Tracking of Merging and Splitting Objects with Application to Storm Data

Curtis Storlie

Department of Statistics, Colorado State University, and
Geophysical Statistics Project, NCAR

About Me

- PhD Student at Colorado State University
  - Plan to graduate spring of 2005
- Supported by NCAR since summer of 2003
  - Work with atmospheric scientists on tracking storms
  - Work with physicists on tracking turbulence structures
- Doug Nychka is a co-advisor on my committee

The Problem

- What is the Underlying Problem?
  - to better understand the dynamics of turbulance
  - to better understand how storms form and evolve over time
  - to validate and improve the storm activity in GCM's (General Circulation Models)
- What is My Part?
  - detect and track vortices in turbulance simulations
  - detect and track storm activity in a doppler radar images
  - detect and track storm activity in GCM output

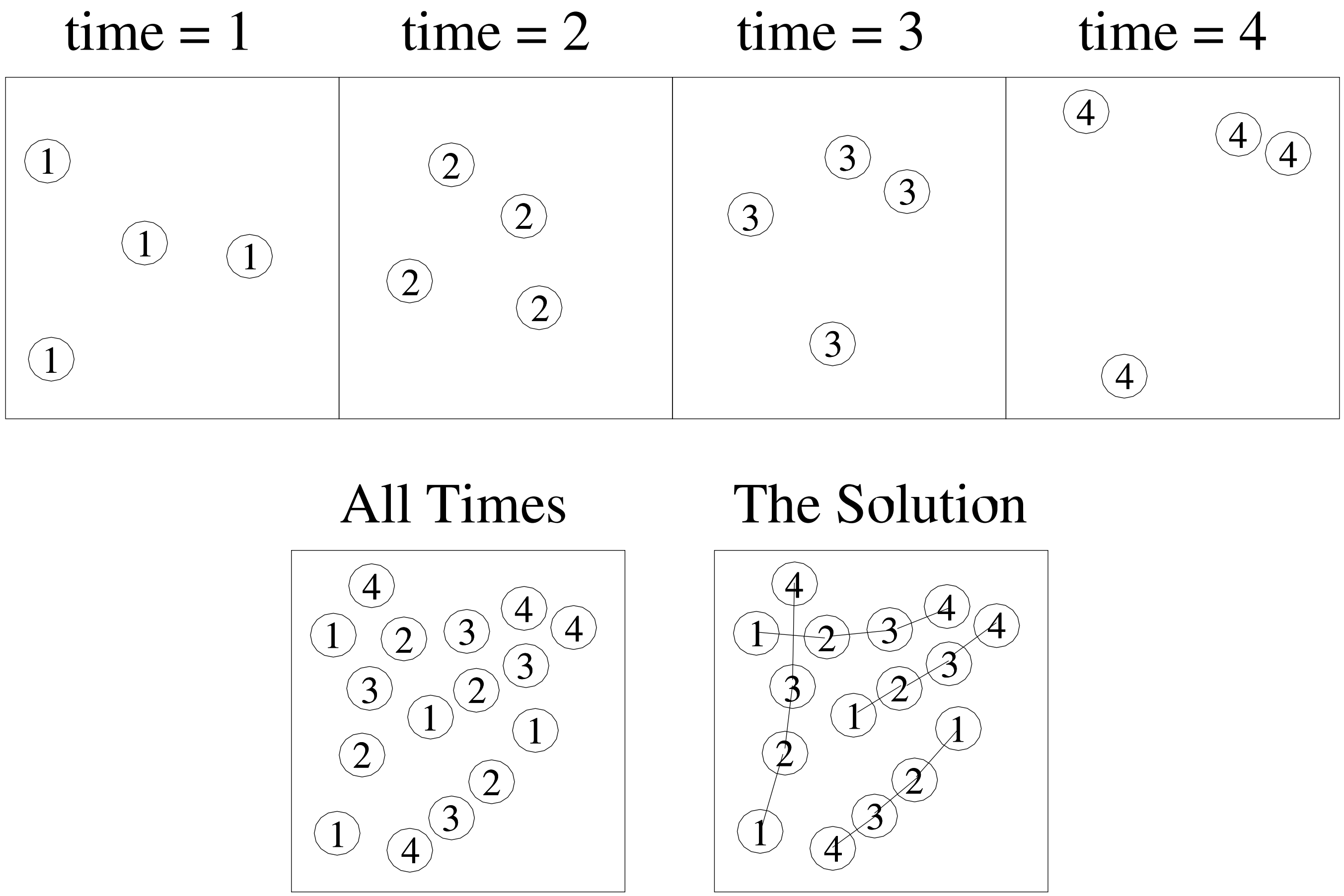
Motivating Videos

Outline

- Description of the tracking problem
- The approach to solve the tracking problem
  1. description of the model
  2. likelihood examples
- Further Work

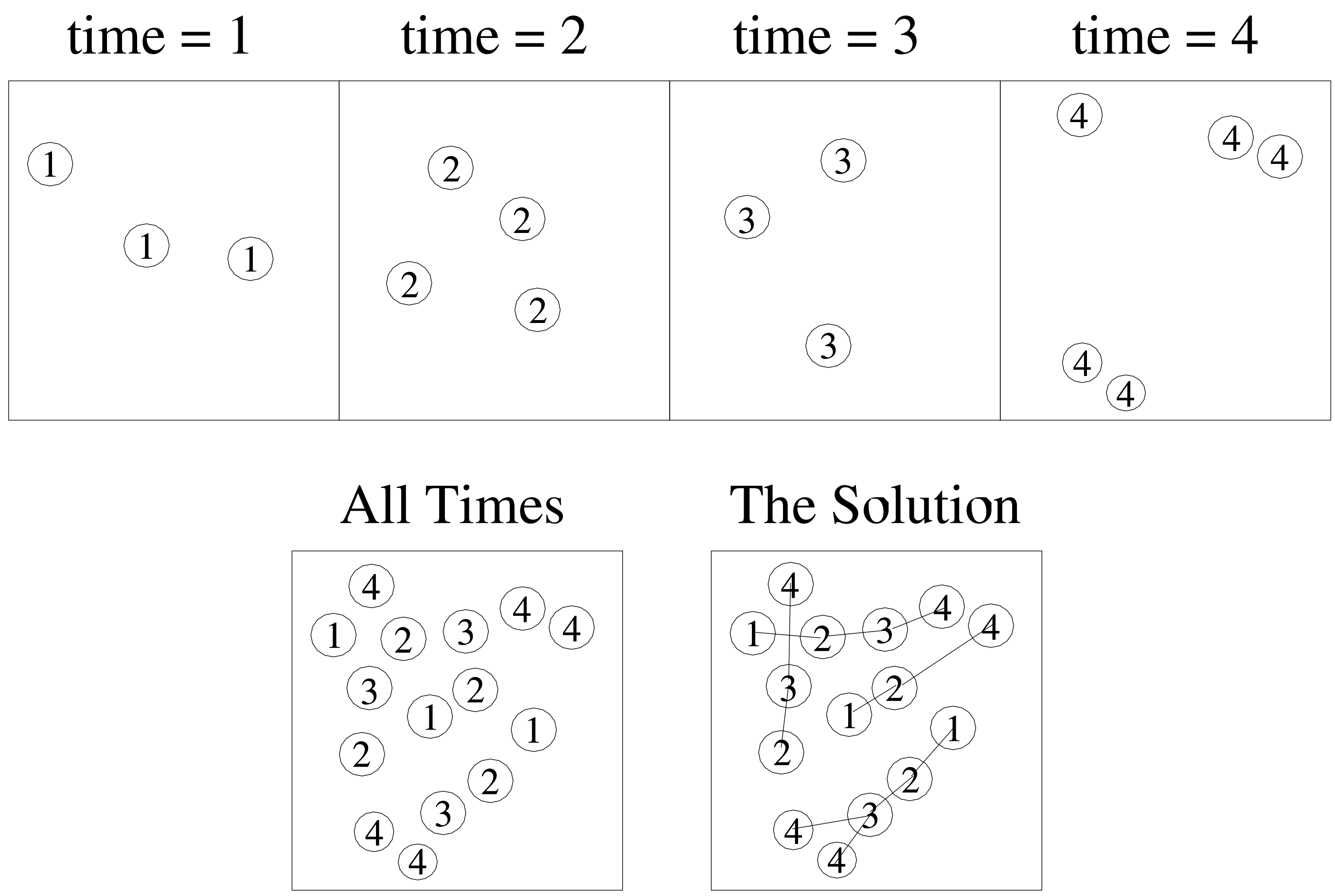
The Tracking Problem

- Given n frames from a sequence of images, find a correspondence between objects from different frames



