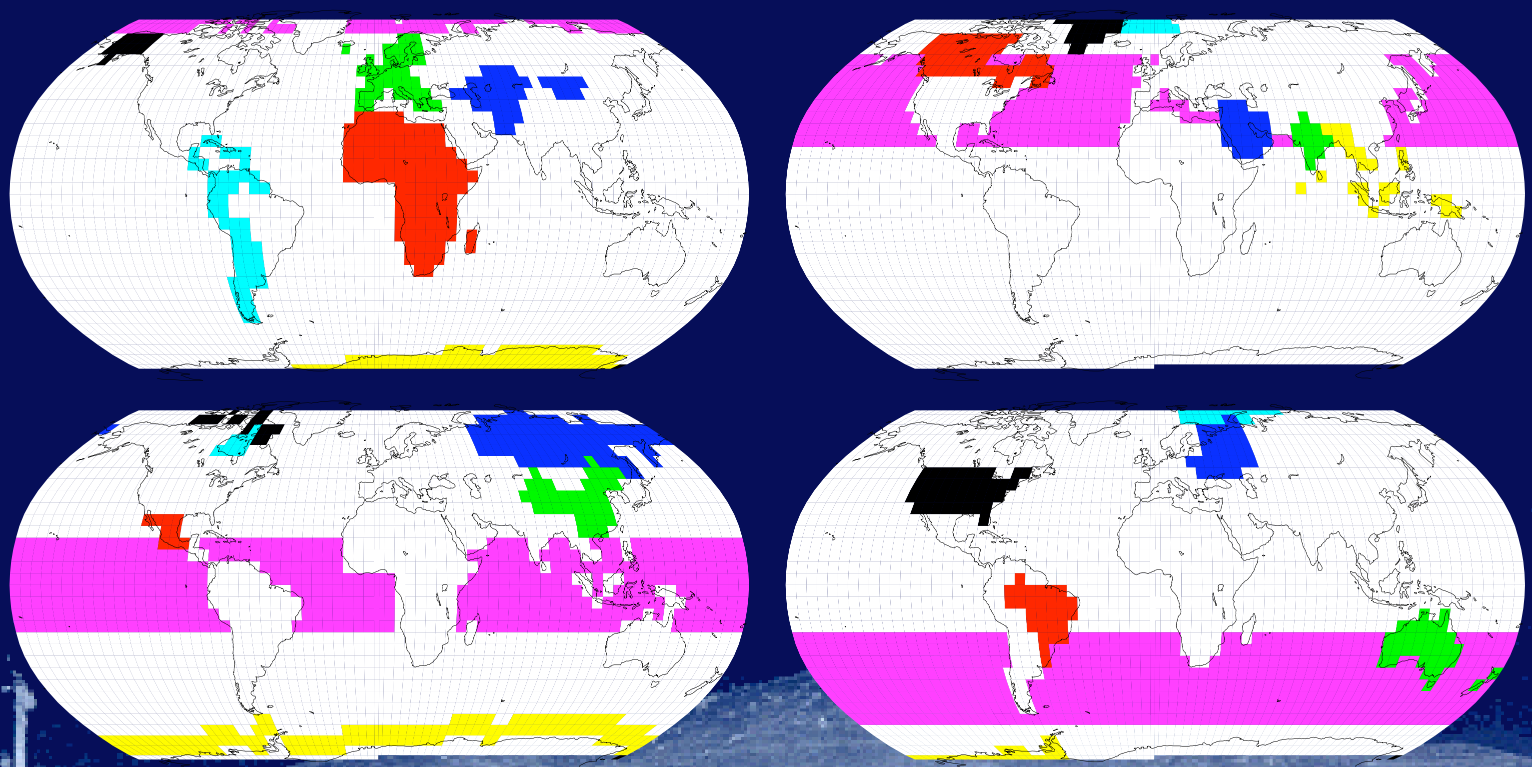
Basis Functions Used in M

{ Data level | Process level | Prior level }

1. Spherical harmonics

2. Indicator functions (28)



Hyperparameters $\xi_1, \ldots, \xi_5$

{ Data level | Process level | Prior level }

To make sure that variability around the truth is smaller than bias and internal variability

