



Case Study II: Extremes

Markov Random Fields and Regional Climate Models

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Goals

- How to model spatial extremes?
- Can we model GEV parameters spatially? In particular the shape parameter?
- What kinds of changes are projected for extreme precipitation?

Extreme Value Distributions

- Let Z denote a block maxima (e.g. annual). Under certain conditions, it can be assumed that Z follows a generalized extreme value distribution given by

$$G(z) = \exp \left[- \left(1 + \xi \frac{z - \mu}{\sigma} \right)^{-1/\xi} \right],$$

for values of z such that $1 + \xi(z - \mu)/\sigma > 0$.

- μ and σ are location and scale parameters.
- If the shape parameter $\xi < 0$, then the distribution has a bounded upper tail. If $\xi = 0$ (taken as the limit), then the distribution has an exponentially decreasing tail, and, if $\xi > 0$, the distribution's tail decays as a power function.

Extreme Value Distributions

- The block maxima approach is limited by using only one data point per block; alternative approaches include models for all points over a threshold.
 - Generalized Pareto Distribution (GPD)
 - Point processes
- From the GEV or GPD, extreme behavior of the distribution can be summarized through return levels or, more simply, values of extreme quantiles implied from the distribution.

A Hierarchical Model for Extremes

- For each grid box, let $Z_{r,i,t}$ denote the precipitation amount recorded for the r th simulation (control or future), the i th grid cell, and the t th day.
- Assume that all $Z_{r,i,t}$ that exceed the threshold $u_{r,i}$ follow the point process model with parameters $\mu_{r,i}$, $\sigma_{r,i}$, and $\xi_{r,i}$ and are conditionally independent of other grid boxes.
- Note: the likelihood for the point process model augmented by the regularization/prior suggested by Martins and Stedinger (2000).

A Hierarchical Model for Extremes

- In the process model, the parameters are modeling via

$$\begin{aligned}\mu_{r,i} &\sim N(X_i' \beta_{r,\mu} + U_{r,i,\mu}, 1/\tau_\mu^2) \\ \log(\sigma_{r,i}) &\sim N(X_i' \beta_{r,\sigma} + U_{r,i,\sigma}, 1/\tau_\sigma^2) \\ \xi_{r,i} &\sim N(X_i' \beta_{r,\xi} + U_{r,i,\xi}, 1/\tau_\xi^2)\end{aligned}$$

- Regression includes elevation, longitude, latitude, ocean, but spatial patterns assumed to be the same across control and future (different intercepts).
- $\mathbf{U} = (\mathbf{U}_{1,\mu}, \mathbf{U}_{1,\sigma}, \mathbf{U}_{1,\xi}, \mathbf{U}_{2,\mu}, \mathbf{U}_{2,\sigma}, \mathbf{U}_{2,\xi})'$ represents a spatial random effect modeled through an intrinsic autoregressive process (IAR) with precision matrix $\mathbf{Q} = \mathbf{T} \otimes \mathbf{W}$.