The Tracking Problem Reloaded

- Now there is birth, missing values, and splitting



Solving the Tracking Problem

- We will assume a statistical model for the objects to be tracked.
- The solution to the tracking problem is the set of paths that maximize the likelihood of this model.
- Our model must account for the possibility of not observing an object at a given time for any of the following reasons
  1. It doesn't exist yet (Birth)
  2. It no longer exists (Death)
  3. It became 2 new paths (Splitting)
  4. It coalesced with another path (Merging)
  5. It is not found by the detection procedure (Missing)

A Statistical Model

The model is broken up into the following parts

- State Model
  - describes when birth, death, splitting and merging occur
- Missing State Model
  - describes when an existing path is missing ($W = 0$) or observable ($W = 1$). It is a continuous time Markov Chain $0 \leftrightarrow 1$.
- Object Size and Orientation
  - treat each object as an ellipse and model the radii (R_1, R_2) as lognormal and the angle of orientation (θ) as VonMises
- Object Location
  - When objects exist, the (X, Y) coordinates are assumed to behave like integrated brownian motion

State Model

- This is a Hidden Model in the sense that the states are not directly observed from the data
- Continuous Time Markov Chain
  1. Births occur with rate λ_1
  2. Deaths occur with rate $N(t)\lambda_2$
     - $N(t)$ is the number of paths in existence at time t
  3. Splits occur with rate $N(t)\lambda_3$
  4. Mergers occur with rate $N(t)\lambda_4$
  5. Births of a false alarm paths occur with rate ρ_1
  6. Deaths of a false alarm paths occur with rate $N_f(t)\rho_2$
     - $N_f(t)$ is the number of false alarms in existence at time t

State Model

- The variables $\boldsymbol{p}$, $\boldsymbol{\xi}$, and $\boldsymbol{\zeta}$ describe the state model
  - p_i is a vector of the parents of the i^{th} path
  - ξ_i is the time of initiation of the i^{th} path
  - ζ_i is the time of termination of the i^{th} path
- $\boldsymbol{p} = (p_1, \ldots, p_M)$, $\boldsymbol{\xi} = (\xi_1, \ldots, \xi_M)$ and $\boldsymbol{\zeta} = (\zeta_1, \ldots, \zeta_M)$
- We observe the process at the times $\underline{T} = (T_1, \ldots, T_n)$ and M is the number of paths and false alarms that exist before time T_n

