A Non-Hydrostatic Dynamical Core in the HOMME Framework [HOMAM]

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Introduction

The High-Order Method Modeling Environment (HOMME), developed at NCAR, is a petascale hydrostatic framework, which employs the cubed-sphere grid system and high-order continuous or discontinuous Galerkin (DG) methods. Recently, the HOMME framework is being extended to a non-hydrostatic (NH) dynamical core, the "High-Order Multiscale Atmospheric Model" (HOMAM). Orography is handled by the terrain-following height-based coordinate system. To alleviate the stringent CFL stability requirement resulting from the vertical aspects of the dynamics, an operator-splitting time integration scheme based on the horizontally explicit and vertically implicit (HEVI) philosophy is adopted for HOMAM.

HOMAM: DG-NH Model

Based on the conservation of mass, momentum and potential temperature, the NH atmospheric model is characterized by the classical compressible Euler System. In 3D generalized curvilinear coordinates (x^j), the governing equations are:

$$\frac{\partial \rho}{\partial t} + \frac{1}{\sqrt{G}}\left[\frac{\partial}{\partial x^j}(\sqrt{G}\,\rho u^j)\right] = 0 \quad \{\text{Summation Implied}\}$$

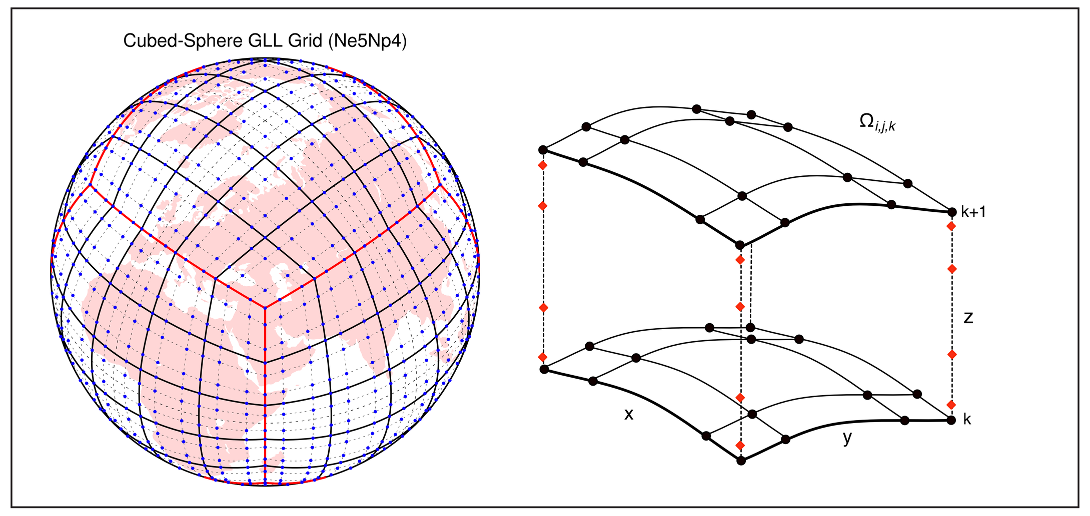
$$\frac{\partial \rho u^i}{\partial t} + \frac{1}{\sqrt{G}}\left[\frac{\partial}{\partial x^j}[\sqrt{G}(\rho u^i u^j + p\,G^{ij})]\right] = -\Gamma^i_{jk}(\rho u^j u^k + p\,G^{jk}) + f\sqrt{G}(u^1 G^{2i} - u^2 G^{1i}) - \rho g\, G^{3i}$$

$$\frac{\partial \rho\theta}{\partial t} + \frac{1}{\sqrt{G}}\left[\frac{\partial}{\partial x^j}(\sqrt{G}\,\rho\theta\, u^j)\right] = 0$$

$$\frac{\partial \rho q}{\partial t} + \frac{1}{\sqrt{G}}\left[\frac{\partial}{\partial x^j}(\sqrt{G}\,\rho q\, u^j)\right] = 0.$$

Where ρ is the air density, θ potential temperature, u^i is the contravariant wind field, f is the Coriolis parameter and q is the tracer field. G_{ij} metric tensor, $\sqrt{G} = |G_{ij}|^{1/2}$ is the Jacobian of the transform, $G^{ij} = (G_{ij})^{-1}$, and $i,j,k \in \{1,2,3\}$. Γ^i_{jk} is the Christoffel symbol (metric terms) associated with coordinate transform.