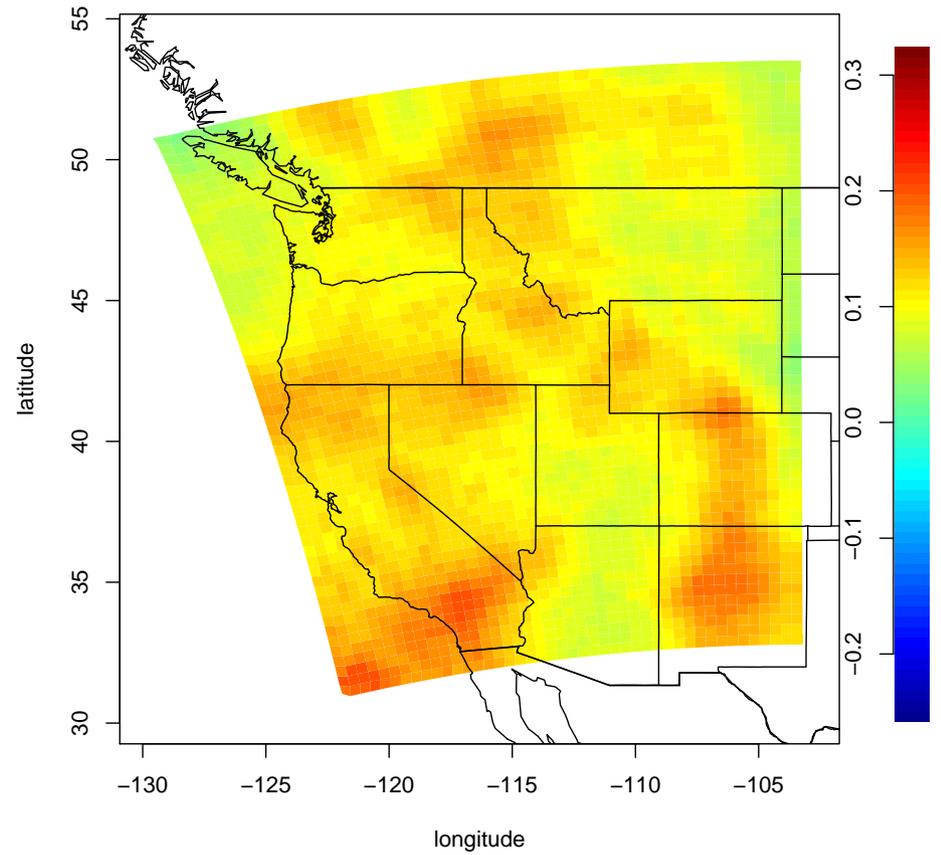
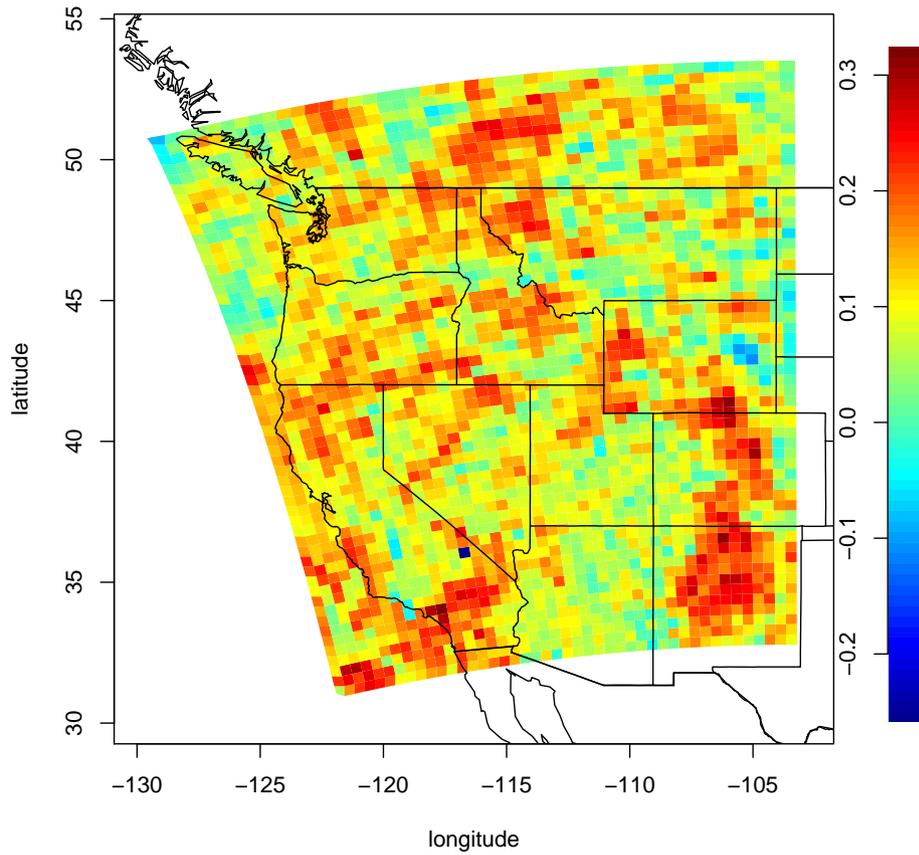
Intrinsic Autogressive Process

- The IAR is an improper version of the CAR model.
- \mathbf{T} is a 6×6 covariance matrix.
- \mathbf{W} has:
 - non-diagonal elements $w_{i,j} = -1$ if grid boxes i and j are neighbors and zero otherwise.
 - $w_{i,i} = -\sum_{j \neq i} w_{i,j}$
- All row sums of \mathbf{W} are zero, imposing the conditions $\sum_i U_{r,i,\theta} = 0$ for $r = 1, 2$ and all $\theta = \mu, \sigma, \xi$.