Model Likelihood

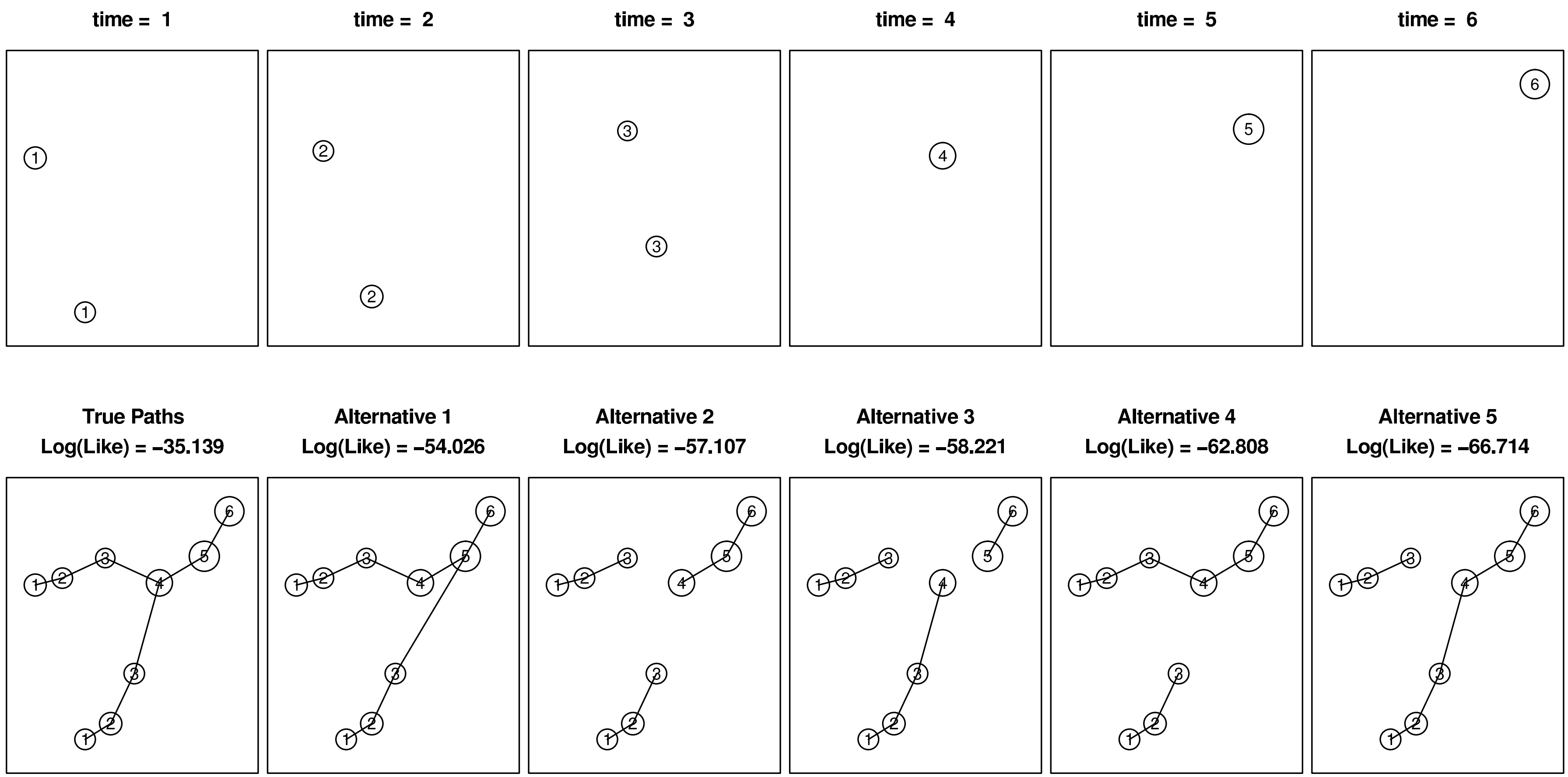
- We can write out a likelihood for $\Phi = (\boldsymbol{p}, \boldsymbol{\xi}, \boldsymbol{\zeta}, \boldsymbol{W}, \boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{R}_{(1)}, \boldsymbol{R}_{(2)}, \boldsymbol{\theta})$
  - The bold variables denote the collection of those variables for all paths at all times
- It factors into several conditional densities

$$
\begin{aligned}
[\Phi] \; = \; & [\boldsymbol{p}, \boldsymbol{\xi}, \boldsymbol{\zeta}] \cdot [\boldsymbol{W} \mid \boldsymbol{p}, \boldsymbol{\xi}, \boldsymbol{\zeta}] \cdot [\boldsymbol{X} \mid \boldsymbol{p}, \boldsymbol{\xi}, \boldsymbol{\zeta}, \boldsymbol{W}] \cdot [\boldsymbol{Y} \mid \boldsymbol{p}, \boldsymbol{\xi}, \boldsymbol{\zeta}, \boldsymbol{W}] \cdot \\
& [\boldsymbol{R}_1, \boldsymbol{R}_2 \mid \boldsymbol{W}] \cdot [\boldsymbol{\theta} \mid \boldsymbol{W}, \boldsymbol{R}_1, \boldsymbol{R}_2]
\end{aligned}
$$