$$\phi_i > \psi_i$$

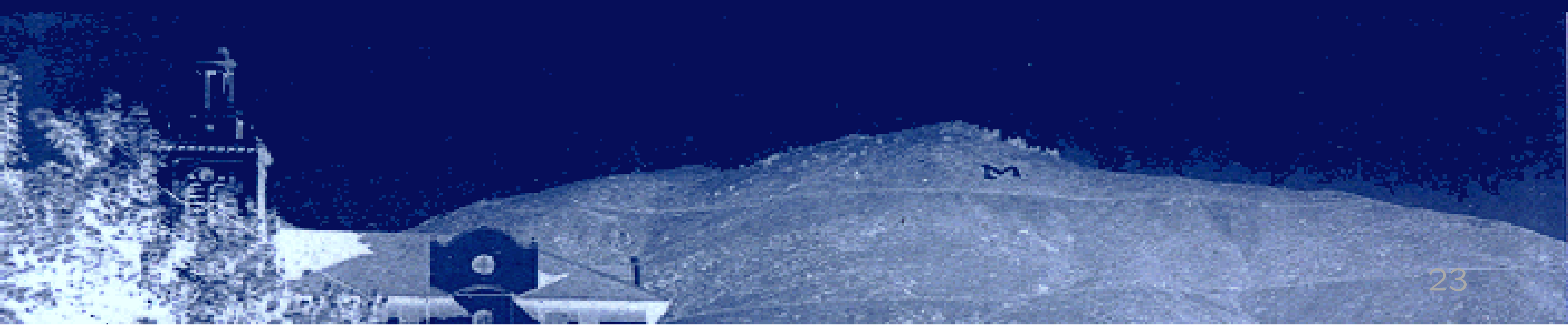
Choose ξ_1, ξ_2, ξ_3 small, $\xi_4 \in [1, 2.5]$, ξ_5 large.

PDF of Climate Change

The goal is the (posterior) PDF of the climate change signal given the AOGCM data and model parameters:

$$[\,\text{climate change}\,|\,\text{AOGCM data}, \text{model parameters}\ldots]$$

$$[\qquad \mathbf{M}\boldsymbol{\nu} \qquad |\quad \mathbf{D}_1, \ldots, \mathbf{D}_N\ , \qquad \ldots \qquad]$$



PDF of Climate Change

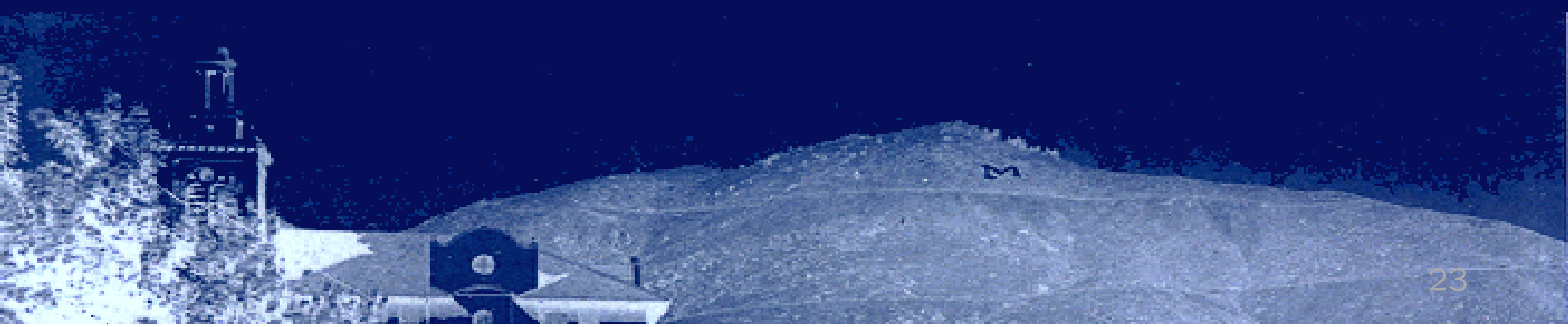
The goal is the (posterior) PDF of the climate change signal given the AOGCM data and model parameters:

$$[\,\text{climate change} \mid \text{AOGCM data}, \text{model parameters} \ldots]$$

$$[\quad \mathbf{M}\boldsymbol{\nu} \quad \mid \quad \mathbf{D}_1, \ldots, \mathbf{D}_N \;, \qquad \ldots \qquad]$$