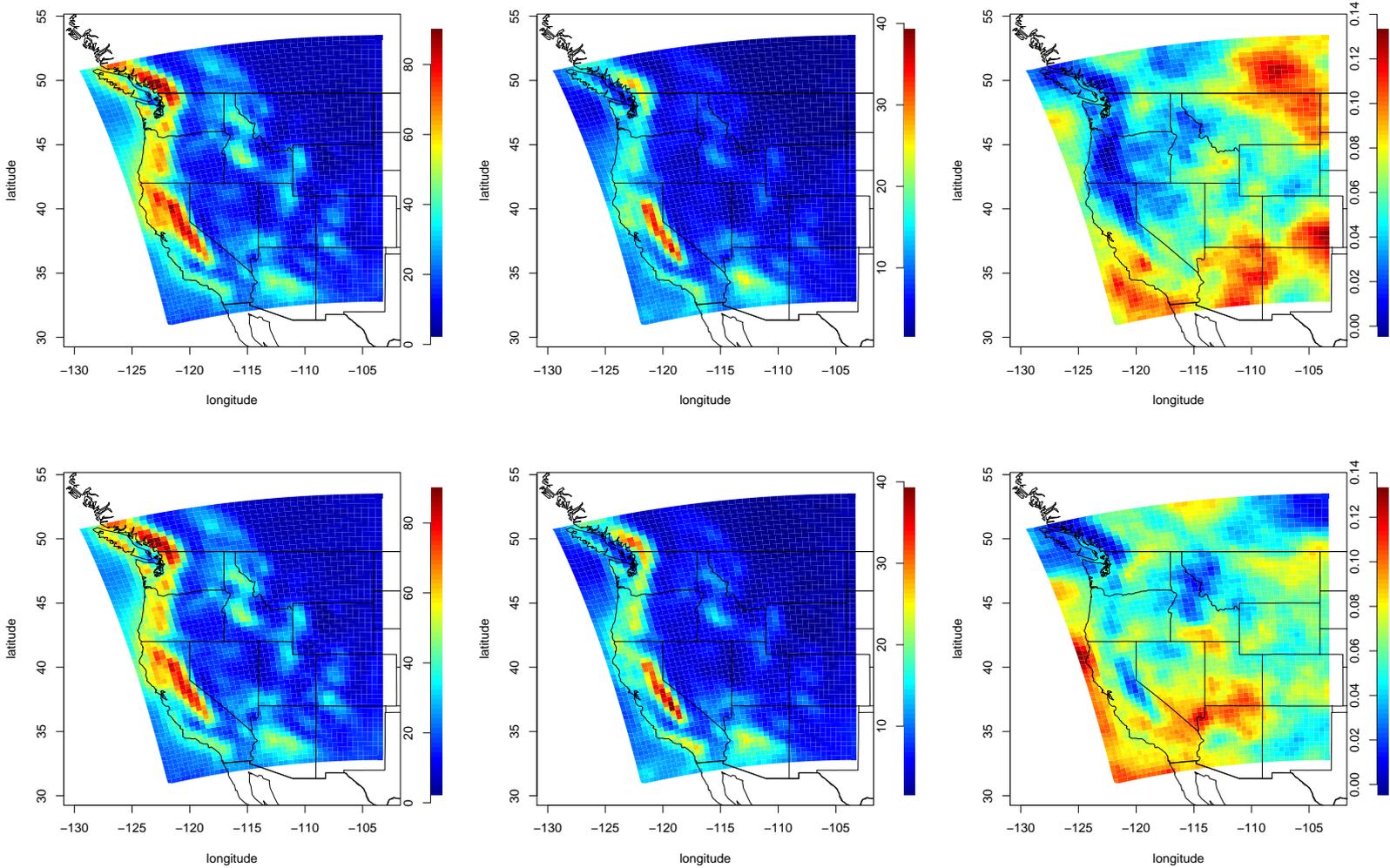
A Hierarchical Model for Extremes

- Conjugate priors for β s (Gaussian) and \mathbf{T} (Whishart) with ranges of variation reflective of exploratory analysis (i.e. MLE).
- Gibbs sampler used to sample from the posterior, with special care taken for the spatial random effect, \mathbf{U} .