- Shallow atmosphere approximation ($x^3 = r + z, z \ll r$). The vertical grid system relies on height-based terrain-following (ζ) coordinates (*Gal-Chen & Sommerville*).
- **Computational Domain:** Dimension-split (2D + 1D) approach allows to treat the 3D atmosphere as a vertically stacked cubed-sphere layers in the ζ direction [1].



- **DG discretization** employs GLL spectral-element grid (HOMME grid system) in the horizontal and 1D Gauss-Legendre (GL) type grid in the vertical direction
- With the prognostic state-vector $\mathbf{U}_h = (\sqrt{G}\rho, \sqrt{G}\rho u^1, \sqrt{G}\rho u^2, \sqrt{G}\rho w, \sqrt{G}\rho\theta)^T$, the DG spatial discretization leads to

$$\frac{\partial \mathbf{U}_h}{\partial t} + \nabla\cdot\mathbf{F}(\mathbf{U}_h) = \mathbf{S}(\mathbf{U}_h) \quad \Rightarrow \quad \frac{d}{dt}\mathbf{U}_h = L(\mathbf{U}_h) \quad \text{in} \quad (0,T)$$

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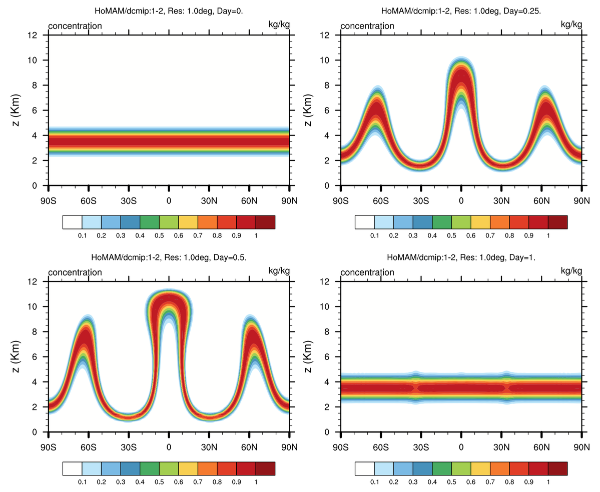
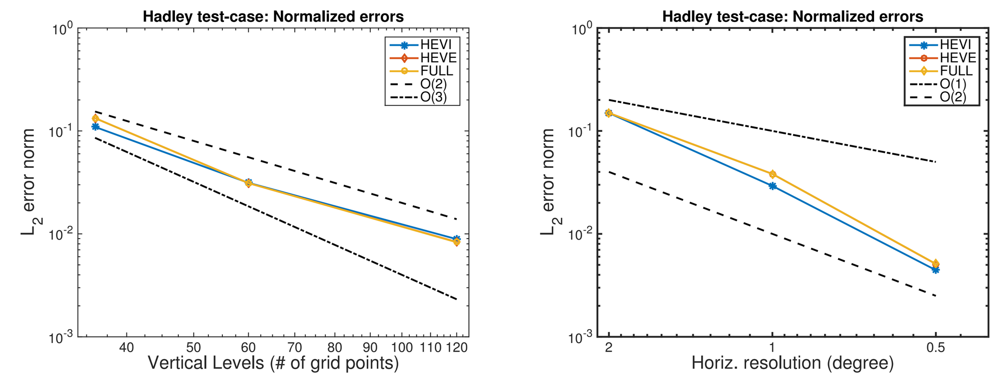
Time Integration Schemes