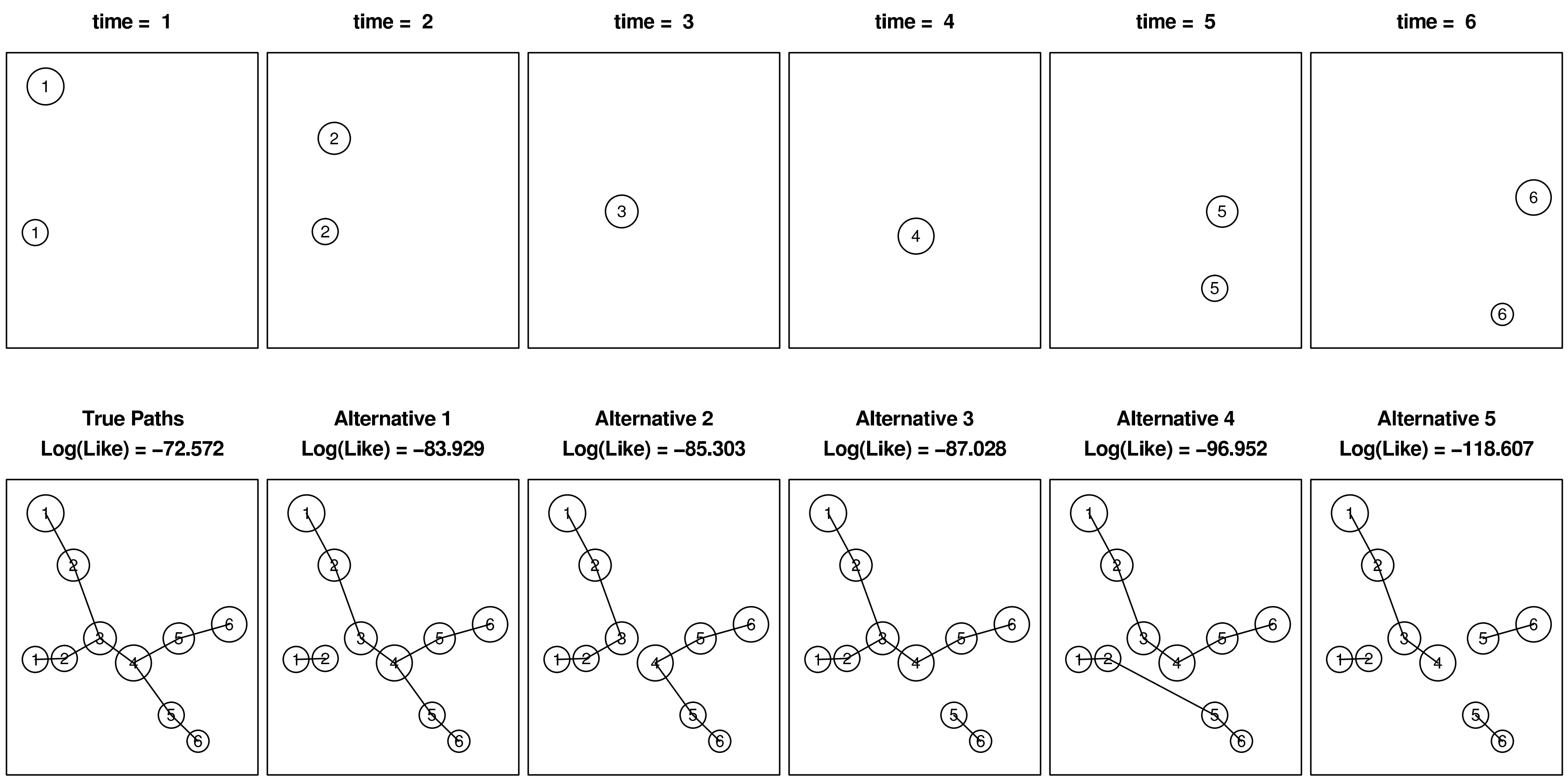
- Each of the densities can be written out seperately