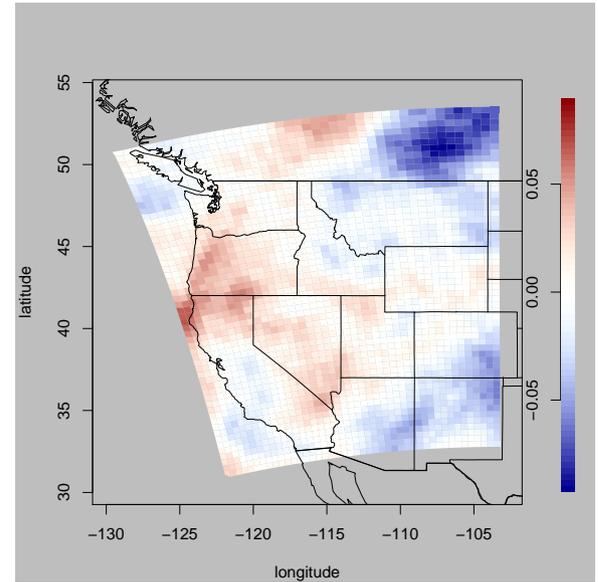
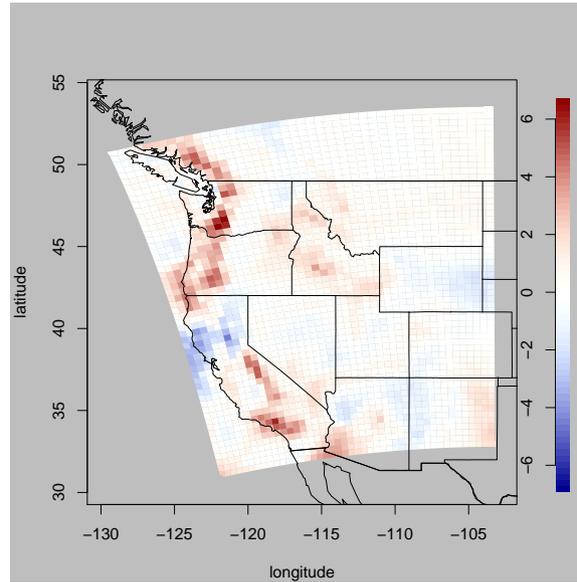
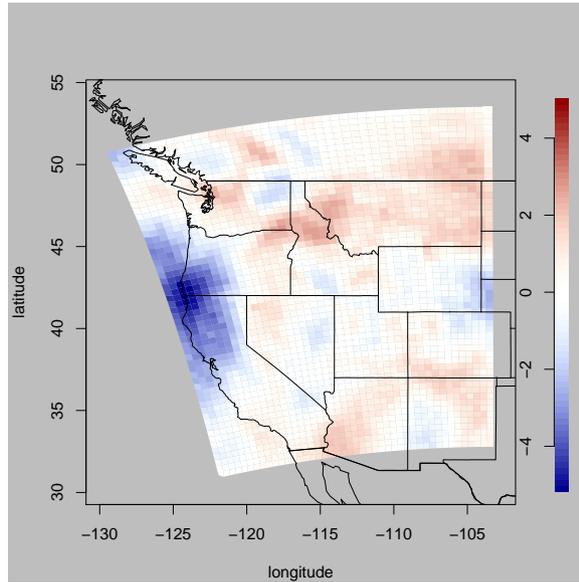
A Comparison of ξ



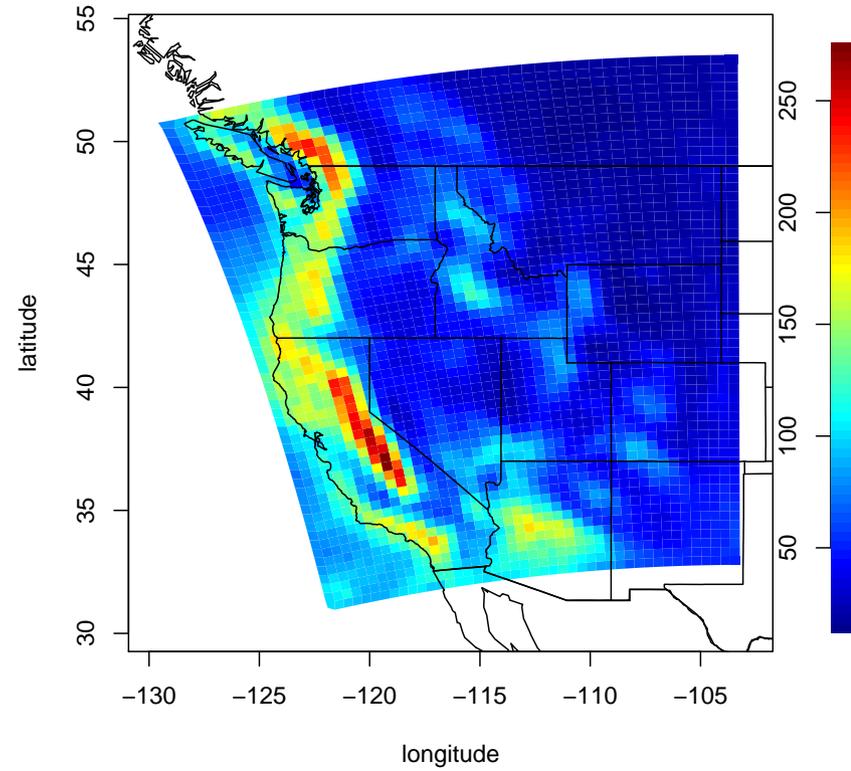
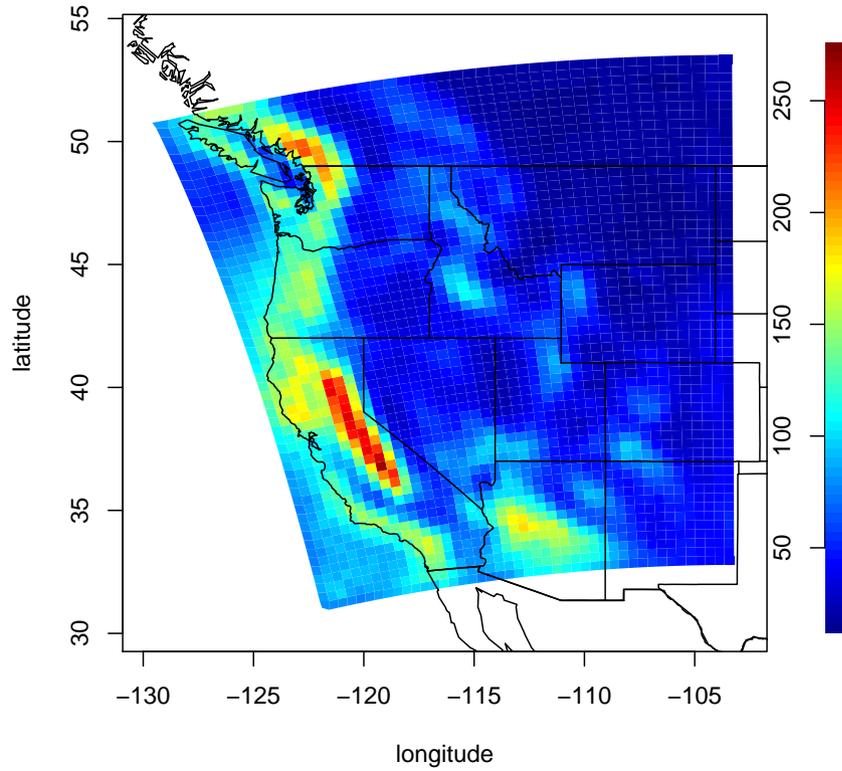
Winter Model Parameters