Via Bayes' theorem, the (posterior) PDF is

$$\begin{aligned} &[\text{ process} \mid \text{data, parameters }] \\ &\qquad \propto [\text{ data} \mid \text{process, parameters }] \\ &\qquad\qquad \cdot [\text{ process} \mid \text{parameters }] \cdot [\text{ parameters }] \end{aligned}$$



Computational Approach

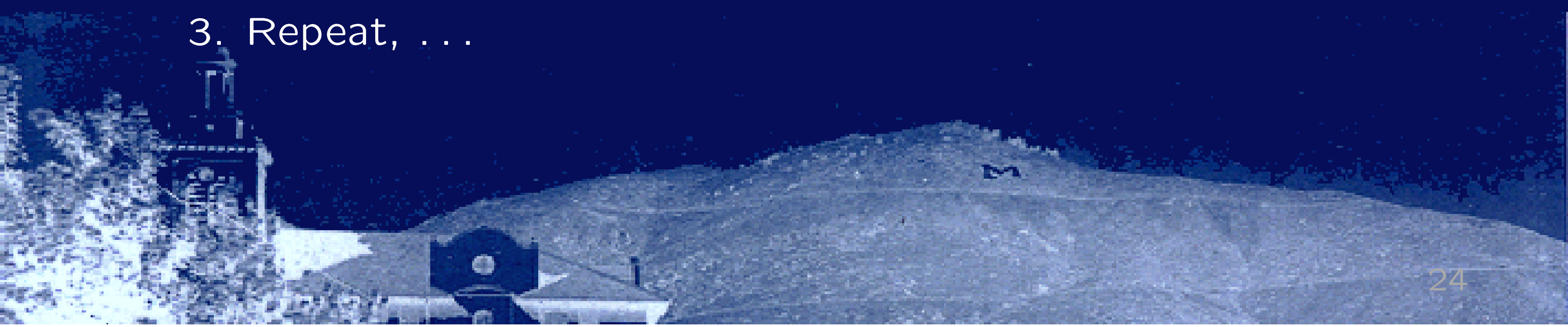
No closed form of the posterior density.

Use a computational approach: Markov Chain Monte Carlo (MCMC), here a Gibbs sampler.

1. Express the distribution of each parameter conditional on everything else (full conditionals).

2. Cycle through the parameters: draw a new value based on the full conditional and the current values of the other parameters.

3. Repeat, . . .



Full Conditionals

Full conditionals for all parameters have been derived:

$$\boldsymbol{\nu} \mid \ldots \sim \mathcal{N}_p(\mathbf{A}^{-1}\mathbf{b},\ \mathbf{A}^{-1})$$

$$\mathbf{A} = \frac{1}{\xi_5}\mathbf{I} + \sum_{i=1}^{N} \frac{1}{\psi_i}\mathbf{I} \qquad \mathbf{b} = \sum_{i=1}^{N} \frac{1}{\psi_i}\boldsymbol{\theta}_i$$

$$i = 1, \ldots, N:\ \boldsymbol{\theta}_i \mid \ldots \sim \mathcal{N}_p(\mathbf{A}^{-1}\mathbf{b},\ \mathbf{A}^{-1})$$

$$\mathbf{A} = \frac{1}{\psi_i}\mathbf{I} + \frac{1}{\phi_i}\mathbf{M}^\mathsf{T}\boldsymbol{\Sigma}^{-1}\mathbf{M} \quad \mathbf{b} = \frac{1}{\psi_i}\boldsymbol{\nu} + \frac{1}{\phi_i}\mathbf{M}^\mathsf{T}\boldsymbol{\Sigma}^{-1}\mathbf{D}_i$$