Example 1: A Merging Event

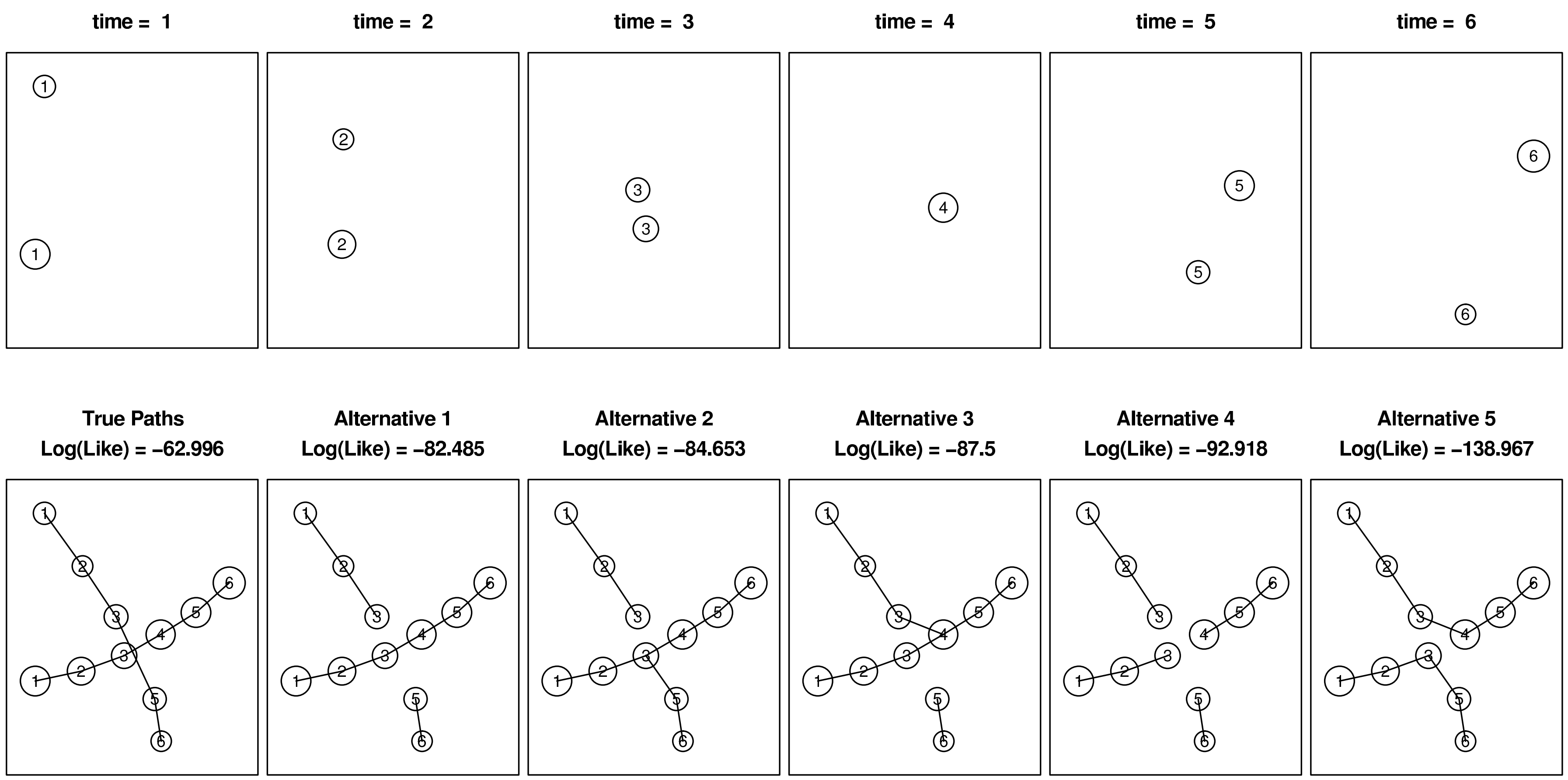
Maximizing the likelihood is a reasonable way to find the solution.