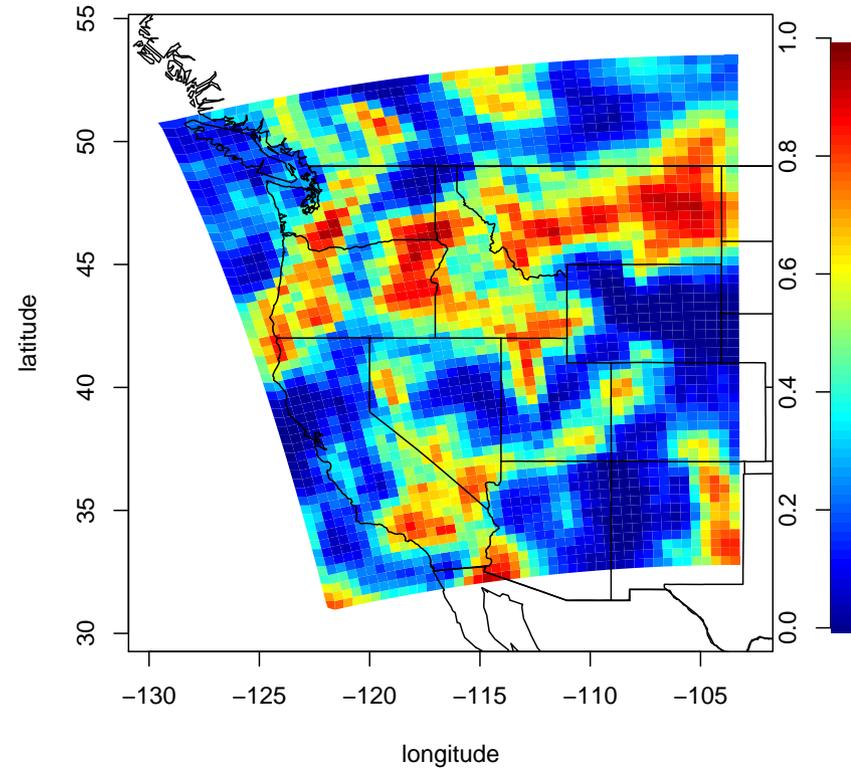
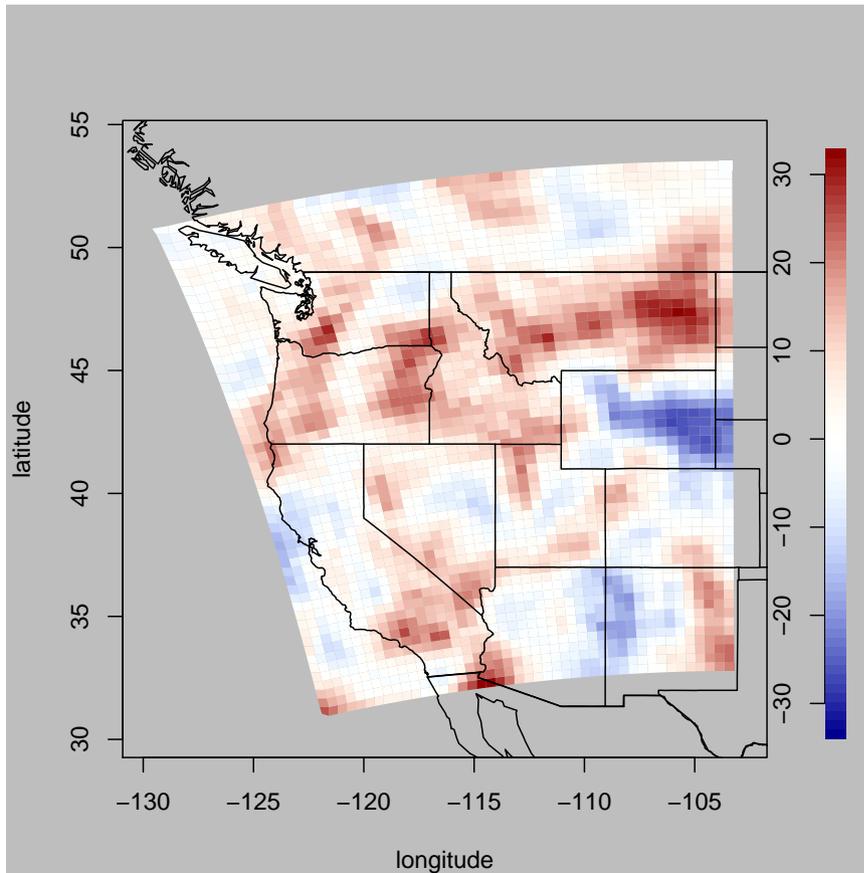
Winter Model Parameters