$$i = 1, \ldots, N:\ \phi_i \mid \ldots \sim \mathrm{I\Gamma}\left(\xi_1 + \frac{n}{2}, \xi_2 + \frac{1}{2}(\mathbf{D}_i - \mathbf{M}\boldsymbol{\theta}_i)^\mathsf{T}\boldsymbol{\Sigma}^{-1}(\mathbf{D}_i - \mathbf{M}\boldsymbol{\theta}_i)\right)$$

$$i = 1, \ldots, N:\ \psi_i \mid \ldots \sim \mathrm{I\Gamma}\left(\xi_3 + \frac{p}{2}, \xi_4 + \frac{1}{2}(\boldsymbol{\theta}_i - \boldsymbol{\nu})^\mathsf{T}(\boldsymbol{\theta}_i - \boldsymbol{\nu})\right)$$

Full Conditionals

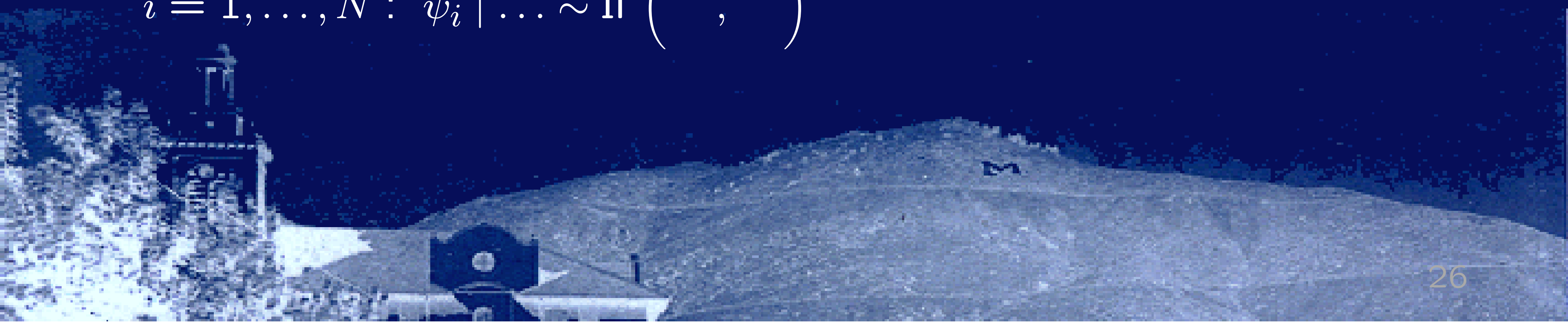
Full conditionals for all parameters have been derived:

$$\boldsymbol{\nu} \mid \ldots \sim \mathcal{N}_p(\quad , \quad)$$

$$i = 1, \ldots, N : \;\; \boldsymbol{\theta}_i \mid \ldots \sim \mathcal{N}_p(\quad , \quad)$$

$$i = 1, \ldots, N : \;\; \phi_i \mid \ldots \sim \mathsf{I}\Gamma\left(\quad , \quad \right)$$

$$i = 1, \ldots, N : \;\; \psi_i \mid \ldots \sim \mathsf{I}\Gamma\left(\quad , \quad \right)$$