- **Dimensional (Operator) Splitting**
  The spatial DG discretization is $L(\mathbf{U}_h)$ is decomposed into the horizontal L^H and vertical L^V parts such that $L(\mathbf{U}_h) = L^H(\mathbf{U}_h) + L^V(\mathbf{U}_h)$.
- **For HEVI:** H-part is solved explicitly via third-order SSP-RK and V-part is solved implicitly via the Diagonally Implicit Runge-Kutta (DIRK).
- "$H - V - H$" sequence of operations, only one implicit solver per time step.
- For the implicit solver: Inner linear solver uses Jacobian-Free GMRES and it usually takes 1 or 2 iterations for the outer Newton solver
- The effective Courant number is only limited by the minimum horizontal grid-spacing [2]. Strang-type operator splitting permits $\mathcal{O}(\Delta t^2)$ accuracy in time.
- The HOMME hydrostatic model relies on explicit time-stepping with excellent petascale scalability. HOMAM may retain the parallel efficiency with 'HEVI'

3D Advection: DCMIP [4] Tests

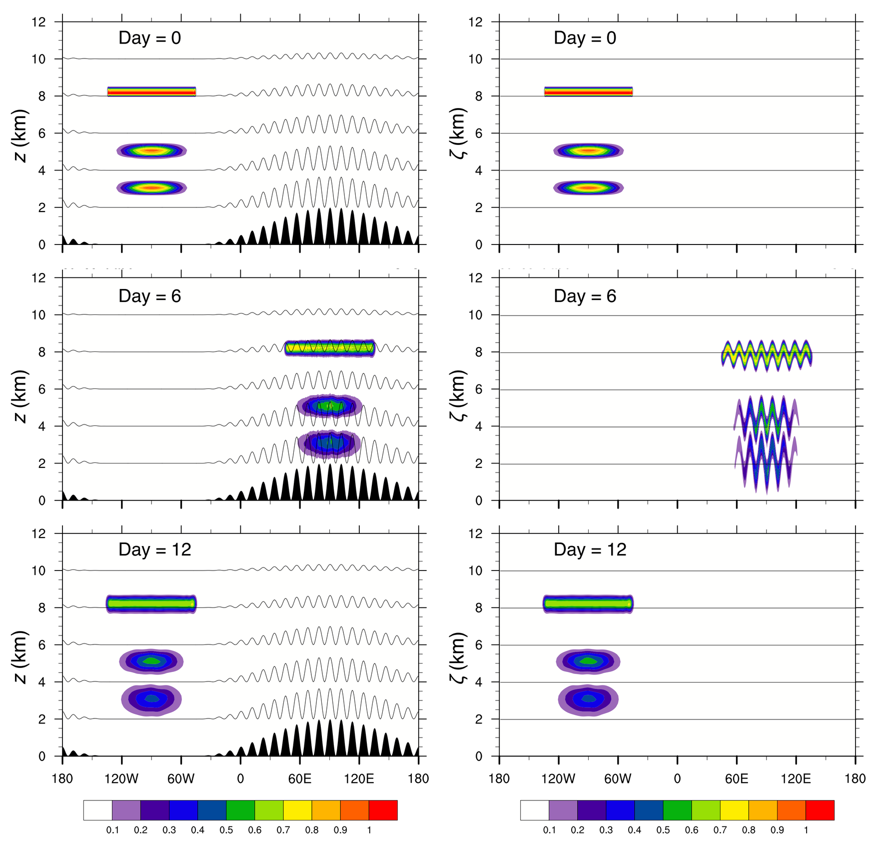
- **DCMIP-12:** A 3D deformational advection test, which mimics "Hadley-like" meridional circulation (*Kent et al.* [3]), for 1 day simulation.
- HOMAM setup for 1°, L60: $N_e = 30, N_p = 4$ (GLL); $V_{nel} = 15, N_g = 4$ (GL), $\Delta t = 60$s. HEVI, HEVE, un-split results are almost identical.





Errors/Models:	Mcore	CAM-FV	ENDGame	CAM-SE	HOMAM
ℓ_1	2.22	1.93	2.18	2.27	2.62
ℓ_2	1.94	1.84	1.83	2.12	2.43
ℓ_∞	1.64	1.66	1.14	1.68	2.16

- **DCMIP-13: Flow Over a Rough Orography** [3].
  A series of steep concentric ring-shaped mountain ranges forms the terrain.



Error Norm	MCore (ref) $1°L120$	HOMAM $1°L120$
ℓ_1	0.83	0.78
ℓ_2	0.55	0.50
ℓ_∞	0.73	0.76

- HOMAM $1°L120$ setup: $N_e = 30, N_p = 4$ (GLL); $V_{nel} = 30, N_g = 4$ (GL).
- Vertical cross-sections along the equator for the tracer fields. The results are simulated with HOMAM using the HEVE/HEVI scheme at a resolution 1° horizontal with 60 vertical levels ($\Delta t = 12$s).