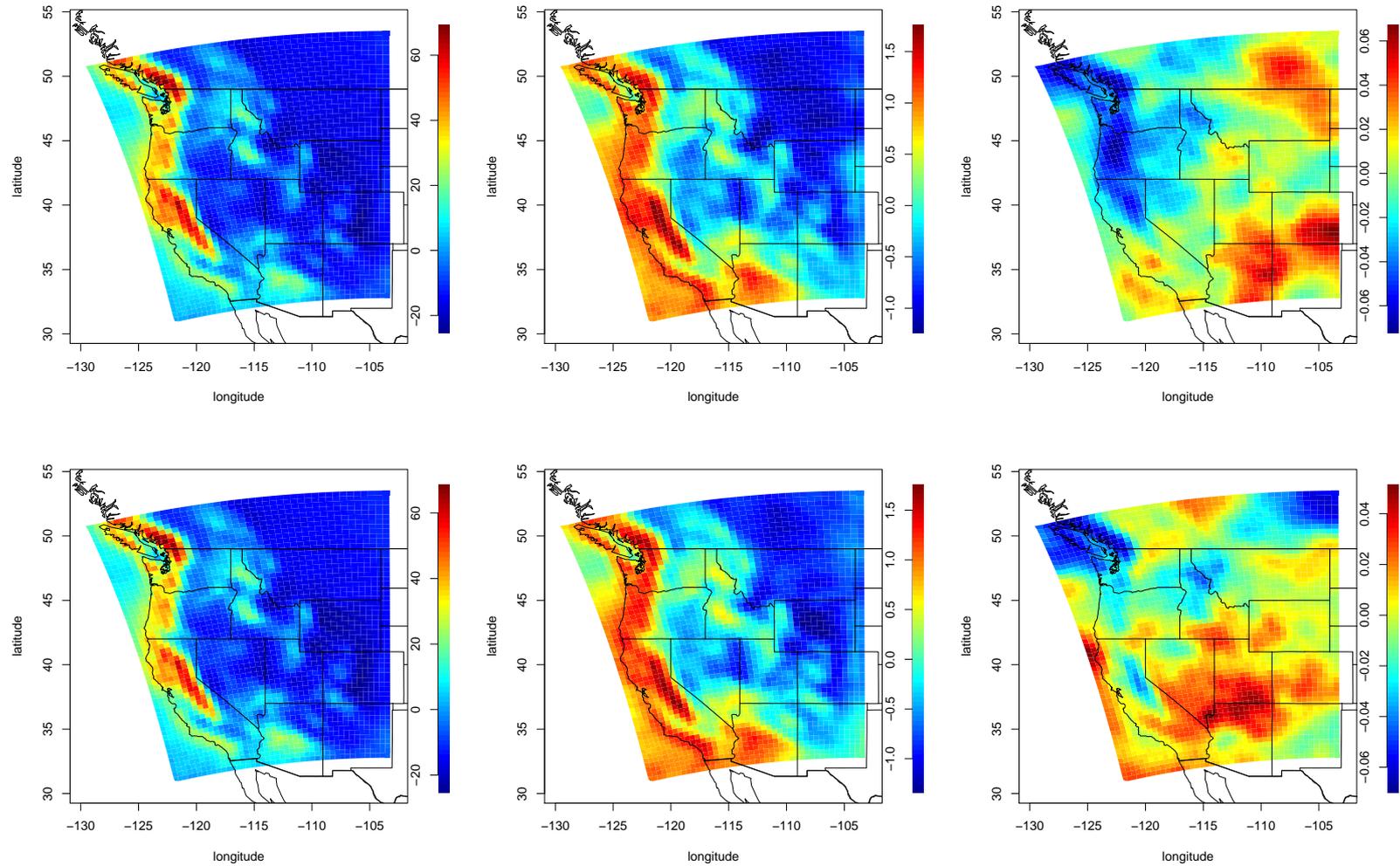
Winter Return Levels



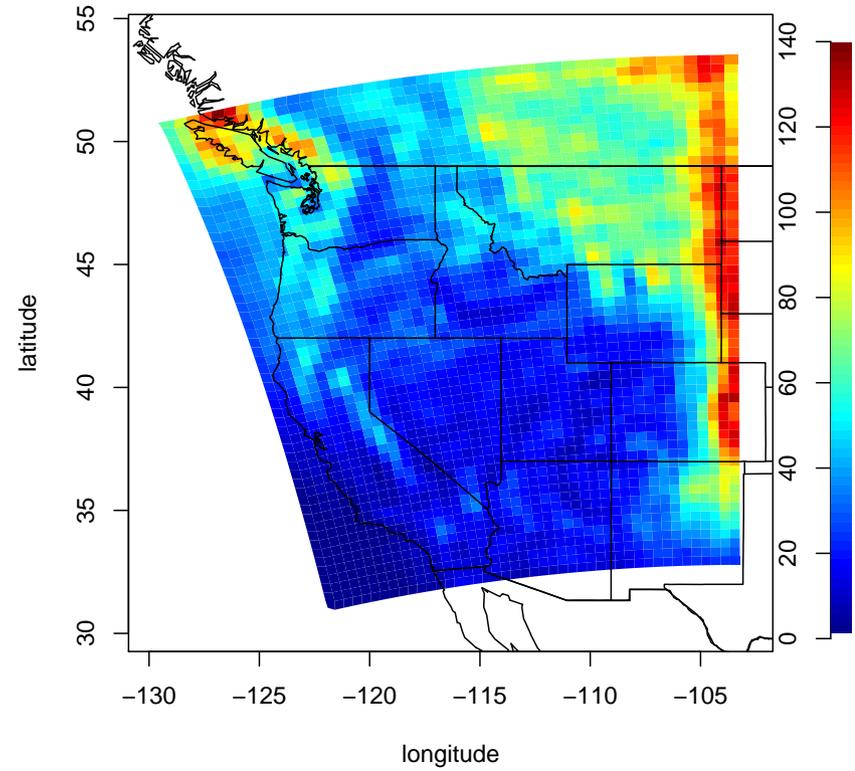