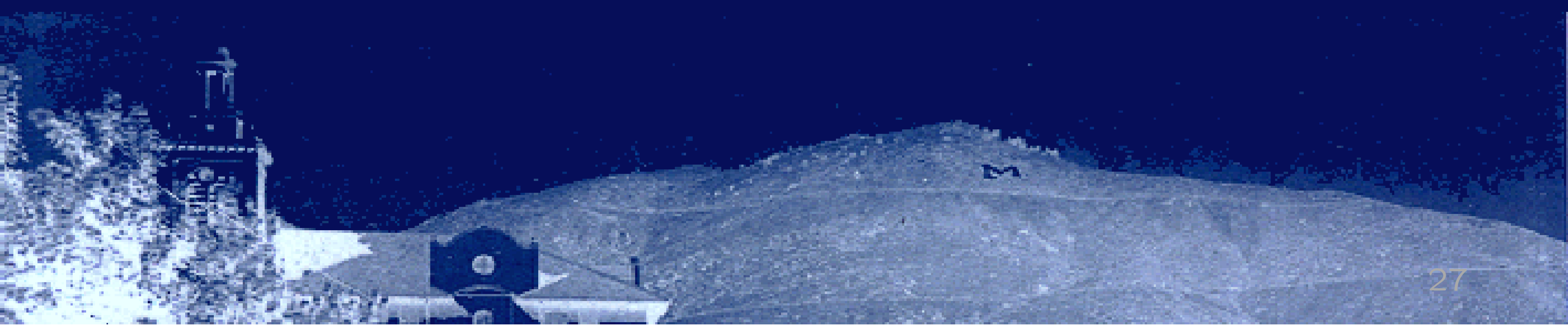


Computational Aspects

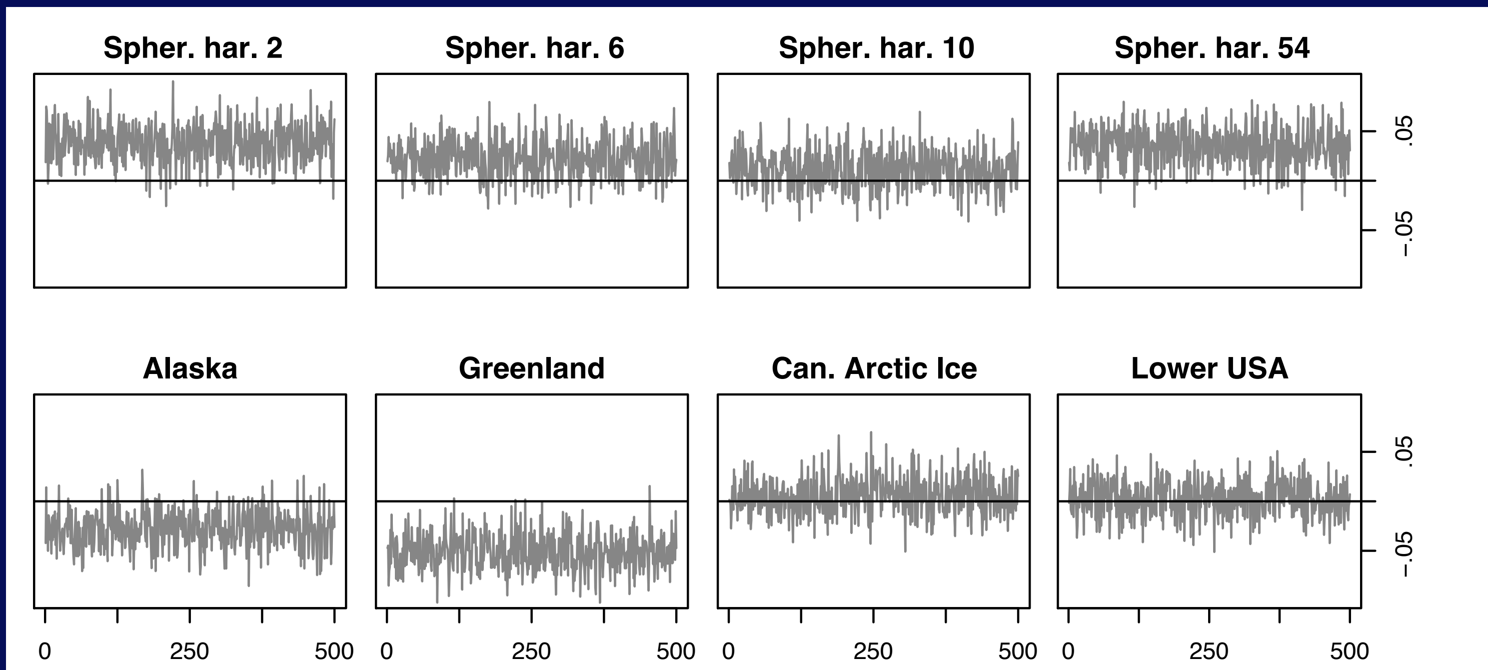
- Gibbs sampler programmed in R
  free software environment for statistical computing and graphics

- Run 20000 iterations
  10000 burn-in, keep every 20th, takes a few hours

- Visual/primitive inspection of convergence



Computational Aspects



and graphics

- Visual/primitive inspection of convergence

Posterior Draws