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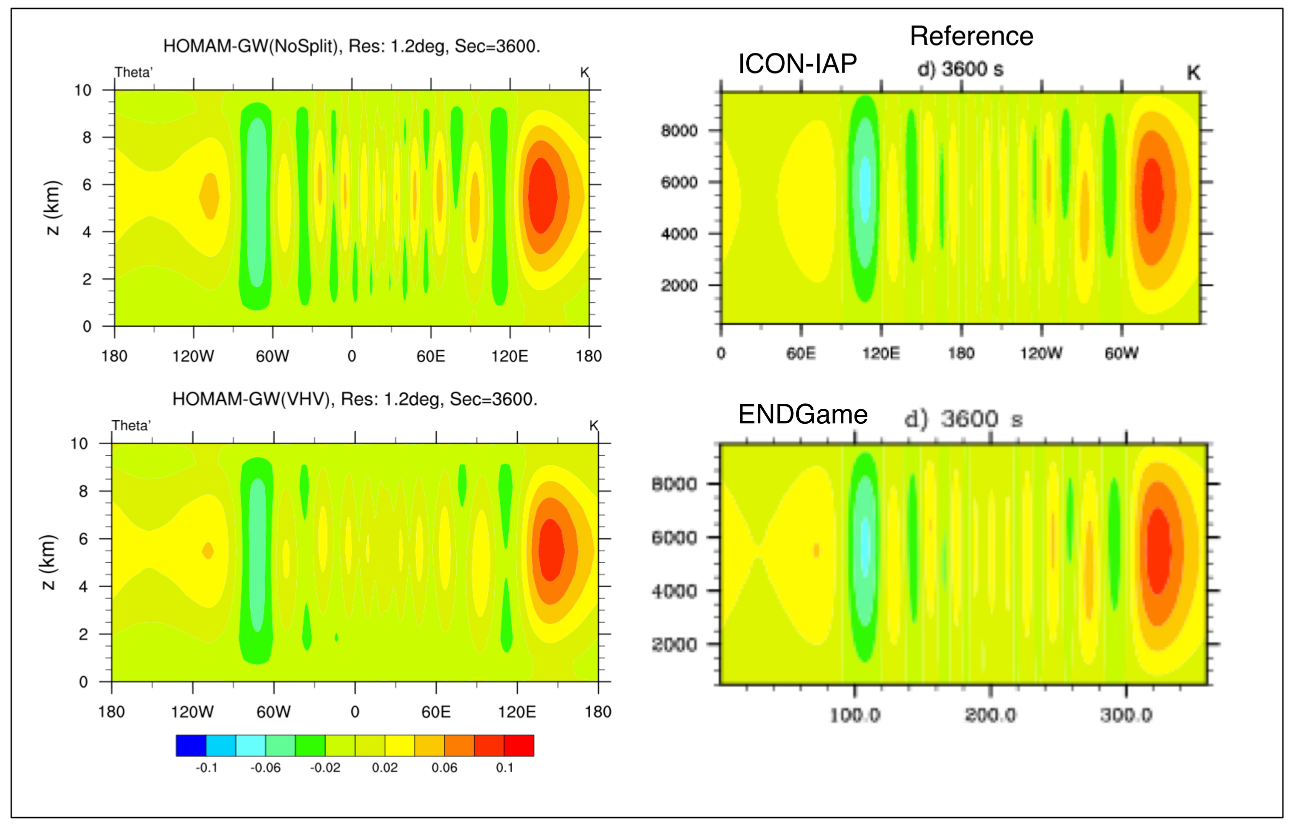
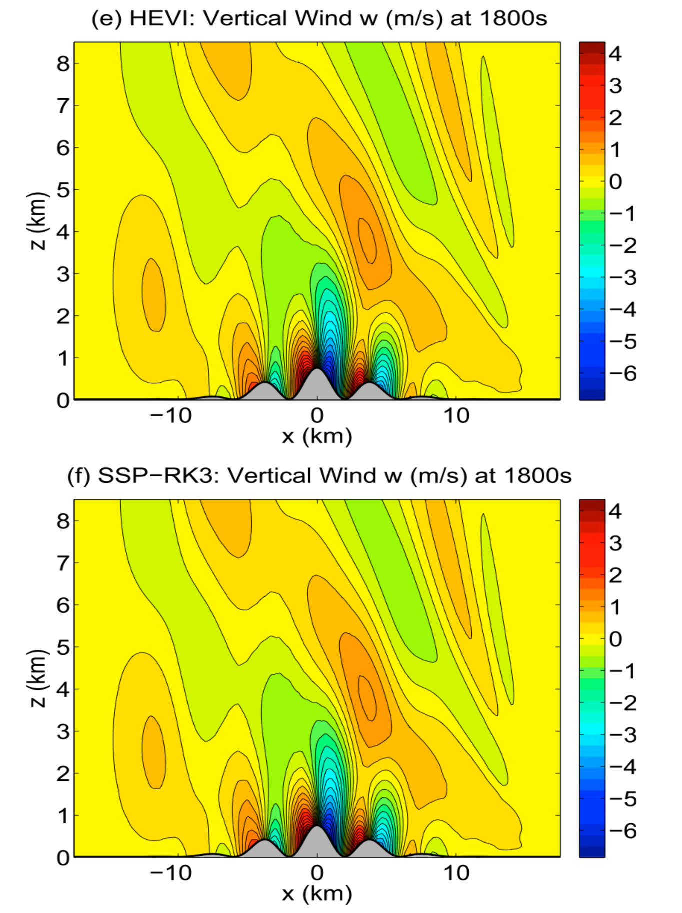
Non-Hydrostatic 3D Gravity Wave Test