Example 2: A Merging and Splitting Event



Example 3: A Crossing Event

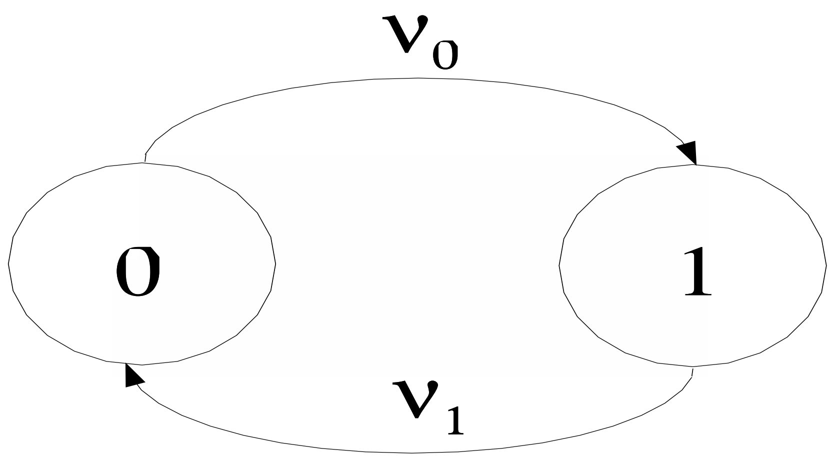


Further Work

- Parameter Estimation
- Apply the tracking algorithm to simulated data
- Apply the tracking algorithm to Doppler Radar Rainfall Data
- Theory
  - convergence to the correct path correspondence
  - Error rate of path classification
- Apply algorithm to 2-D turbulence problem

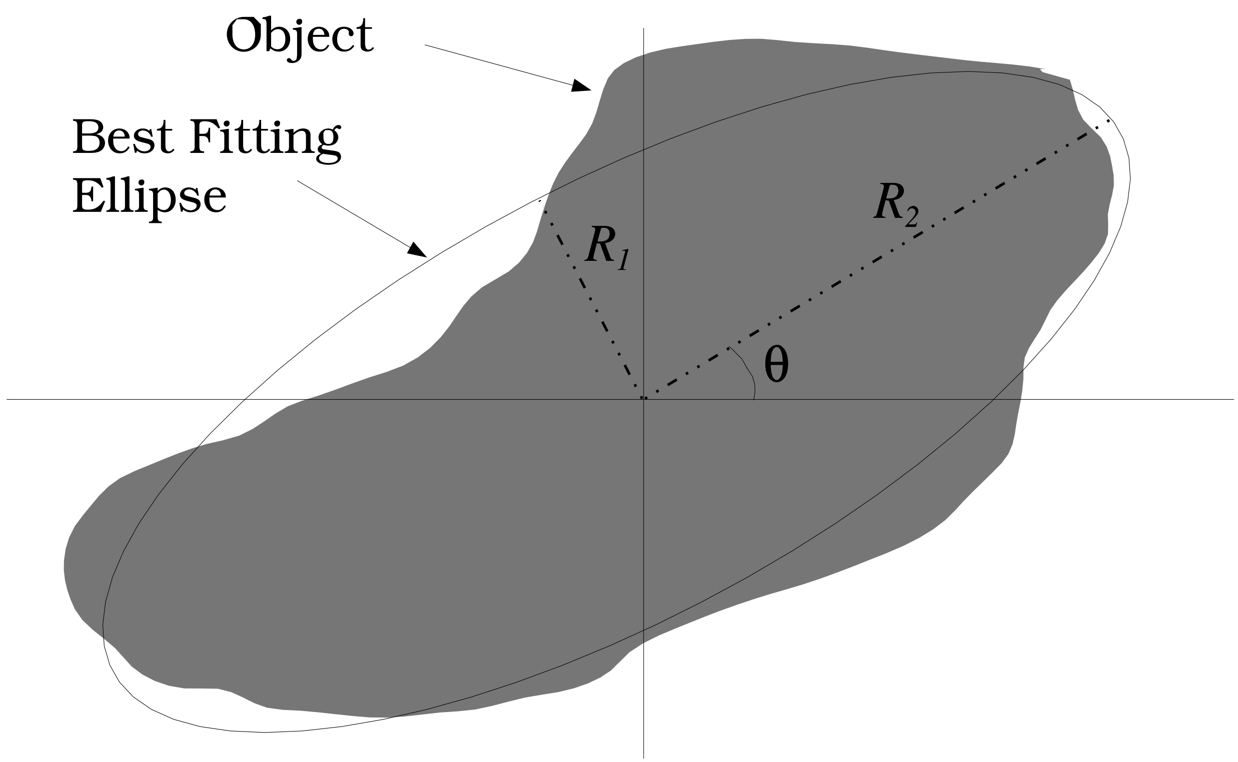
Missing State Model

- The State Model accounts for everything except missing observations
- Use another Markov Chain with state variable, W, that has only 2 states, missing ($W(t) = 0$) and observable ($W(t) = 1$)



Object Size and Orientation

- Model the radii, $R_{1,i}$ and $R_{2,i}$, of the best fitting ellipse to the i^{th} object with lognormal distributions
- Also model the orientation, θ_i with a VonMises distribution.