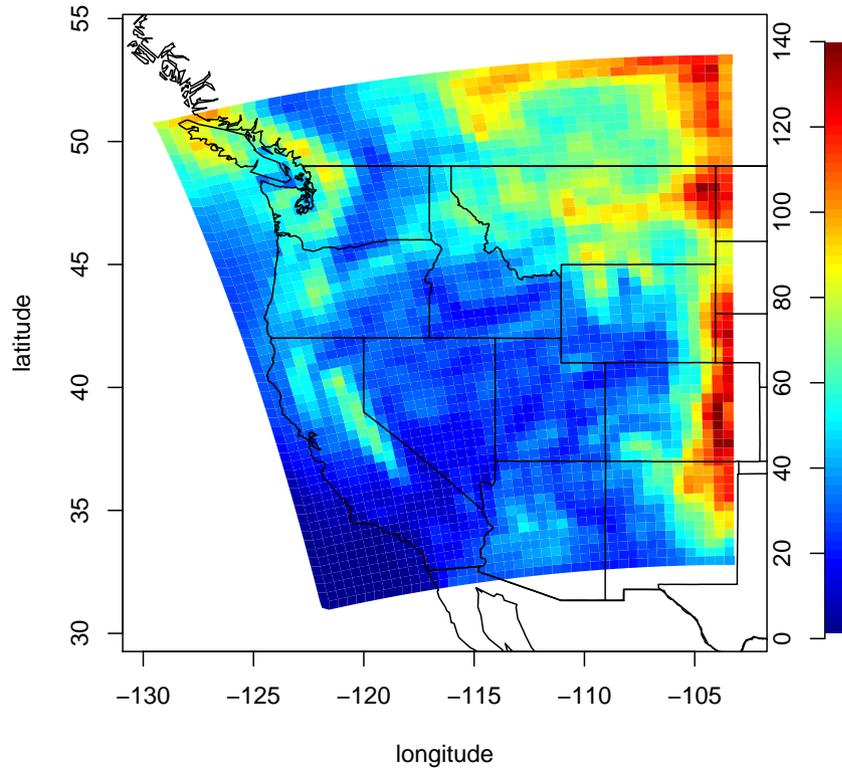
Winter Return Levels