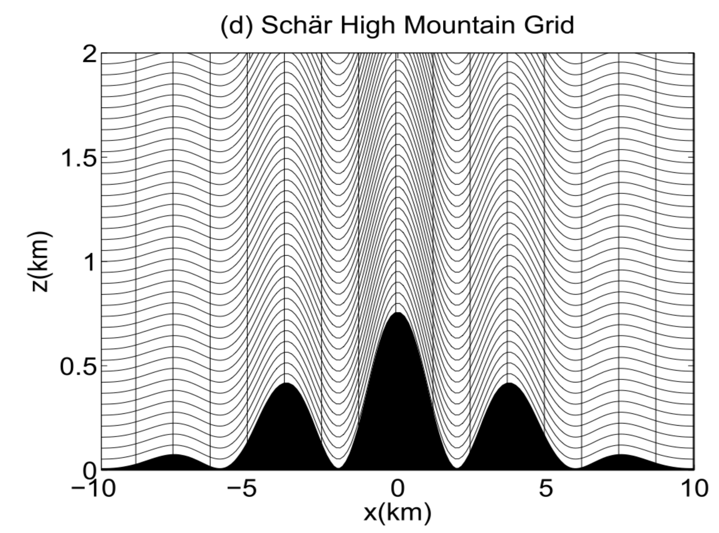
- **DCMIP-31:** NH gravity wave test on a reduced planet ($X = 125$). θ' after 3600s. HOMAM setup: $N_e = 25, N_p = 4, N_g = 4$ ($\Delta x \approx \Delta z \approx 1$ km), $\Delta t = 0.20$s
- The initial state is hydrostatically balanced and in gradient-wind balance. An overlaid potential temperature perturbation triggers the evolution of gravity waves.



Schär Mountain Test (2D)



- Schär mountain test with Higher elevation (slope ≈ 55%)
- w after 1800 s, with $dt = 0.175s$, with p^3-DG (GL) [2]
- Virtually identical results with explicit RK3 and HEVI



- **Work In Progress:** More benchmark tests including DCMIP
- IMEX time integration method. Design of a proper preconditioner for accelerating implicit time solver.
- Efficient explicit time solver for horizontal dynamics (sub-cycling).

References

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[3] J. Kent, P. A. Ullrich, and C. Jablonowski. Dynamical core model intercomparison project: Tracer transport test cases. *Q. J. Roy. Meteol. Soc.*, 140:1279–1293, 2014.

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