Object Location

- We will present the model for each of the 4 cases
  1. Path resulting form a birth
  2. Path resulting from a split
  3. Path resulting from a merger
  4. False alarm
- Let $X_i(t)$ be the x-coordinate of the i^{th} path at time t
- The model for $Y_i(t)$ will be identical and independent of $X_i(t)$
- Recall p_i contains the indices of the parents of the i^{th} path

Object Location (Birth)

- The location of a path resulting from birth is described by

$$X_i(t) = X_i(\xi_i) + X_i'(\xi_i)(t - \xi_i) + \sigma_i Z_i(t - \xi_i)$$

- $X_i'(t)$ is the velocity of the path at time t
- $Z_i(t)$ is an IBM, $Z_i(t) = \int_0^t B_i(s)ds$, where $B_i(t)$ is the Brownian Motion driving the i^{th} path
- For the initial position and velocity

$$X_i(\xi_i) \sim N\left(\mu_{X_0}, \sigma^2_{X_0}\right) \text{ and } X_i'(\xi_i) \sim N\left(\mu_{X_0'}, \sigma^2_{X_0'}\right)$$

Object Location (Split)

- The location of a path resulting from a split is described by

$$X_i(t) = X_{p_{i,1}}(\xi_i) + \phi_i + \left[X'_{p_{i,1}}(\xi_i) + \phi'_i\right](t - \xi_i) + \sigma_i Z_i(t - \xi_i)$$

- $\phi_i \sim N(0, \sigma^2_{X_s})$ and $\phi'_i \sim N(0, \sigma^2_{X'_s})$
- Conservation of momentum condition during a split
  - Let c_i contain the indices of the paths involved in the i^{th} splitting event, $i = 1, \ldots, N_s$. $c_{i,1}$ is the parent.
  - Change in momentum after the i^{th} split is

$$C_i = ES_{c_{i,2}} X'_{c_{i,2}}(\xi_{c_{i,2}}) + ES_{c_{i,3}} X'_{c_{i,3}}(\xi_{c_{i,2}}) - ES_{c_{i,1}} X'_{c_{i,1}}(\xi_{c_{i,2}})$$

  - Condition the model on $C_i = 0$ for $i = 1, \ldots, N_s$

Object Location (Merger)

- The location of a path resulting from merger is described by

$$X_i(t) = \frac{ES_{p_{i,1}}}{ES_i} X_{p_{i,1}}(\xi_i) + \frac{ES_{p_{i,2}}}{ES_i} X_{p_{i,2}}(\xi_i) + \left[\frac{ES_{p_{i,1}}}{ES_i} X'_{p_{i,1}}(\xi_i) + \frac{ES_{p_{i,2}}}{ES_i} X'_{p_{i,2}}(\xi_i)\right](t - \xi_i) + \sigma_i Z_i(t - \xi_i)$$

- Conservation of momentum is built into the term in brackets
- Need to force the paths close together before merger
  - Let d_i contain the indices of the paths involved in the i^{th} merging event, $i = 1, \ldots, N_m$. $d_{i,1}$, $d_{i,2}$ are the parents.
  - Difference in location before the i^{th} merger plus an error is

    $$D_i = X_{d_{i,1}}(\xi_{d_{i,3}}) - X_{d_{i,2}}(\xi_{d_{i,3}}) + \psi_i$$

    where $\psi_i \sim N(0, \sigma^2_{X_m})$
  - Condition the model on $D_i = 0$ for $i = 1, \ldots, N_m$

Object Location (False Alarm)

- The location of a false alarm path is described by

$$X_i(t) = X_i(\xi_i) + \sigma_i B_i(t - \xi_i)$$

- The initial position follows the same distribution as that for a true path

$$X_i(\xi_i) \sim N\left(\mu_{X_0}, \sigma_{X_0}^2\right)$$