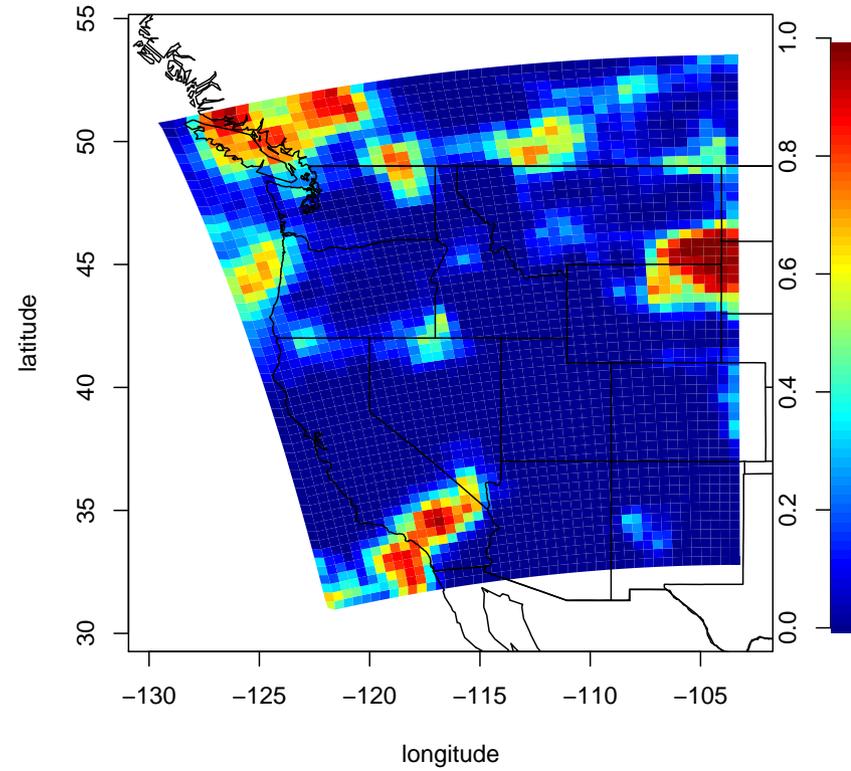
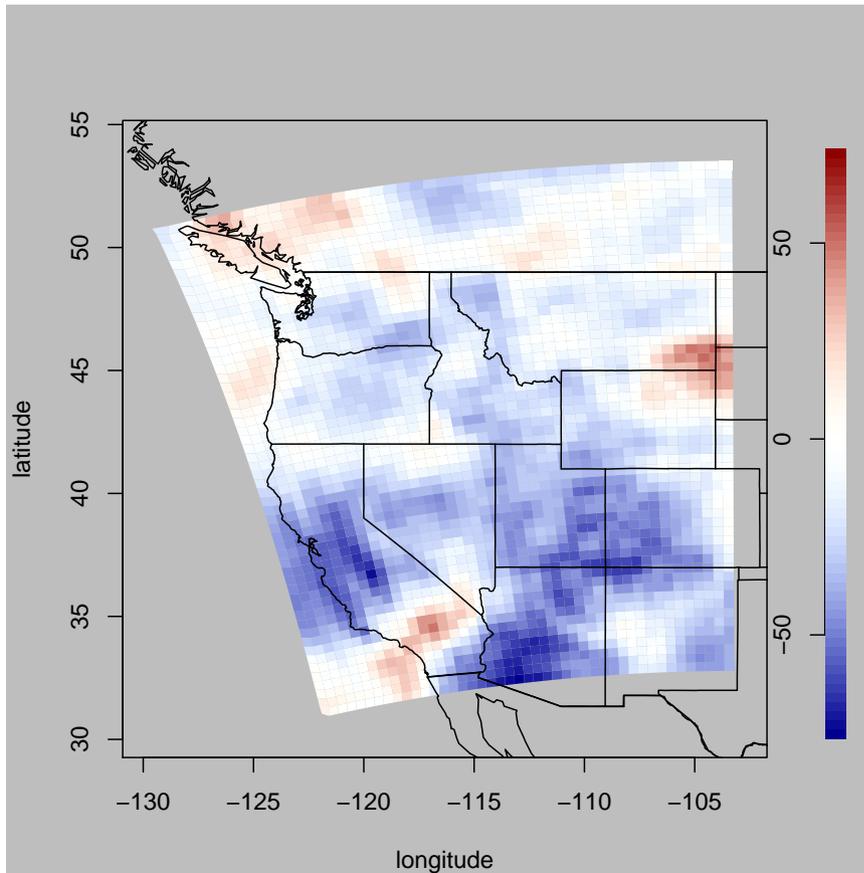
Winter Spatial Effect



Summer Return Levels



Summer Return Levels



A Big Issue

